WEB APPENDIX A: VARIANCES AND COVARIANCES IN EQUATION 4 FOR A BALANCED DESIGN

$$var(A) = \frac{1}{Nm_z m_w \hat{\mu}_{101}^2}\begin{bmatrix}\hat{\mu}_{202} + (m_w - 1)\hat{\mu}_{2,0,11} + (m_z - 1)\hat{\mu}_{11,0,2} + (m_w - 1)(m_z - 1)\hat{\mu}_{11,0,11} \\ -m_w m_z \hat{\mu}_{101}^2\end{bmatrix} \quad \text{(A.1)}$$

$$var(B) = \frac{1}{Nm_x m_w \hat{\mu}_{011}^2}\begin{bmatrix}\hat{\mu}_{022} + (m_w - 1)\hat{\mu}_{0,2,11} + (m_x - 1)\hat{\mu}_{0,11,2} + (m_w - 1)(m_x - 1)\hat{\mu}_{0,11,11} \\ -m_x m_w \mu_{011}^2\end{bmatrix} \quad \text{(A.2)}$$

$$var(C) = \frac{4}{N_2 m_w^2 (m_w - 1)^2 \mu_{001,11}^2}\begin{Bmatrix}\frac{m_w(m_w-1)}{2}\left[\hat{\mu}_{0,0,22} - \hat{\mu}_{0,0,11}^2\right] + m_w(m_w - 1)(m_w - 2) \\ \left[\hat{\mu}_{0,0,211} - \hat{\mu}_{0,0,11}^2\right] + \frac{m_w(m_w-1)(m_w-2)(m_w-3)}{4}\left(\hat{\mu}_{0,0,1111} - \hat{\mu}_{0,0,11}^2\right)\end{Bmatrix} \quad \text{(A.3)}$$

$$var(D) = \frac{1}{Nm_z \hat{\mu}_{200}^2}\left[\hat{\mu}_{400} + (m_z - 1)\hat{\mu}_{22,0,0} - m_z \hat{\mu}_{200}^2\right] \quad \text{(A.4)}$$

$$cov(A, B) = \frac{1}{Nm_w \hat{\mu}_{101}\hat{\mu}_{011}}\left[\hat{\mu}_{112} + \left(m_w - 1\right)\hat{\mu}_{11(1,1)} - m_w \hat{\mu}_{101}\hat{\mu}_{011}\right] \quad \text{(A.5)}$$

$$cov(A, C) = \frac{1}{Nm_w \hat{\mu}_{101}\hat{\mu}_{0,0,11}}\left\{2[\hat{\mu}_{1,0,21} - \hat{\mu}_{101}\hat{\mu}_{0,0,11}] + \left(m_w - 2\right)[\hat{\mu}_{1,0,111} - \hat{\mu}_{101}\hat{\mu}_{0,0,11}]\right\} \quad \text{(A.6)}$$

$$cov(A, D) = \frac{1}{Nm_z \hat{\mu}_{101}\hat{\mu}_{200}}\left[\hat{\mu}_{301} - \hat{\mu}_{101}\hat{\mu}_{200} + (m_z - 1)\left(\hat{\mu}_{21,0,1} - \hat{\mu}_{101}\hat{\mu}_{200}\right)\right] \quad \text{(A.7)}$$

$$cov(B, C) = \frac{1}{Nm_w \hat{\mu}_{011}\hat{\mu}_{0,0,11}}\left\{2[\hat{\mu}_{1,0,21} - \hat{\mu}_{011}\hat{\mu}_{0,0,11}] + \left(m_w - 2\right)[\hat{\mu}_{0,1,111} - \hat{\mu}_{011}\hat{\mu}_{0,0,11}]\right\} \quad \text{(A.8)}$$

$$cov(B, D) = \frac{1}{N\hat{\mu}_{011}\hat{\mu}_{200}}\left(\hat{\mu}_{211} - \hat{\mu}_{011}\hat{\mu}_{200}\right) \quad \text{(A.9)}$$

$$cov(C, D) = \frac{1}{N\hat{\mu}_{0,0,11}\hat{\mu}_{200}}\left(\hat{\mu}_{2,0,11} - \hat{\mu}_{0,0,11}\hat{\mu}_{200}\right) \quad \text{(A.10)}$$

