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Constraints of thermal noise on the effects of weak 60-Hz magnetic fields acting on biological magnetite

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ABSTRACT **Previous calculations of limits imposed by thermal noise on the effects of weak 60-Hz magnetic fields on biological magnetite are generalized and extended to consider multiple signals, the possibility of anomalously large magnetosome structures, and the possibility of anomalously small cytoplasm viscosities. The results indicate that the energies transmitted to the magnetite elements by fields less than 5 μT, characteristic of the electric power distribution system, will be much less than thermal noise energies. Hence, the effects of such weak fields will be masked by that noise and cannot be expected to affect biology or, therefore, the health of populations.**

Biological magnetite, Fe_3O_4, a ferrimagnetic metallic compound, with a saturation magnetism about 30% of that of iron, is usually found in single-domain units covered with a thin membrane, called magnetosomes. Low-frequency magnetic fields induce a torque on these domains that might affect biology in a manner that could result in consequences important to public health (8, 14, 15). However, if that field is so small that the induced torques are much less than the stochastic Brownian-motion torques from thermal agitation, the torques from the fields will be masked by that noise and cannot be expected to be biologically significant. In particular, we consider affects of a canonical 60-Hz 5-μT field, one-tenth of the earth's DC field, but appreciably greater than the maximum exposures to populations from the electric power distribution system. In considering the details of the interactions and the limits on the effects of such fields, it is convenient to consider two kinds of magnetite elements, composite systems and single magnetosomes.

Magnetosomes are generally found in chain-like assemblies, with their magnetic moments aligned, in the interior of cells. The individual magnetosomes are tied to cell bodies, probably through the cytoskeleton. In such an assembly, torques on the magnetosomes are transmitted to the cell body through structures that secure the magnetosome to that body. Hence the magnetic field acts to rotate the whole cell through forces on the individual magnetosomes. The movement of these composite systems, typically whole cells, is strongly constrained by the impedance of the surrounding environment that, for cells interior to the body, will not be less than that of the viscosity of water.

Although individual magnetosomes not rigidly tied to the whole cell structure do not seem to have been identified, I assume that such magnetosomes exist and, though secured to a place in the cell, may be free to rotate in that place and generate biological effects through that rotation. The rotation of these individual magnetosomes must also be constrained by the impedance of their environment that will be no less than that of the viscosity of the cytoplasm in which they are immersed. Since the energy transfers to quasi-free individual magnetosomes are less strongly constrained than that of the composite systems, such transfers must be considered in analyses of possible biological effects of 60-Hz magnetic fields. Consequently, most of this discussion is directed toward the establishment of limits on the possible interactions of hypothetical mechanisms.

If the rotation of the composite system or single magnetosome is to affect biology, the magnetite system must be coupled to an element through some harness so that motion of the magnetite system changes the biologically sensitive conformation of that element. That transition must require an energy in excess of kT, where k is Boltzmann's constant and T is the absolute temperature, if it were not to occur regularly through thermal agitation in the absence of any coupling to the 60-Hz magnetic field. The viscous impedance of a harness and the addition of a substantial activation energy add to the minimum energy transfer required if the rotation of the magnetosome is to affect biology. But both the coupling and the minimum energy requirement are model-dependent; hence, in these limiting calculations, we treat the magnetic system as if it were free and conclude, conservatively, that the system cannot affect biology if the energy transfer to the free system from the 60-Hz magnetic field is less than kT.

Energy Transfers to Biological Elements

A time-varying magnetic field, $\mathbf{B}(t) = \mathbf{B}_0\cos\,\omega t$, will, in general, induce varying torques on a magnetite system with a moment of inertia I and a magnetic moment, μ, residing in equilibrium in an environment that may not be isotropic. That driving torque will be countered by binding and resistive torques that will vary with the angular displacement, θ, and its time derivative. I write the total torque as a vector with components;

$$T_i = -\kappa_{ij}\theta_j - \beta_{ij}\dot{\theta}_j + [\mathbf{B}(t) \times \boldsymbol{\mu}]_i + \chi_i(t), \qquad j = 1, 3. \quad [1]$$

Here κ and β are tensors and $\kappa_{ij}\theta_j$, $\beta_{ij}\dot{\theta}_j$, and $[\mathbf{B}(t) \times \boldsymbol{\mu}]_i$ are components of vectors that are not, in general, aligned; $\chi_i(t)$ represents the effects of thermal agitation in the direction $\hat{\mathbf{i}}$.

By taking the direction of the total torque $\mathbf{T}$ as the base direction, I write the linear equation,

$$T = I\ddot{\theta} = -\beta\dot{\theta} - \kappa\theta + B_0\mu\,\cos\,\omega t\,\cos\,\phi\,\sin\,\zeta + \chi(t), \quad [2]$$

where the terms at the right are the components of the vector torques in the direction of the resultant $\mathbf{T}$, κ and β are now scalar coefficients, and θ, $\dot{\theta}$, and $\ddot{\theta}$ are the components of the angular displacement, angular velocity, and angular acceleration, respectively, in the direction of the total torque. The angle ϕ is the angle between the direction of the imposed torque and the final acceleration and ζ is the angle between the imposed field, $\mathbf{B}_0\cos\,\omega t$, and the magnetic moment, μ, of the element.

