

Dynamics of nutrient uptake strategies: Lessons from
the tortoise and the hare
Electronic Supplementary Material
Theoretical Ecology

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Keywords: nutrient limitation, plasticity, co-limitation, ecosystem theory, dynamics

December 9, 2010

1 Online Supplementary Materials: Equilibrium solutions

The equilibrium expressions for equations 1-4 are

$$\bar{A}_2 = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \quad (24)$$

$$a_1 = \frac{k_2\varepsilon_2}{\mu_0 + \psi c} \quad (25)$$

$$a_2 = \frac{I_1\varepsilon_1 - I_2\varepsilon_2}{\mu_0 + \psi c} - \frac{k_1\varepsilon_1}{\omega_1\nu_1} - \frac{k_2\varepsilon_2}{\omega_2\nu_2} \quad (26)$$

$$a_3 = \frac{I_2\varepsilon_2 - I_1\varepsilon_1}{\omega_2\nu_2} \quad (27)$$

$$\varepsilon_i = \frac{\omega_i(m_i + \phi_i)}{\phi_i} \quad (28)$$

$$\bar{A}_1 = \frac{1}{\omega_1\nu_1 \left(\frac{1}{\mu_0 + \psi c} - \frac{1}{\omega_2\nu_2\bar{A}_2} \right)} \quad (29)$$

$$\bar{s} = \bar{s}^* = \frac{1}{1 + \frac{\omega_1\nu_1\bar{A}_1}{\omega_2\nu_2\bar{A}_2}} \quad (30)$$

$$\bar{B} = (I_i - k_i\bar{A}_i) \varepsilon_i \quad (31)$$

$$\bar{D}_i = \frac{(\mu_0 + \psi c)(I_i - k_i\bar{A}_i)}{\phi_i} \quad (32)$$

Both branches of equation 24 are real numbers because the discriminant is always positive. Although we have not proven it, the parameter values we have examined yield a single positive equilibrium (from the + branch of equation 24) rather than two.

2 Online Supplementary Materials: Slow invasion figure

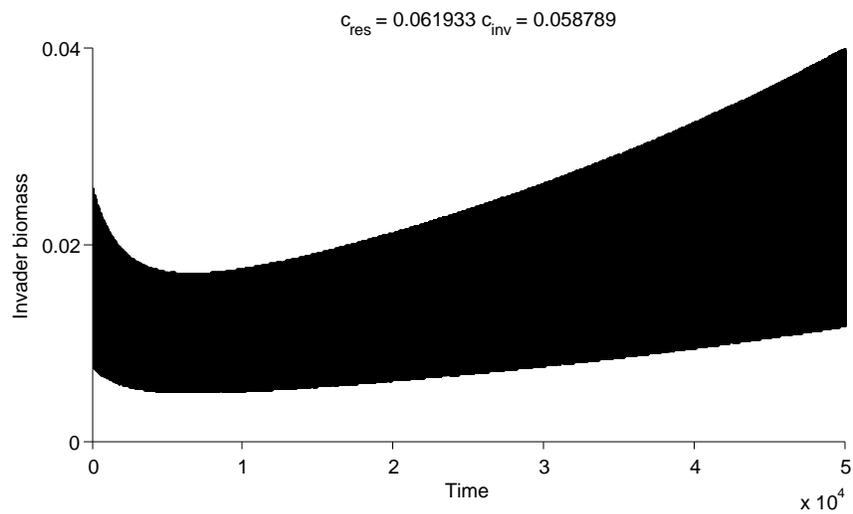


Figure 1: Dynamics of invader biomass. This case is one from the pairwise invasibility plot (Fig. 6) where the initial biomass of the invader decreases before eventually invading. Such cases are common near the evolutionary repellor in Fig. 6, as well as near other transitions, hence the necessity for longer simulation periods in these cases to determine invasion results. Details of the simulation are as in all pairwise invasions in Fig. 6, with the specific Fig. 6 pixel represented (c values for resident and invader) given on the figure. Similar dynamics resulted when the initial invader biomass was up to a few orders of magnitude lower and when resident biomass was unchanged or replaced by the initial invader biomass.