The Human Brain Encodes Event Frequencies While Forming Subjective Beliefs

Supplementary Information

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1 Bayesian Solution

1.1 Ball Betting Task

Here we show all the steps leading to the Bayesian formulas. The random variables for the ball betting task are the following. θ indicates the proportion of red balls in the bin, an outcome of the random variable Θ. k is the outcome of a binomial random variable and denotes the number of red balls observed in n drawings.

At the beginning of each trial, Θ follows a uniform distribution between $]a,b[$, where a and b denote the positions of the triangles on the screen ($0 < a < b < 1$). The probability that Θ takes the value θ equals the probability density function of the uniform distribution (we use the symbol P for both probability density and mass functions):

$$P(\Theta = \theta) = \begin{cases} \frac{1}{b-a} & a < \theta < b \\ 0 & \text{Otherwise} \end{cases}$$

The probability of observing k red balls in n drawings given a certain proportion of red balls θ is given by the density function of the binomial distribution ($0 \leq k \leq n$):

$$P(\text{`k in n'} | \Theta = \theta) = \binom{n}{k} (1-\theta)^{n-k} \theta^k$$

The probability that the bin contains a proportion θ of red balls and that k red balls are observed in n drawings is given by:

$$P(\text{`k in n'} \cap \Theta = \theta) = P(\Theta = \theta) P(\text{`k in n'} | \Theta = \theta)$$

To get the probability of observing k red balls in n drawings (not knowing Θ), one integrates over Θ:

$$P(\text{`k in n'}) = \int_a^b P(\Theta = \theta) P(\text{`k in n'} | \Theta = \theta) \, \mathrm{d}\theta$$

The result of the integration gives:

$$P(\text{`k in n'}) = \frac{\binom{n}{k} (\text{Beta}(a, k+1, n-k+1) - \text{Beta}(b, k+1, n-k+1))}{a-b}$$

where Beta denotes the incomplete Beta function.

To calculate the posterior distribution of Θ, given the observed data, we apply Bayes' rule:

$$P(\Theta | \text{`k in n'}) = \frac{P(\text{`k in n'} \cap \Theta = \theta)}{P(\text{`k in n'})}$$

which gives:

$$P(\Theta | \text{`k in n'}) = -\frac{(1-\theta)^{n-k} \theta^k}{\text{Beta}(a, k+1, n-k+1) - \text{Beta}(b, k+1, n-k+1)}$$

The probability of observing another red ball after recording k red balls in n previous draws is given by the expected value of the posterior distribution:

$$P(\text{'red ball'}|\text{'k in n'}) = \int_a^b \theta P(\Theta|\text{'k in n'})\,\mathrm{d}\theta$$

which gives:

$$P(\text{'red ball'}|\text{'k in n'}) = \frac{\mathrm{Beta}(a,k+2,n-k+1) - \mathrm{Beta}(b,k+2,n-k+1)}{\mathrm{Beta}(a,k+1,n-k+1) - \mathrm{Beta}(b,k+1,n-k+1)}$$

1.2 Bin Betting Task

For the bin betting task, the relevant random variables are the following. Let U denote a variable following a Bernoulli distribution with parameter θ indicating if the left ($U=0$) or right ($U=1$) bin was selected to draw balls. θ is the outcome of a random variable Θ defined on the unit interval. k is a random variable following a Binomial distribution indicating the number of green balls observed in n drawings. The parameter of this Binomial distribution equaled $\frac{3}{10}$ if the left bin was selected ($U=0$) and $\frac{7}{10}$ if the right bin was selected ($U=1$).

At the beginning of each trial, Θ follows a uniform distribution between $]a,b[$, with a and b given by the position of the triangles on the screen ($0<a<b<1$). The probability that Θ takes the value θ equals the probability density function of the uniform distribution:

$$P(\Theta=\theta) = \begin{cases} \frac{1}{b-a} & a<\theta<b \\ 0 & \text{Otherwise} \end{cases}$$

The probability that the bin u $(=0,1)$ is selected given θ equals the probability mass function of the Bernoulli distribution:

$$P(U=u|\Theta=\theta) = \begin{cases} 1-\theta & u=0 \\ \theta & u=1 \end{cases}$$

The probability of observing k green balls in n drawings given u (right or left bin) equals the probability density function of the binomial distribution ($0 \le k \le n$):

$$P(\text{'k in n'}|U=u) = \begin{cases} \binom{n}{k}(1-L)^{n-k}L^k & u=0 \\ \binom{n}{k}(1-R)^{n-k}R^k & u=1 \end{cases}$$

where R and L denote the probability of drawing a green ball when the right or left bin is selected, respectively.