WEB APPENDIX B: VARIANCES AND COVARIANCES IN EQUATION 4 FOR AN UNBALANCED DESIGN

$$var(A) = \frac{1}{m_z(2n_2+n_1)^2\hat{\mu}_{101}^2}\{(2n_2+n_1)(\hat{\mu}_{202}-\hat{\mu}_{101}^2)+2n_2(\hat{\mu}_{2,0,11}-\hat{\mu}_{101}^2)+(m_z-1)(2n_2+n_1)(\hat{\mu}_{11,0,2}-\hat{\mu}_{101}^2)+(m_z-1)(2n_2)(\hat{\mu}_{11,0,11}-\hat{\mu}_{101}^2)\} \quad (B.1)$$

$$var(B) = \frac{1}{m_x(2n_2+n_1)^2\hat{\mu}_{011}^2}\{(2n_2+n_1)(\hat{\mu}_{022}-\hat{\mu}_{011}^2)+2n_2(\hat{\mu}_{0,2,11}-\hat{\mu}_{011}^2)+(m_x-1)(2n_2+n_1)(\hat{\mu}_{0,11,2}-\hat{\mu}_{011}^2)+(m_x-1)(2n_2)(\hat{\mu}_{0,11,11}-\hat{\mu}_{011}^2)\} \quad (B.2)$$

$$var(C) = \frac{1}{\hat{\mu}_{0,0,11}^2 n_2}(\hat{\mu}_{0,0,22}-\hat{\mu}_{0,0,11}^2) \quad (B.3)$$

Note that in the special case where $b_i = 2$, we have

$$\hat{\mu}_{0,0,11} = \sum_{i=1}^{n_2}\frac{(w_{i1}-\overline{w})(w_{i2}-\overline{w})}{n_2}, \hat{\mu}_{0,0,22} = \sum_{i=1}^{n_2}\frac{(w_{i1}-\overline{w})^2(w_{i2}-\overline{w})^2}{n_2}$$

$$var(D) = \frac{1}{Nm_z\hat{\mu}_{200}^2}\left[(\hat{\mu}_{400}-\hat{\mu}_{200}^2)+(m_z-1)(\hat{\mu}_{22,0,0}-\hat{\mu}_{200}^2)\right] \quad (B.4)$$

$$cov(A,B) = \frac{1}{(2n_2+n_1)\hat{\mu}_{101}\hat{\mu}_{011}}\left[(\hat{\mu}_{112}-\hat{\mu}_{101}\hat{\mu}_{011})+\frac{2n_2}{(2n_2+n_1)}(\hat{\mu}_{1,1,11}-\hat{\mu}_{101}\hat{\mu}_{011})\right] \quad (B.5)$$

$$cov(A,C) = \frac{2}{(2n_2+n_1)\hat{\mu}_{101}\hat{\mu}_{0,0,11}}(\hat{\mu}_{1,0,21}-\hat{\mu}_{101}\hat{\mu}_{0,0,11}) \quad (B.6)$$

$$cov(A,D) = \frac{1}{Nm_z\hat{\mu}_{101}\hat{\mu}_{200}}\left[(\hat{\mu}_{301}-\hat{\mu}_{101}\hat{\mu}_{200})+(m_z-1)(\hat{\mu}_{21,0,1}-\hat{\mu}_{101}\hat{\mu}_{200})\right] \quad (B.7)$$

$$cov(B,C) = \frac{2}{(2n_2+n_1)\hat{\mu}_{011}\hat{\mu}_{0,0,11}}(\hat{\mu}_{0,1,21}-\hat{\mu}_{011}\hat{\mu}_{0,0,11}) \quad (B.8)$$

$$cov(B,D) = \frac{1}{N\hat{\mu}_{011}\hat{\mu}_{200}}(\hat{\mu}_{211}-\hat{\mu}_{011}\hat{\mu}_{200}) \quad (B.9)$$

$$cov(C,D) = \frac{1}{N\hat{\mu}_{0,0,11}\hat{\mu}_{200}}(\hat{\mu}_{2,0,11}-\hat{\mu}_{0,0,11}\hat{\mu}_{200}) \quad (B.10)$$

Note that in the special case where $b_i = 1$ or 2, the

$$\hat{\mu}_{2,0,11} = \sum_{i=1}^{n_2}\sum_{j=1}^{m_z}(z_{ij}-\bar{z})^2(w_{i1}-\overline{w})(w_{i2}-\overline{w})/(n_2\, m_z)$$

WEB APPENDIX C: COMPONENTS OF VAR$\left[ln(\hat{\lambda}_{x|Z})\right]$ AS A FUNCTION OF θ

