

Table S1. Hypothesis tests for two-sample survival data

No.	Name	Test statistic	Notations
1	Weighted Log-rank	$Z = \frac{[\sum_{j=1}^{\tau} W(t_j)(d_{1j} - n_{1j} \left(\frac{d_j}{n_j}\right))]^2}{\sum_{j=1}^{\tau} W(t_j)^2 \left(\frac{n_{1j}}{n_j}\right) \left(1 - \frac{n_{1j}}{n_j}\right) \left(\frac{n_j - d_j}{n_j - 1}\right) d_j} \sim \chi^2(1)$	<p>$W(t_j)=1$, Logrank test (LR);</p> <p>$W(t_j)=n_j$, Gehan-Wilcoxon test (GW);</p> <p>$W(t_j)=\sqrt{n_j}$, Tarone-Ware test (TW);</p> <p>$W(t_j)=[S(t_{j-1})]^\rho [1-S(t_{j-1})]^\gamma$ with $\rho \geq 0, \gamma \geq 0$.</p> <p>Fleming-Harrington test ($G^{\rho,\gamma}$),</p> <p>where d_{ij} is the number of events in the ith group ($i=1, 2$) out of n_{ij} individuals (at risk) at time t_j and $d_j=d_{1j}+d_{2j}$, $n_j=n_{1j}+n_{2j}$.</p>
2	Renyi(RY)	$Q = \sup\{ \tilde{Z}(t_j) , t_j \leq \tau\} / \sigma(\tau)$	$\tilde{Z}(t_j) = \sum_{t_k \leq t_j} W(t_k) \left[d_{1k} - n_{1k} \left(\frac{d_k}{n_k}\right) \right], j = 1, 2, \dots, \tau,$ $\sigma^2(\tau) = \sum_{t_k \leq \tau} W(t_k)^2 \left(\frac{n_{1k}}{n_k}\right) \left(\frac{n_{2k}}{n_k}\right) \left(\frac{n_k - d_k}{n_k - 1}\right) d_k$
3	Modified Kolmogorov-Smirnov(MKS)	$y = \sup_{0 \leq t \leq T} Y(t) $	$Y_{N_1, N_2}(t) = \frac{1}{2} \{ \hat{S}_1(t) + \hat{S}_2(t) \} \int_0^t \left\{ \frac{N_1 \hat{C}_1(s-) N_2 \hat{C}_2(s-)}{N_1 \hat{C}_1(s-) + N_2 \hat{C}_2(s-)} \right\}^{\frac{1}{2}} \times I_{\{N_1(s)N_2(s) > 0\}} d\{\hat{H}_1(s) - \hat{H}_2(s)\}$ $T = \max\{t_j : N_1(t_j)N_2(t_j) > 0\}$
4	Cramer-von Mises(CVM)	$\int_0^\tau \left[\int_0^t K(s) d\{\hat{A}(s) - \hat{A}_0^*(s)\} \right]^2 dH(s)$	<p>CVM1 statistic uses untransformed process, which is asymptotic to a standard Brownian motion process, another CVM2 statistic converges to a Brownian bridge process</p>