The probability that the bin u was selected and that k greens balls were observed in n drawings, not knowing the value of θ, is given by the integral:

$$P(\text{'k in n'} \cap U=u) = \int_0^1 P(\Theta=\theta)P(U=u|\Theta=\theta)P(\text{'k in n'}|U=u)\,\mathrm{d}\theta$$

The integration results in:

$$P(\text{'k in n'} \cap U=u) = \begin{cases} -\frac{1}{2}\binom{n}{k}(a+b-2)(1-L)^{n-k}L^k & u=0 \\ \frac{1}{2}\binom{n}{k}(a+b)(1-R)^{n-k}R^k & u=1 \end{cases}$$

To calculate the probability that the right bin was selected, given the observed data, we apply Bayes' rule:

$$P(U=1|\text{‘k in n’}) = \frac{P(\text{‘k in n’} \cap U=1)}{P(\text{‘k in n’} \cap U=1) + P(\text{‘k in n’} \cap U=0)}$$

Which produces:

$$P(U=1|\text{‘k in n’}) = \frac{\frac{a+b}{2}(1-R)^{n-k}R^k}{\frac{a+b}{2}(1-R)^{n-k}R^k + (1-\frac{a+b}{2})(1-L)^{n-k}L^k}$$

The probability to observe a green ball after sampling is given by:

$$P(\text{‘green ball’}|\text{‘k in n’}) = P(U=1|\text{‘k in n’})R + P(U=0|\text{‘k in n’})L$$

2 Optimal Bet

To find the optimal betting strategy in the second price auction, we define p as the probability to draw the ball associated with the one dollar payoff in the ball betting task or the probability that the bin associated with the one dollar payoff was used to draw balls in the bin betting task. Thus p is the probability to receive the fixed (one dollar) payoff. Conditional on winning the auction, the expected value of the random payoff X is:

$$E(X|\text{‘win auction’}) = p$$

The participant places a bet b. The computer draws a price Y between 0 and 1 from a uniform distribution. The participant wins the auction if the price falls below the bet. This probability is equal to the bet:

$$P(\text{‘win action’}) = b$$

The expected value of the price Y conditional on winning the auction is half the bet:

$$E(Y|\text{‘win auction’}) = \frac{b}{2}$$

The net payoff Z is the payoff X minus the price Y. So the expected net payoff conditional on winning the auction is:

$$E(Z|\text{‘win auction’}) = E(X|(\text{‘win auction’}) - E(Y|\text{‘win auction’}) = p - \frac{b}{2}$$

To calculate the unconditional expected net payoff, one multiples the conditional expected net payoff with the probability of winning the auction:

$$E(Z) = P(\text{‘win auction’}) \cdot E(Z|\text{‘win auction’}) = \frac{b}{p} - \frac{b^2}{2}$$

The derivative of this function with respect to the bet is:

$$E(Z)' = p - b$$

The maximum expected net payoff is reached when this derivative equals 0. This obtains when the bet equals the probability of receiving the payoff in the gamble ($b = p$). Thus, it is optimal for participants to place a bet equal to the probability of winning the gamble. For instance, if the one dollar payoff is associated to the blue ball, the optimal bet is the probability of drawing a blue ball in the ball betting task. In the bin betting task, if the one dollar payoff is associated with the right bin, the optimal bet is the probability that the right bin is used by the computer to draw balls.

In case the bet is optimal, the expected net payoff is a quadratic function of the probability p:

$$E(Z|\text{'optimal bet'}) = \frac{p^2}{2} \tag{1}$$

3 Multiple Comparison Correction

Results for the voxel-based GLMs are reported both at the cluster and peak-activation levels in the tables. The False Discover Rate (FDR) computed by SPM8 for each level is reported as well (cluster-wise and peak-wise corrections; Chumbley and Friston, 2009; Chumbley et al., 2010). The principal results discussed in the article were significant after correction for multiple comparisons at the cluster level (FDR $< .05$).

4 Results

4.1 Participant Choices

Participant bets was first regressed on the Bayesian probabilities. Bets and probabilities were centered at 0 by removing 0.5 to their values ($p - 0.5$). The condition was centered at 0 as well (Condition = -1 when the blue ball/left bin was rewarded; Condition = 1 when the red ball/right bin was rewarded). Results for the ball betting task are shown in Table S1. Results for the bin betting task are shown in Table S2. Then frequencies were entered as an additional independent variable in the regressions. Results for the ball betting task are shown in Table S3. Results for the bin betting task are shown in Table S4. Tables also show the effect of the condition (whether the reward was associated to the red ball/right bin). The t-values of tables S3 and S4 are displayed in Figure 3 of the article.

Data of the two tasks were merged to assess the overall effect of Frequentist and Bayesian probabilities. A factor taking the value of 0 for the ball betting task and 1 for the bin betting task was defined to distinguish between the two tasks. Results showed a larger effect of Bayesian probabilities, but the effect of frequencies was significant as well. The interaction revealed that the effect of Bayesian probabilities was smaller in the bin betting task compared to the ball betting task. No significant difference between the tasks was found for frequencies. Results are shown in Table S5.

4.2 Brain Activation

4.2.1 Prior Information Epoch

A voxel-based analysis was conducted to test the effect of the left-right position of the triangles on the screen in the prior information period. Results for the triangles moving to the right are shown in Table S6. Results for the triangles moving to the left are shown in Table S7. Significantly activated voxels are shown in Figure 4a of the article. Results for the effect of the expected net payoff calculated based on the prior information can be found in Table S8. This table corresponds to Figure 5a in the article.

4.2.2 Sampling Epoch

The effect of stimulus probabilities was first assessed by computing two GLMs. One with the frequency as covariate of the stimulus presentation (GLM1). The other with Bayesian probability as covariate of the same event (GLM2). The positive and negative effects of frequencies are shown in Table S9 and S10 respectively (GLM1). These results are shown in Figure 6a+b of the article. The negative and positive effects of Bayesian probabilities are shown in Table S11 and S12 respectively. These results are displayed in Figure 7a+b of the article.