After some algebra, the variance and covariances provided in Appendix B can be written as a function of θ as follows:

$$var(A) = f_{1A} + f_{2A}\theta \tag{C.1}$$

where

$$\theta = 2n_2/(n_1 + 2n_2)$$

$$f_{1A} = \frac{\hat{\mu}_{202} - \hat{\mu}_{101}^2 + (m_z - 1)\left(\hat{\mu}_{11,0,2} - \hat{\mu}_{101}^2\right)}{m_z\hat{\mu}_{101}^2 M}$$

$$f_{2A} = \frac{\hat{\mu}_{2,0,11} - \hat{\mu}_{101}^2 + (m_z - 1)\left(\hat{\mu}_{11,0,11} - \hat{\mu}_{101}^2\right)}{m_z\hat{\mu}_{101}^2 M}$$

$$var(B) = f_{1B} + f_{2B}\theta \tag{C.2}$$

where

$$f_{1B} = \frac{\hat{\mu}_{022} - \hat{\mu}_{011}^2 + (m_x - 1)\left(\hat{\mu}_{0,11,2} - \hat{\mu}_{011}^2\right)}{m_x\hat{\mu}_{011}^2 M}$$

$$f_{2B} = \frac{\hat{\mu}_{0,2,11} - \hat{\mu}_{011}^2 + (m_x - 1)\left(\hat{\mu}_{0,11,11} - \hat{\mu}_{011}^2\right)}{m_x\hat{\mu}_{011}^2 M}$$

$$var(C) = f_C/\theta \tag{C.3}$$

where

$$f_C = \frac{2\left(\hat{\mu}_{0,0,22} - \hat{\mu}_{0,0,11}^2\right)}{\hat{\mu}_{0,0,11}^2 M}$$

$$var(D) = f_D/(2 - \theta) \tag{C.4}$$

where

$$f_D = \frac{2}{Mm_z\hat{\mu}_{200}^2}\left[(\hat{\mu}_{400} - \hat{\mu}_{200}^2) + (m_z - 1)\left(\hat{\mu}_{22,0,0} - \hat{\mu}_{200}^2\right)\right]$$

$$cov(A,B) = f_{1,AB} + \theta f_{2,AB} \tag{C.5}$$

where

$$f_{1,AB} = \frac{\hat{\mu}_{112} - \hat{\mu}_{101}\hat{\mu}_{011}}{M\hat{\mu}_{101}\hat{\mu}_{011}}$$

$$f_{2,AB} = \frac{\hat{\mu}_{1,1,11} - \hat{\mu}_{101}\hat{\mu}_{011}}{M\hat{\mu}_{101}\hat{\mu}_{011}}$$

$$cov(A,C) = f_{AC} \tag{C.6}$$

where

$$f_{AC} = \frac{2\left(\hat{\mu}_{1,0,21} - \hat{\mu}_{101}\hat{\mu}_{0,0,11}\right)}{M\hat{\mu}_{101}\hat{\mu}_{0,0,11}}$$

$$cov(A,D) = f_{AD}/(2-\theta) \tag{C.7}$$

where

$$f_{AD} = \frac{2}{Mm_z\hat{\mu}_{101}\hat{\mu}_{200}}\left[(\hat{\mu}_{301} - \hat{\mu}_{101}\hat{\mu}_{200}) + (m_z - 1)\left(\hat{\mu}_{21,0,1} - \hat{\mu}_{101}\hat{\mu}_{200}\right)\right]$$

$$cov(B,C) = f_{BC} \tag{C.8}$$

where

$$f_{BC} = \frac{2\left(\hat{\mu}_{0,1,21} - \hat{\mu}_{011}\hat{\mu}_{0,0,11}\right)}{M\hat{\mu}_{011}\hat{\mu}_{0,0,11}}$$

$$cov(B,D) = f_{BD}/(2-\theta) \tag{C.9}$$

where

$$f_{BD} = \frac{2}{M\hat{\mu}_{011}\hat{\mu}_{200}}(\hat{\mu}_{211} - \hat{\mu}_{011}\hat{\mu}_{200})$$

$$cov(C, D) = f_{CD}/(2 - \theta) \tag{C.10}$$

where

$$f_{CD} = \frac{2}{M\hat{\mu}_{0,0,11}\hat{\mu}_{200}}\left(\hat{\mu}_{2,0,11} - \hat{\mu}_{0,0,11}\hat{\mu}_{200}\right)$$