5	Weighted Kaplan-Meier (WKM)	$V_{wkm} = \sqrt{\frac{N_1 N_2}{N_1 + N_2}} \int_0^{T_m} \tilde{w}(t) [\widehat{S}_2(t) - \widehat{S}_1(t)] dt$ $\widehat{\sigma}_{wkm}^2 = -\int_0^{T_m} \left\{ \int_t^{T_m} [\tilde{w}(s) \widehat{S}(s)] ds \right\}^2 \frac{N_1 \widehat{C}_1(t-) + N_2 \widehat{C}_2(t-)}{(N_1 + N_2) \widehat{C}_1(t-) \widehat{C}_2(t-)} \frac{d\widehat{S}(t)}{\widehat{S}(t) \widehat{S}(t-)}$	$T_m = \sup\{t : \min\{\widehat{C}_1(t), \widehat{C}_2(t), \widehat{S}_1(t), \widehat{S}_2(t)\} > 0\}$ $\tilde{w}(t) = \frac{(N_1 + N_2) \widehat{C}_1(t-) \widehat{C}_2(t-)}{N_1 \widehat{C}_1(t-) + N_2 \widehat{C}_2(t-)}$
6	Maximum Weighted Kaplan-Meier (MKM)	$\tilde{V} = \max \tilde{V}_\ell $	$V_\ell = \sqrt{\frac{N_1 N_2}{N_1 + N_2}} \int_0^\tau f\{\widehat{S}(t-)\} \tilde{w}(t) [\widehat{S}_1(t) - \widehat{S}_2(t)] dt$ $f\{S(t-)\} = [S(t-)]^\rho [1 - S(t-)]^\gamma, \rho = 0, 1, 2; \gamma = 0, 1, 2$ $\tilde{V}_\ell = V_\ell / (\widehat{\sigma}_{\ell, \ell})^{1/2}, \ell = 1, \dots, m (m = 9)$
7	Lee's versatile test(SHL)	$SHL1: Z_1 + Z_2 / 2$ $SHL2: (Z_1 + Z_2) / 2$ $SHL3: \max(Z_1 , Z_2)$	$Z_1 \text{ and } Z_2 \text{ are calculated as } Z_1 \text{ with weight function } \rho=0, \gamma=1; Z_2 \text{ with weight function } \rho=1, \gamma=0;$
8	Lin and Wang's test(LW)	$T^* = \frac{\Delta - E(\Delta)}{\sqrt{Var(\Delta)}} \sim N(0, 1)$	$\Delta = \sum_{j=1}^{\tau} [d_{1j} - E(d_{1j})]^2$ $E(\Delta) = \sum_{j=1}^{\tau} \frac{n_{1j} n_{2j} d_j (n_j - d_j)}{n_j^2 (n_j - 1)}$ $Var(\Delta) = \sum_{j=1}^{\tau} \{E(d_{1j}^4) - 4E(d_{1j}^3)E(d_{1j}) + 6E(d_{1j}^2)(E(d_{1j}))^2 - 3(E(d_{1j}))^4 - (Var(d_{1j}))^2\}$

9	Lin and Xu's test (LX)	$\Delta^* = \frac{\Delta - \widehat{E}(\Delta)}{\sqrt{\widehat{Var}(\Delta)}} \sim N(0,1)$	$\Delta = \sum_{t_j < \tau} \widehat{S}_1(t_j) - \widehat{S}_2(t_j) (t_{j+1} - t_j)$ $\widehat{E}(\Delta) = \sum_{j t_j < \tau} \left\{ \frac{2}{\pi} [\widehat{\sigma}_{S_1}^2(t_j) + \widehat{\sigma}_{S_2}^2(t_j)] \right\}^{1/2} (t_{j+1} - t_j)$ $\widehat{Var}(\Delta) = \sum_{j t_j < \tau} (t_{j+1} - t_j)^2 \left(1 - \frac{2}{\pi}\right) [\widehat{\sigma}_{S_1}^2(t_j) + \widehat{\sigma}_{S_2}^2(t_j)] + \sum_{j < j' t_j, t_{j'} < \tau} 2\rho_{j,j'} (t_{j+1} - t_j)(t_{j'+1} - t_{j'}) \left(1 - \frac{2}{\pi}\right) \times \{[\widehat{\sigma}_{S_1}^2(t_j) + \widehat{\sigma}_{S_2}^2(t_j)][\widehat{\sigma}_{S_1}^2(t_{j'}) + \widehat{\sigma}_{S_2}^2(t_{j'})]\}^{1/2}$
10	Two-stage(TS)	$V = \sup_{\tau_\varepsilon \leq m \leq \tau - \tau_\varepsilon} (V_m)$	$V_m = \sum_{j=1}^{\tau} w_j^{(m)} \left(d_{1j} - n_{1j} \frac{d_j}{n_j} \right) / \sqrt{\sum_{j=1}^{\tau} (w_j^{(m)})^2 \left(\frac{n_{1j}}{n_j} \right) \left(\frac{n_{2j}}{n_j} \right) \left(\frac{n_j - d_j}{n_j - 1} \right) d_j}$ $w_j^{(m)} = \begin{cases} -1, & j = 1, 2, \dots, m \\ c_m, & j = m+1, \dots, \tau \end{cases}$
11	Adaptive Neyman's Smooth tests (NY)	$T_d = U(\tau)^T \widehat{\sigma}(\tau)^{-1} U(\tau)$ $T_s = \max \{ V_C(\tau)^T \sigma_{CC}(\tau)^{-1} V_C(\tau) : C \in \zeta, C = d^* \}$	T_d : fixed-dimensional test(NY1) T_s : data-driven test(NY2) $V_C(\tau)$ and $\widehat{\sigma}_{CC}(\tau)$ are the subvector and submatrix of $V(\tau)$ and $\sigma(\tau)$ corresponding to the subset C . Schwarz's criterion is given as $S = \arg \max_{C \in \zeta} \{ T_C - C \log n \}.$