Emergence of disassortative mixing from pruning nodes in growing scale-free networks

Supplementary Information

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We here provide more numerical evidences in support of our main conclusion that pruning largest-degree nodes on a growing scale-free (SF) network leads to the emergence of a disassortative mixing pattern.

First, we consider SF networks with $N = 10^3$ and $\langle k \rangle = 4$. We divide all nodes into 50 classes. Precisely, each class is composed by 20 nodes. All the 20 smallest-degree nodes compose class 1, all the 20 largest-degree nodes compose class 50, while all other intermediate classes are composed by sets of 20 nodes according to increasing degree ranking. Our simulations refer to removing an entire class (or rank) of 20 nodes, and monitor the changes in the degree correlation coefficient r under this action. Figure S1 shows how the degree correlation coefficient r behaves as a function of the removed class. The figure shows that removal of any class or rank of nodes produces just very slight fluctuations of r, whereas when the nodes of largest rank (namely, the nodes of largest-degree nodes) are removed, the degree correlation coefficient r drops substantially, implying the setting of a disassortative degree mixing pattern.

Another significant question is whether the decline of degree correlation coefficient caused by the deletion of largest-degree nodes is robust against different network size and average degree. Figure S2 reveals the effect of removing largest-degree nodes on the degree correlation coefficient r for different network size and different average degree. For simplicity, we fix the fraction of removing nodes at f=0.02. One can see that the degree correlation coefficient r keeps nearly constant, irrespective of the size and average degree. Thus, these results suggest that pruning the largest-degree nodes is universally effective in yielding disassortative networks.

Another fundamental issue to be addressed is whether the drop in r due to removal of largest-degree nodes is intimately connected with the growing mechanism through which the network structure is constructed. While, indeed, the Barabási-Albert (BA) algorithm is very effective [1] in producing SF topologies, it generally implies that the age of the nodes (the instant at which they enter the network during the growth process) is directly proportional to their final degree, i.e. early existing nodes have larger degree. It is therefore natural to ask what would be the situation when considering other generating models for SF networks. To examine this issue, the *configuration model* becomes an ideal candidate [2,3]. We then construct SF networks by means of the configuration model, and monitor the effect of removing the largest-degree nodes on the degree correlation (Fig. S3). While, similar to the growing SF networks, the power-law degree distribution remains almost unchanged as the fraction of removed nodes increases (inset of Fig. S3), the impact of removing largest-degree nodes on the degree correlation coefficient is, instead, negligible. Actually, removing more and more of the largest-degree nodes in the configuration model, has even the result of making r closer to zero. For this point, it is similar to previous methods of constructing uncorrelated SF networks based on *configuration model*, where large-degree nodes above a certain threshold will be removed [4,5]. This is because the bias of *configuration model* is fully different from the one caused by preferential attachment in BA model [6].

References

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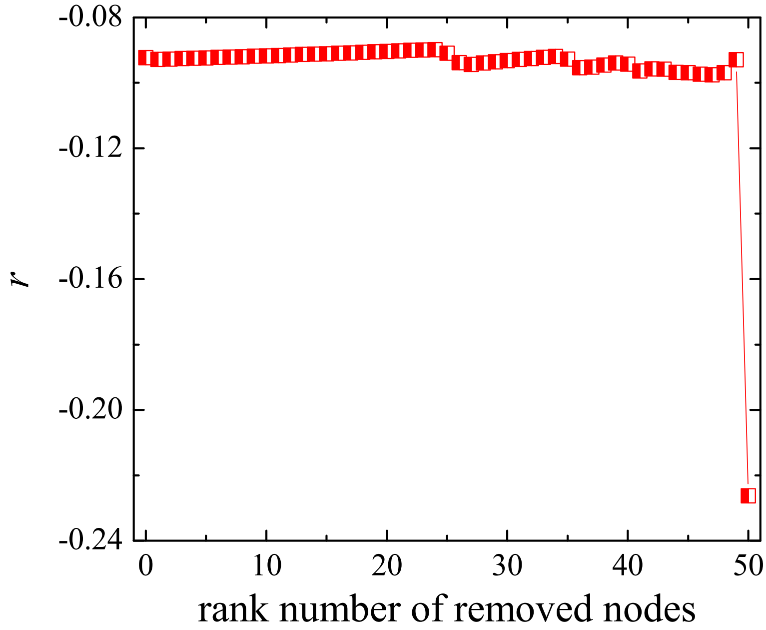

Figure S1. The degree correlation coefficient r as a function of the rank number of the class of removed nodes. All the results are obtained on the networks with $N = 10^3$ and $\langle k \rangle = 4$.