Frequentist and Bayesian probabilities were entered in the same regression to explain BOLD activity in each ROI. The regression was computed in R. All independent variables were scaled, but not the dependent variable (the beta estimated for each event in SPM8). Results for the Frequentist ROIs are found in Table S13 for the left angular gyrus, S14 for the right angular gyrus, S15 for the posterior cingulate, S16 for the left middle frontal gyrus, and S17 for the medial prefrontal cortex. Barplots presenting t-values are found in Figure 6c. Results for the Bayesian ROIs are found in Table S18 for the left inferior frontal gyrus, S19 for the right inferior frontal gyrus, S20 for the right supramarginal gyrus, and S21 for the supplementary motor cortex. Barplots presenting t-values of are found in Figure 7c of the article.

To test if the effect of frequentist and Bayesian probabilities was significantly stronger in the frequentist vs Bayesian ROIs, an additional mixed-linear regression was computed in R. A factor was defined to indicate if a ROI was frequentist or Bayesian. The complementary of the Bayesian probability was used as a dependent variable (1-p). Frequencies were entered in the regression without transformation. In the first regression, this factor was set to Frequentist ROIs = 0 and Bayesian ROIs = 1. Thus the betas for the frequentist and Bayesian probabilities indicate their effects in the frequentist ROIs. Results are shown in Table S22. The row labeled “Stimulus (in Freq ROI)” is the intercept. “Stimulus x Bay ROI” indicates the quantity which needs to be added to the intercept in order to get the effect of the stimulus in the Bayesian ROIs. Then the ROI location factor was set to Frequentist ROIs = 1 and Bayesian ROIs = 0. Thus the betas for the frequentist and Bayesian probabilities indicate their effects in the Bayesian ROIs. Results are shown in Table S23. The row labeled “Freq Prob x Freq ROI” indicates the quantity which needs to be added to the main effect of the frequencies – in the Bayesian ROIs – to get their effect in the frequentist ROIs (this quantity is positive). The row labeled “Bay Prob x Freq ROI” indicates the quantity which needs to be added to the main effect of the Bayesian probabilities – in the Bayesian ROIs – to get their effect in the frequentist ROIs (this quantity is negative).

Two connectivity analyzes (PPI) were performed in SPM8. The first with the bilateral angular gyrus ROIs as seed regions. The second with the bilateral inferior frontal gyrus as seed regions. Results of the contrast Angular minus Inferior frontal gyrus are shown in Table S24. Voxels identified as significant are thus more connected to the angular gyrus than to the inferior frontal gyrus. Results of the contrast Inferior frontal minus Angular gyrus are shown in Table S25. Voxels identified as significant are thus more connected to the inferior frontal gyrus than to the angular gyrus. Results of the two contrasts are shown in Figure 8b+c of the article.

4.2.3 Betting, Auction, and Final Outcome Epoch

For the betting period, the position of the prior information triangles was defined as a covariate in the GLM. The effect of the triangles moving to the right can be found in Table S26. The effect of the triangles moving to the left can be found in Table S27. The two contrasts are displayed in Figure 4b of the article.

The hand used to place the bet was defined as a covariate at the group level of analysis. The effect of using the right hand is shown in Table S28. The effect of using the left hand is shown in Table S29. The two contrasts are displayed in Figure 4c of the article.

The computer price and the net payoff were defined as covariates in the GLM. Table S30 shows the effect of the price and Table S31 the effect of the net payoff. Voxels significantly activated are displayed in Figure 5b and Figure 5c of the article.

5 Tables

Table S1: Bet regressed on Bayesian probabilities (ball betting task)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Intercept	−0.004	−0.017	0.009	0.007	388	−0.63	0.529
Condition	−0.001	−0.018	0.016	0.009	388	−0.10	0.919
Bay Prob	1.150∗∗∗	1.059	1.242	0.047	388	24.60	0.000
Random effect (SD)							
Intercept	0.011	–	–	–	–	–	–
Condition	0.032	–	–	–	–	–	–
Bay Prob	0.188	–	–	–	–	–	–
Error	0.123	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 416.

Table S2: Bet regressed on Bayesian probabilities (bin betting task)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Intercept	0.005	−0.009	0.020	0.007	388	0.73	0.463
Condition	0.002	−0.018	0.022	0.010	388	0.18	0.855
Bay Prob	0.816∗∗∗	0.745	0.886	0.036	388	22.64	0.000
Random effect (SD)							
Intercept	0.000	–	–	–	–	–	–
Condition	0.037	–	–	–	–	–	–
Bay Prob	0.155	–	–	–	–	–	–
Error	0.146	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 416.

Table S3: Bet regressed on Bayesian and frequentist probabilities (ball betting task)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Intercept	−0.002	−0.015	0.011	0.006	387	−0.31	0.756
Condition	−0.001	−0.018	0.016	0.009	387	−0.10	0.920
Bay Prob	0.855∗∗∗	0.737	0.974	0.060	387	14.16	0.000
Freq Prob	0.224∗∗∗	0.138	0.310	0.044	387	5.14	0.000
Random effect (SD)							
Intercept	0.014	–	–	–	–	–	–
Condition	0.033	–	–	–	–	–	–
Bay Prob	0.113	–	–	–	–	–	–
Freq Prob	0.117	–	–	–	–	–	–
Error	0.115	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 416.

