Hypothesis: Bones Toughness Arises from the Suppression of Elastic Waves

November 18, 2014

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1 Hierarchal Topology in Random Fuse Networks

The analysis of Sethna et al assumes infinite variation in the strength of the fuses within a topologically homogenous diamond lattice for percolation like damage; which is clearly not physically possible because it requires an infinite variety of individual fuses [1]. However a pair of heterogeneous units (Collagen and Hydroxyapatite) arranged with a hierarchal topology can be equivalent to a regular diamond array of fuses with a power law distribution of strength as used by Sethna. Consider Fuse Arranged in Parallel and Series like in a Diamond Lattice:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{parallel}}$$

$$R_1 + R_2 = R_{Series}$$

Therefore the resistance of a infinite number of resistors in Series can be Infinitely Large and the resistance of a large Number of resistors in parallel is always less than the resistance of resistor with lowest resistance. Thus a combinations of resistors in series can be expressed as single resistor with an arbitrarily large resistance/strength that can in turn form the basis of a resistor network with even greater resistance/strength. Conversely resistors in parallel can be designed to have very very small strengths which in turn can form subsequent networks with small strengths. It is thus possible for a resistor network with two heterogeneous resistor types at the base level and a hierarchal structure to exhibit behaviour akin to infinite disorder percolation limit as outlined by Sethna et al. This structure that heterogeneous at every length scale is exactly what is seen in bone with soft and hard phases at each structural level.

2 Impedance of Bones Components

	Density (Kgm^{-3})	Wave Velocity (ms^{-1})	Impedance (Rayls)
HA	3190	5598	17860571
Collagen	1400	1133	1587450
Collagen Fibril	2474	3812	9433145
Collagen Fibre	2044	2741	5604323

3 Reflectance of Bones Components

	Reflectance	Lengthscale
HA-Collagen	.8367	10's nm
Fibril (60:40) and Non-Collagenous Protien	0.7119	100's nms
Collagen Fibre and Non-Collagenous Protein	0.5585371	micron's

Note that if a wave encounters 5 interfaces a reflectance of .8367 then only .16 of wave energy is transmitted. Thus 3 reflections lead to only .5 % of the initial energy propogating.

4 Evidence of Damage Localisation in Bone over Multiple Length-scales

Experimental investigations confirm that bone is heterogeneous at every length scale with regions of locally high strain (Figure 3). AFM mapping at the collagen fibril level showed variations in stiffness (that correlates with reflectance) of 10 GPa over 100nm periods have been observed using AFM and linked to bones improved mechanical properties [2]. Such a large reflectance and spatial variation in materials properties at fibrils suppress the propagation of phonons with a wavelength $l \sim 100$nm and explain how fibrillar sliding and dilatational band formation, major toughening mechanism, are so widespread but do not transition to catastrophic cracks [3, 4, 5]. At next hierarchal level, the lamellar unit, alternating "soft" non-collagneous protein rich inter-lamellar and "hard" intra-lamellar regions could also confine in theory scatter phonons[6]. Consequently strain localisation and extensive micro-cracking occur in inter-lamellar regions have been observed [6, 7]. At the osteonal level cement lines, which are hyper-mineralised, could in theory reflect large wavelength phonons. However because large wavelength phonons are lower in energy the importance of phonon suppression at larger scales are reduced and the effect of extrinsic toughening mechanisms is more important. Bone and other flaw tolerant natural materials toughness can be further increased because the extensive diffuse damage amplifies the effect extrinsic toughening mechanisms. Extrinsic toughening mechanisms require the presence of a crack consequently more cracks means more crack bridging, deflection, twisting, and ligament bridging and hence greater energy dissipation and improved toughness [4].

References

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