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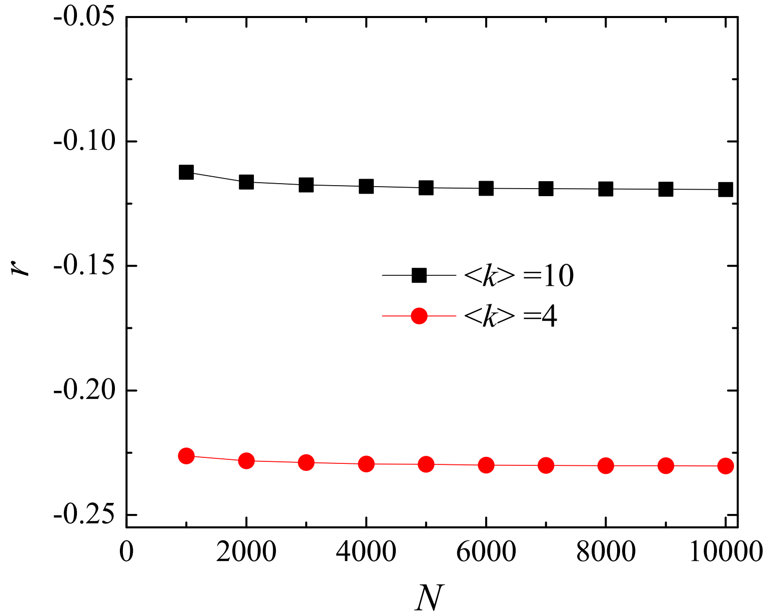

Figure S2. The degree correlation coefficient r as a function of the network size N, for different average degree $\langle k\rangle$. The fraction of removed node is here fixed at $f = 0.02$.

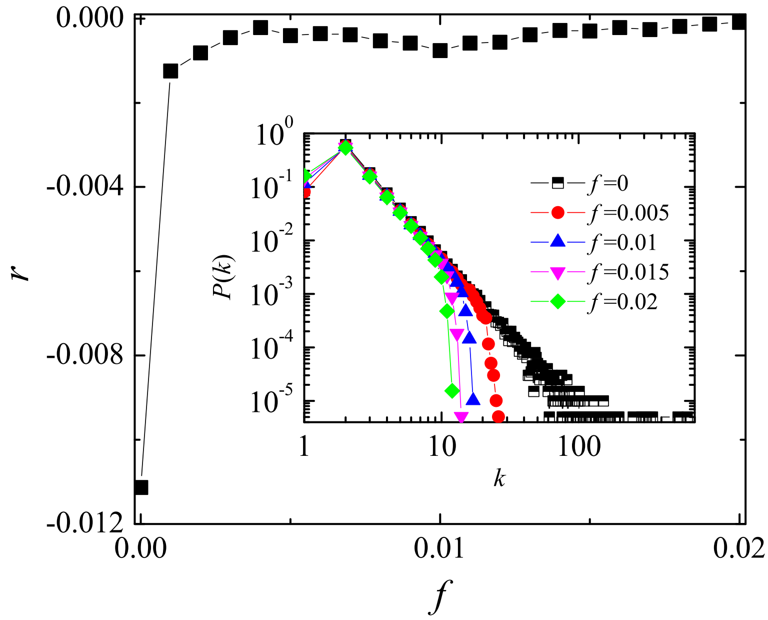

Figure S3. The correlation coefficient r as a function of the fraction of removed largest-degree nodes f in the configuration model of SF networks. The inset shows the degree distribution of the networks obtained by removing the nodes at different f values. All results are obtained for networks with $N = 10^4$ and $\langle k\rangle = 4$.