Table S4: Bet regressed on Bayesian and frequentist probabilities (bin betting task)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Intercept	0.006	−0.008	0.020	0.007	387	0.87	0.386
Condition	0.002	−0.017	0.021	0.010	387	0.20	0.839
Bay Prob	0.619∗∗∗	0.511	0.726	0.055	387	11.33	0.000
Freq Prob	0.222∗∗	0.088	0.356	0.068	387	3.26	0.001
Random effect (SD)							
Intercept	0.000	–	–	–	–	–	–
Condition	0.035	–	–	–	–	–	–
Bay Prob	0.041	–	–	–	–	–	–
Freq Prob	0.186	–	–	–	–	–	–
Error	0.140	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 416.

Table S5: Bet regressed on Bayesian and frequentist probabilities (both tasks)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Intercept	−0.002	−0.014	0.011	0.006	799	−0.30	0.763
Task	0.008	−0.010	0.026	0.009	799	0.88	0.377
Condition	−0.001	−0.018	0.016	0.009	799	−0.13	0.899
Bay Prob	0.859∗∗∗	0.739	0.979	0.061	799	14.05	0.000
Freq Prob	0.225∗∗∗	0.129	0.321	0.049	799	4.60	0.000
Task x Condition	0.002	−0.018	0.022	0.010	799	0.23	0.820
Task x Bay Prob	−0.234∗∗	−0.392	−0.077	0.080	799	−2.92	0.004
Task x Freq Prob	−0.011	−0.150	0.127	0.070	799	−0.16	0.871
Random effect (SD)							
Intercept	0.000	–	–	–	–	–	–
Task	0.000	–	–	–	–	–	–
Condition	0.031	–	–	–	–	–	–
Bay Prob	0.000	–	–	–	–	–	–
Freq Prob	0.140	–	–	–	–	–	–
Task x Condition	0.023	–	–	–	–	–	–
Task x Bay Prob	0.075	–	–	–	–	–	–
Task x Freq Prob	0.115	–	–	–	–	–	–
Error	0.128	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 832.

Table S6: Voxel-based analysis: BOLD regressed on prior position (left to right)

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	*t*	*z*	p_{unc}	*x*	*y*	*z*
R Occipital	0.000	464	0.000							
R Calcarine				0.015	6.40	4.84	0.000	12	−80	4
R Lingual				0.071	5.05	4.13	0.000	16	−68	−6

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.018.

Table S7: Voxel-based analysis: BOLD regressed on prior position (right to left)

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	*t*	*z*	p_{unc}	*x*	*y*	*z*
L Occipital	0.000	1030	0.000							
L Lingual				0.005	7.71	5.42	0.000	−8	−68	−4
L Calcarine				0.034	6.38	4.83	0.000	−12	−78	6
L Occipital Superior				0.165	5.38	4.32	0.000	−14	−82	28

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.018.

Table S8: Voxel-based analysis: BOLD regressed on Bayesian expected net payoff

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
L Caudate	0.029	192	0.004							
L Caudate				0.110	5.70	4.49	0.000	−8	22	2
L Caudate				0.231	4.97	4.08	0.000	−10	22	−6
L Caudate				0.461	4.50	3.80	0.000	−18	28	−8

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.026.

Table S9: Voxel-based analysis: BOLD regressed on frequentist probabilities

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
L Mid Frontal	0.001	358	0.000							
L Mid Frontal				0.015	8.11	5.58	0.000	−40	18	50
Post Cingulate	0.000	2181	0.000							
Post Cingulate				0.016	7.34	5.27	0.000	−2	−28	42
Post Cingulate				0.016	7.28	5.24	0.000	−14	−54	28
Post Cingulate				0.016	7.20	5.21	0.000	6	−46	28
L Angular	0.000	1212	0.000							
L Angular				0.016	7.19	5.20	0.000	−48	−66	26
L Angular				0.067	5.96	4.63	0.000	−58	−60	30
L Angular				0.078	5.83	4.56	0.000	−46	−60	34
Medial Prefrontal	0.000	2042	0.000							
Sup Medial Prefrontal				0.016	7.15	5.18	0.000	−8	42	50
Sup Medial Prefrontal				0.059	6.25	4.77	0.000	−14	46	40
Sup Medial Prefontal				0.061	6.05	4.67	0.000	4	50	30
R Angular	0.000	614	0.000							
R Angular				0.171	5.36	4.31	0.000	44	−66	24
R Angular				0.249	4.97	4.08	0.000	62	−52	34
R Angular				0.269	4.86	4.01	0.000	46	−64	36
R Occipital	0.077	115	0.013							
R Fusiform				0.392	4.52	3.81	0.000	34	−48	−12
R Fusiform				0.432	4.35	3.70	0.000	30	−64	−6
R Fusiform				0.686	3.87	3.38	0.000	32	−56	−8
L Orbito Frontal	0.077	115	0.013							
L Orbito Frontal				0.405	4.45	3.76	0.000	−42	38	−16
L Orbito Frontal				0.427	4.40	3.73	0.000	−42	56	4
L Orbito Frontal				0.432	4.35	3.70	0.000	−34	42	−10

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.019.

Table S10: Voxel-based analysis: BOLD regressed on frequentist improbabilities

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
Suppl Motor	0.075	121	0.011							
Suppl Motor				0.216	5.35	4.30	0.000	−10	6	60
R Supramarginal	0.075	115	0.013							
R Supramarginal				0.216	5.29	4.26	0.000	44	−34	46

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.019.

Table S11: Voxel-based analysis: BOLD regressed on Bayesian improbabilities

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
L Inf Frontal	0.000	592	0.000							
L Inf Precentral				0.066	6.62	4.95	0.000	−46	4	22
L Inf Precentral				0.356	5.06	4.13	0.000	−40	0	38
L Inf Precentral				0.498	4.68	3.90	0.000	−54	4	40
R Supramarginal	0.003	244	0.000							
R Supramarginal				0.191	5.79	4.54	0.000	42	−34	46
R Supramarginal				0.500	4.63	3.88	0.000	50	−34	44
R Supramarginal				0.555	4.31	3.67	0.000	46	−28	38
R Inf Frontal	0.000	381	0.000							
R Pars Opercularis				0.191	5.72	4.50	0.000	50	8	28
R Pars Opercularis				0.243	5.40	4.32	0.000	54	10	16
R Inf Precentral				0.289	5.23	4.23	0.000	40	2	36
Suppl Motor Cortex	0.063	111	0.010							
Suppl Motor				0.555	4.44	3.76	0.000	−6	10	52
Suppl Motor				0.647	4.07	3.51	0.000	−12	4	62

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.014.

Table S12: Voxel-based analysis: BOLD regressed on Bayesian probabilities

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
L Mid Frontal	0.001	288	0.000							
L Mid Frontal				0.050	6.83	5.04	0.000	−40	18	52
L Mid Prefrontal				0.050	6.28	4.79	0.000	−42	22	42
Sup Medial Prefrontal	0.000	376	0.000							
Sup Medial Prefrontal				0.050	6.26	4.77	0.000	−8	48	48
Sup Medial Prefrontal				0.234	4.87	4.02	0.000	−6	40	54
Sup Medial Prefrontal				0.250	4.79	3.97	0.000	−14	38	48
Post Cingulate/Precuneus	0.006	175	0.002							
Precuneus				0.118	5.57	4.42	0.000	−10	−54	30
Post Cingulate				0.724	3.94	3.42	0.000	−6	−44	28
Post Cingulate				0.889	3.58	3.17	0.001	6	−46	28
L Angular	0.000	569	0.000							
L Angular				0.141	5.31	4.28	0.000	−46	−60	26
L Angular				0.234	4.91	4.04	0.000	−50	−66	38
L Angular				0.283	4.64	3.88	0.000	−48	−54	34

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.014.

Table S13: ROI analysis: BOLD Response in left angular gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	−0.712**	−1.246	−0.177	0.273	4253	−2.61	0.009
Nbr of Draws	0.143	−0.051	0.337	0.099	4253	1.45	0.147
Freq Prob	0.499***	0.279	0.719	0.112	4253	4.45	0.000
Bay Prob	−0.048	−0.210	0.114	0.083	4253	−0.58	0.563
Random effect (SD)							
Stimulus	1.359	–	–	–	–	–	–
Nbr of Draws	0.364	–	–	–	–	–	–
Freq Prob	0.340	–	–	–	–	–	–
Bay Prob	0.001	–	–	–	–	–	–
Error	3.724	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S14: ROI analysis: BOLD Response in right angular gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	−0.090	−0.547	0.366	0.233	4253	−0.39	0.698
Nbr of Draws	0.352***	0.175	0.530	0.091	4253	3.89	0.000
Freq Prob	0.395***	0.242	0.548	0.078	4253	5.06	0.000
Bay Prob	−0.232**	−0.371	−0.094	0.071	4253	−3.29	0.001
Random effect (SD)							
Stimulus	1.163	–	–	–	–	–	–
Nbr of Draws	0.360	–	–	–	–	–	–
Freq Prob	0.116	–	–	–	–	–	–
Bay Prob	0.089	–	–	–	–	–	–
Error	3.087	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S15: ROI analysis: BOLD response in posterior cingulate

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	−0.353*	−0.683	−0.023	0.168	4253	−2.10	0.036
Nbr of Draws	0.139	−0.013	0.292	0.078	4253	1.79	0.074
Freq Prob	0.298***	0.180	0.417	0.060	4253	4.95	0.000
Bay Prob	−0.095	−0.202	0.013	0.055	4253	−1.73	0.085
Random effect (SD)							
Stimulus	0.839	–	–	–	–	–	–
Nbr of Draws	0.336	–	–	–	–	–	–
Freq Prob	0.126	–	–	–	–	–	–
Bay Prob	0.110	–	–	–	–	–	–
Error	2.267	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S16: ROI analysis: BOLD response in left middle frontal gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	−0.165	−0.505	0.175	0.173	4253	−0.95	0.343
Nbr of Draws	0.476***	0.203	0.750	0.140	4253	3.41	0.001
Freq Prob	0.422***	0.198	0.647	0.115	4253	3.69	0.000
Bay Prob	0.052	−0.145	0.249	0.100	4253	0.51	0.607
Random effect (SD)							
Stimulus	0.820	–	–	–	–	–	–
Nbr of Draws	0.593	–	–	–	–	–	–
Freq Prob	0.269	–	–	–	–	–	–
Bay Prob	0.190	–	–	–	–	–	–
Error	4.196	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S17: ROI analysis: BOLD response in medial prefrontal cortex

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	−1.241∗∗∗	−1.782	−0.700	0.276	4253	−4.50	0.000
Nbr of Draws	0.499∗∗	0.199	0.800	0.153	4253	3.26	0.001
Freq Prob	0.476∗∗∗	0.256	0.696	0.112	4253	4.25	0.000
Bay Prob	−0.053	−0.295	0.189	0.123	4253	−0.43	0.670
Random effect (SD)							
Stimulus	1.361	–	–	–	–	–	–
Nbr of Draws	0.656	–	–	–	–	–	–
Freq Prob	0.125	–	–	–	–	–	–
Bay Prob	0.364	–	–	–	–	–	–
Error	4.512	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S18: ROI analysis: BOLD response in left inferior frontal gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	1.250∗∗∗	0.803	1.697	0.228	4253	5.48	0.000
Nbr of Draws	0.544∗∗∗	0.349	0.739	0.100	4253	5.46	0.000
Freq Prob	−0.053	−0.240	0.134	0.095	4253	−0.56	0.579
Bay Prob	−0.241∗∗	−0.389	−0.094	0.075	4253	−3.20	0.001
Random effect (SD)							
Stimulus	1.131	–	–	–	–	–	–
Nbr of Draws	0.396	–	–	–	–	–	–
Freq Prob	0.247	–	–	–	–	–	–
Bay Prob	0.000	–	–	–	–	–	–
Error	3.398	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S19: ROI analysis: BOLD Response in right inferior frontal gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	0.790∗∗∗	0.372	1.207	0.213	4253	3.71	0.000
Nbr of Draws	0.865∗∗∗	0.652	1.078	0.109	4253	7.96	0.000
Freq Prob	0.021	−0.191	0.232	0.108	4253	0.19	0.848
Bay Prob	−0.288∗∗∗	−0.439	−0.138	0.077	4253	−3.76	0.000
Random effect (SD)							
Stimulus	1.052	–	–	–	–	–	–
Nbr of Draws	0.449	–	–	–	–	–	–
Freq Prob	0.348	–	–	–	–	–	–
Bay Prob	0.000	–	–	–	–	–	–
Error	3.458	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S20: ROI analysis: BOLD response in right supramarginal gyrus

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	0.532∗∗	0.191	0.874	0.174	4253	3.06	0.002
Nbr of Draws	0.446∗∗∗	0.275	0.617	0.087	4253	5.12	0.000
Freq Prob	−0.109	−0.247	0.028	0.070	4253	−1.56	0.120
Bay Prob	−0.091	−0.205	0.023	0.058	4253	−1.57	0.116
Random effect (SD)							
Stimulus	0.863	–	–	–	–	–	–
Nbr of Draws	0.370	–	–	–	–	–	–
Freq Prob	0.151	–	–	–	–	–	–
Bay Prob	0.000	–	–	–	–	–	–
Error	2.623	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S21: ROI analysis: BOLD response in supplementary motor cortex

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus	1.119∗∗∗	0.545	1.693	0.293	4253	3.82	0.000
Nbr of Draws	0.259∗∗	0.069	0.448	0.097	4253	2.68	0.007
Freq Prob	−0.292∗∗	−0.500	−0.084	0.106	4253	−2.76	0.006
Bay Prob	−0.103	−0.272	0.065	0.086	4253	−1.20	0.230
Random effect (SD)							
Stimulus	1.469	–	–	–	–	–	–
Nbr of Draws	0.376	–	–	–	–	–	–
Freq Prob	0.341	–	–	–	–	–	–
Bay Prob	0.209	–	–	–	–	–	–
Error	3.389	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 4282.

Table S22: ROI analysis: Interaction Probability x ROI (Freq ROI = 0; Bay ROI = 1)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus (in Freq ROI)	−0.709∗∗∗	−1.063	−0.356	0.180	25659	−3.93	0.000
Stimulus x Bay ROI	2.007∗∗∗	1.624	2.389	0.195	25659	10.29	0.000
Nbr of Draws (in Freq ROI)	0.380∗∗∗	0.183	0.577	0.101	25659	3.78	0.000
Freq Prob (in Freq ROI)	0.584∗∗∗	0.406	0.762	0.091	25659	6.42	0.000
Bay Prob (in Freq ROI)	−0.101	−0.218	0.016	0.060	25659	−1.70	0.090
Nbr of Draws x Bay ROI	0.341∗∗∗	0.208	0.474	0.068	25659	5.02	0.000
Freq Prob x Bay ROI	−0.569∗∗	−0.917	−0.221	0.178	25659	−3.20	0.001
Bay Prob x Bay ROI	0.415∗∗∗	0.217	0.612	0.101	25659	4.12	0.000
Random effect (SD)							
Stimulus (in Freq ROI)	0.901	–	–	–	–	–	–
Stimulus x Bay ROI	0.897	–	–	–	–	–	–
Nbr of Draws (in Freq ROI)	0.483	–	–	–	–	–	–
Freq Prob (in Freq ROI)	0.331	–	–	–	–	–	–
Bay Prob (in Freq ROI)	0.172	–	–	–	–	–	–
Nbr of Draws x Bay ROI	0.180	–	–	–	–	–	–
Freq Prob x Bay ROI	0.710	–	–	–	–	–	–
Bay Prob x Bay ROI	0.275	–	–	–	–	–	–
Error	3.640	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 25692.

Table S23: ROI analysis: Interaction Probability x ROI (Bay ROI = 0; Freq ROI = 1)

Variable	*Estimate*	*Lower*	*Upper*	*SE*	*Df*	*t*	*p*
Fixed effect							
Stimulus (in Bay ROI)	1.291∗∗∗	0.930	1.652	0.184	25659	7.01	0.000
Stimulus x Freq ROI	−1.997∗∗∗	−2.383	−1.610	0.197	25659	−10.12	0.000
Nbr of Draws (in Bay ROI)	0.710∗∗∗	0.515	0.905	0.099	25659	7.14	0.000
Freq Prob (in Bay ROI)	0.021	−0.183	0.226	0.104	25659	0.21	0.837
Bay Prob (in Bay ROI)	0.315∗∗∗	0.180	0.450	0.069	25659	4.58	0.000
Nbr of Draws x Freq ROI	−0.323∗∗∗	−0.463	−0.184	0.071	25659	−4.54	0.000
Freq Prob x Freq ROI	0.570∗∗∗	0.263	0.877	0.157	25659	3.64	0.000
Bay Prob x Freq ROI	−0.422∗∗∗	−0.623	−0.220	0.103	25659	−4.10	0.000
Random effect (SD)							
Stimulus (in Bay ROI)	0.856	–	–	–	–	–	–
Stimulus x Freq ROI	0.911	–	–	–	–	–	–
Nbr of Draws (in Bay ROI)	0.446	–	–	–	–	–	–
Freq Prob (in Bay ROI)	0.270	–	–	–	–	–	–
Bay Prob (in Bay ROI)	0.000	–	–	–	–	–	–
Nbr of Draws x Freq ROI	0.211	–	–	–	–	–	–
Freq Prob x Freq ROI	0.567	–	–	–	–	–	–
Bay Prob x Freq ROI	0.296	–	–	–	–	–	–
Error	3.640	–	–	–	–	–	–

*0 not included in the 95% Confidence Interval; Obs = 25692.

Table S24: Connectivity analysis: Angular minus inferior frontal gyrus

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
L Angular Gyrus	0.000	2461	0.000							
L Angular Gyrus				0.000	15.68	7.65	0.000	−42	−62	32
L Angular Gyrus				0.000	15.45	7.61	0.000	−48	−70	32
R Angular Gyrus	0.000	1443	0.000							
R Angular Gyrus				0.000	11.52	6.73	0.000	58	−60	32
R Angular Gyrus				0.003	7.50	5.38	0.000	46	−56	24
Posterior Cingulate	0.000	2249	0.000							
Precuneus				0.001	8.65	5.83	0.000	0	−70	32
Posterior Cingulate				0.001	8.27	5.69	0.000	−4	−54	32
R Mid Cingulate				0.002	7.87	5.53	0.000	10	−50	32
Medial/Sup/Middle/L Orbito Frontal	0.000	8472	0.000							
L Middle Frontal				0.001	8.65	5.83	0.000	−38	20	52
R Sup Frontal				0.003	7.60	5.42	0.000	22	32	54
Sup Medial Frontal				0.005	7.24	5.27	0.000	12	36	54
L Middle Temporal	0.000	769	0.000							
L Middle Temporal				0.004	7.35	5.31	0.000	−60	−22	−18
L Temporal Pole				0.006	6.67	5.01	0.000	−54	0	−30
L Middle Temporal				0.006	6.57	4.96	0.000	−52	−24	−14
R Lat Orbito Frontal	0.001	301	0.000							
R Orbito Frontal				0.005	7.00	5.16	0.000	42	32	−18
R Orbito Frontal				0.012	6.03	4.70	0.000	52	42	−14
R Orbito Frontal				0.073	4.94	4.09	0.000	32	26	−20
R Middle Temporal	0.000	486	0.000							
R Temporal Pole				0.005	6.87	5.10	0.000	60	0	−28
R Middle Temporal				0.018	5.82	4.58	0.000	64	−12	−16
R Middle Temporal				0.025	5.58	4.46	0.000	64	−34	−6
Cerebelum	0.023	134	0.010							
Cerebelum				0.052	5.13	4.20	0.000	−4	−40	−4
L Lingual				0.148	4.57	3.86	0.000	−10	−34	−8
R Lingual				0.220	4.35	3.72	0.000	10	−36	−10

Height threshold: T = 3.45, p = 0.001; Extent threshold: k = 100 voxels, p = 0.022.

Table S25: Connectivity analysis: Inferior frontal minus angular gyrus

Cluster	p_{FDR}	k_E	p_{unc}							
Local Max				p_{FDR}	t	z	p_{unc}	x	y	z
Post/Precentral	0.000	39397	0.000							
L Precentral				0.000	16.88	Inf	0.000	−50	4	34
R Pars Opercularis				0.000	15.93	7.70	0.000	54	10	26
L Precentral				0.000	13.39	7.19	0.000	−48	−4	40
L Occipital/Temporal	0.001	389	0.000							
L Inf Temporal				0.040	5.37	4.34	0.000	−46	−54	−10
L Inf Occipital				0.056	5.18	4.23	0.000	−40	−76	−8
L Inf Temporal				0.061	5.14	4.20	0.000	−46	−64	−4
R Mid Temporal	0.008	233	0.001							
R Mid Temporal				0.040	5.36	4.33	0.000	48	−58	−2

Height threshold: T = 3.45, p = 0.001; Extent threshold: k = 100 voxels, p = 0.022.

Table S26: Voxel-based analysis: BOLD regressed on prior position (left to right)

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
R Occipital	0.000	778	0.000							
R Lingual				0.005	7.18	5.20	0.000	12	−74	0
R Calcarine				0.020	6.10	4.70	0.000	16	−82	4
R Sup Occipital				0.226	4.51	3.80	0.000	24	−98	10
R Occipital	0.007	145	0.007							
R Sup Occipital				0.060	5.34	4.29	0.000	22	−92	24

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.021.

Table S27: Voxel-based analysis: BOLD regressed on prior position (right to left)

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
L Occipital	0.000	1252	0.000							
L Lingual				0.061	6.14	4.72	0.000	−12	−78	−6
L Lingual				0.061	6.09	4.69	0.000	−10	−70	−6
L Calcarine				0.066	5.80	4.54	0.000	−10	−82	4

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.021.

Table S28: Voxel-based analysis: BOLD regressed on right minus left hand

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
L Motor Cortex	0.000	1239	0.000							
L Postcentral				0.000	9.82	6.17	0.000	−34	−22	46
L Precentral				0.003	7.57	5.36	0.000	−34	−22	62
L Postcentral				0.060	5.74	4.51	0.000	−48	−18	56
R Cerebelum	0.001	299	0.000							
R Cerebelum				0.060	5.64	4.46	0.000	12	−50	−12
R Cerebelum				0.221	4.81	3.98	0.000	8	−58	−8
R Cerebelum				0.523	4.03	3.49	0.000	20	−46	−24
Medial Motor/Mid Cingulate	0.044	106	0.012							
Medial Motor				0.217	4.89	4.04	0.000	−6	−18	50
Mid Cingulate				0.367	4.47	3.78	0.000	−6	−26	48

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.014.

Table S29: Voxel-based analysis: BOLD regressed on left minus right hand

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
R Motor Cortex	0.000	1305	0.000							
R Precentral				0.000	10.93	6.49	0.000	42	−16	62
R Precentral				0.000	10.38	6.34	0.000	40	−28	64
R Precentral				0.000	9.69	6.13	0.000	36	−20	50

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.014.

Table S30: Voxel-based analysis: BOLD regressed on the auction price

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
Bilat Caudate	0.000	510	0.000							
L Caudate				0.027	7.40	5.29	0.000	−8	20	4
R Caudate				0.126	6.14	4.72	0.000	6	16	0
R Caudate				0.126	6.04	4.67	0.000	8	8	−4
Occipital	0.001	399	0.000							
R Linguale				0.197	5.19	4.21	0.000	6	−78	−6
R Calcarine				0.285	4.87	4.02	0.000	12	−96	6
R Sup Occipital				0.285	4.85	4.01	0.000	20	−98	16
R Cerebelum	0.019	194	0.002							
R Cerebelum				0.218	5.07	4.14	0.000	18	−76	−18
R Cerebelum				0.445	4.35	3.70	0.000	30	−76	−18
R Cerebalum				0.495	4.22	3.61	0.000	36	−72	−22

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.019.

Table S31: Voxel-based analysis: BOLD regressed on the net payoff

Cluster Local Max	p_{FDR}	k_E	p_{unc}	p_{FDR}	t	z	p_{unc}	x	y	z
R Striatum	0.000	780	0.000							
R Putamen				0.001	9.38	6.02	0.000	18	12	−2
R Caudate				0.244	5.13	4.17	0.000	8	2	−4
R Putamen				0.244	5.10	4.15	0.000	30	6	−2
L Striatum	0.000	984	0.000							
L Caudate				0.005	7.81	5.46	0.000	−10	12	−4
L Caudate				0.017	6.99	5.11	0.000	−16	12	18
L Putamen				0.017	6.97	5.11	0.000	−20	12	−2
L Putamen	0.009	198	0.001							
L Putamen				0.208	5.31	4.28	0.000	−30	−12	4
L Putamen				0.643	4.29	3.66	0.000	−26	−20	10
L Putamen				0.643	4.20	3.60	0.000	−30	−32	10
Ventral Medial Prefrontal	0.009	197	0.001							
Ventral Medial Prefrontal				0.301	4.96	4.07	0.000	−6	46	−10
Ventral Medial Prefrontal				0.643	4.34	3.69	0.000	−2	42	−16
Ventral Medial Prefrontal				0.643	4.21	3.61	0.000	2	44	−2

Height threshold: T = 3.47, p = 0.001; Extent threshold: k = 100 voxels, p = 0.011.

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