I ignored torques proportional to higher powers of displacement and velocity. In the spirit of a Taylor's expansion, for small displacements and velocities, the linear term must dominate; for larger displacements, more complex behavior is possible. However, since κ is left a free parameter, any

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variation of the binding force through high-order effects cannot affect the conclusions. And, since β cannot be less than the value derived from the linear effect of viscosity, higher-order resistive effects can only add to the resistance and decrease the energy transfer.

I consider the four terms at the right in Eq. 2:

The Driving Torque. The energy transferred to the system will be greatest for $\phi = 0$ and $\zeta = \pi/2$, and I adopt those values.

An appropriate conjunction of four vectors—the 60-Hz magnetic field, the earth's field, the magnetic moment of the magnetite element, and the biologically defined direction of the effective action—must obtain if the effect of the field is to be large. If the biological effect of the torque is proportional to $\hat{\mathbf{T}}\cdot\hat{\mathbf{n}}$, where the effective direction, $\hat{\mathbf{n}}$, is randomly set with respect to the direction of the torque, the energy transfer, proportional to $(\hat{\mathbf{T}}\cdot\hat{\mathbf{n}})^2$, will be greater than 50% of the maximum for only about 15% of the elements.

Since moveable magnetosomes tend to be aligned with the earth's field and the resultant vector torque T is proportional to $[\mathbf{B}(t) \times \boldsymbol{\mu}]$, 60-Hz fields in the direction of the earth's field may be least effective in transferring energy to magnetic systems. Hence, fields from power lines, generally near vertical outside of the right-of-way and partially aligned with the earth's field, can be expected to have their effects reduced by factors of the order of 5 from the maximum energy transfers derived here.

Binding Torques. Magnetosomes secured in place, but otherwise free to rotate, will be aligned with the earth's field B_e and the binding torque will be $B_e\mu \sin\theta$. Since $\sin\theta < \theta$, for large angles, in the linear approximation, $\kappa < B_e\mu$. For other plausible situations, further constraints may exist and $\kappa \geq B_e\mu$. I retain generality by leaving κ, the retaining force constant, as a free parameter.

Resistive Torques. Since viscosity is complex on the microscopic level, it must be incoherent with other processes; hence, the dissipation factor, β, cannot be less than that from the viscous resistance of the fluid in which the elements are immersed. Hence, we choose, minimally, $\beta = 6\eta\nu$, the Stokes' Law value for a sphere with a volume ν equal to the volume of the element, where η is the viscosity of the medium in which the element is imbedded. For elongated bodies with the magnetic moment in the direction of the longest axis, this form is conservative inasmuch as it will somewhat understate the resistance to the rotation of the bodies about their minor axes and overstate the maximum energy transfer to such systems. For the rotation of whole cells in the tissue plasma, we take the viscosity as that of water, $\eta_w = 0.0007$ N·s/m^2. For the rotation of individual magnetosomes in the interior of the cell, we consider a range of viscosities from that of water to the value of about 0.1 N·s/m^2, measured for rotating systems in the aqueous cytoplasm of eukaryote plant and mammalian cells (1, 2). Since the energy transfer to magnetite systems is largely bounded by the local viscosity, the uncertainties in that viscosity are important.

Thermal Noise. The magnetosome will undergo Brownian-like thermal agitation represented by $\chi(t)$.

Signal and Noise Amplitudes

Since Eq. 2 is linear, we can write the solution to the equation as $\theta(t) = \theta_B(t) + \theta_{kT}(t)$, where $\theta_B(t)$ is the solution in the absence of noise, $\chi(t) = 0$, and $\theta_{kT}(t)$ is the solution with noise but no perturbing field, $B(t) = 0$.

Here, $\theta_B(t) = \theta_0\cos(\omega t + \phi)$, where ϕ is an irrelevant phase angle, and

$$\theta_0^2 = \frac{(B_0\mu)^2}{I^2(\omega_0^2 - \omega^2)^2 + \beta^2\omega^2}, \quad \text{where} \quad \omega_0^2 = \frac{\kappa}{I}. \qquad [3]$$

From statistical mechanics and the Langevin equation,

$$\overline{\cos\theta_{kT}} = \coth\left(\frac{\kappa}{kT}\right) - \frac{kT}{\kappa}, \quad \text{or} \quad \overline{\theta_{kT}^2} \xrightarrow{\kappa > kT} \frac{2\,kT}{\kappa}. \qquad [4]$$

Since the noise amplitude, θ_{kT}, and the signal amplitude, θ_B, are orthogonal, the time-average intensities simply add; $\overline{\theta^2(t)} = \overline{\theta_B^2(t)} + \overline{\theta_{kT}^2(t)}$. We write, for convenience, $\theta_{rms}^2 = \theta_0^2/2 = \overline{\theta_B^2(t)}$ and $\theta_{kT}^2 = \overline{\theta_{kT}^2(t)}$.

The dominant role that energy plays in statistical mechanics, and in information theory (3), follows largely from the orthogonality of noise amplitudes. Hence, we define the signal-to-noise (S/N) ratio in terms of energies and compare the mean kinetic and potential energies the system acquires from the external "signal" magnetic field with the characteristic "noise" energy, kT;

$$W_T = \frac{1}{2} I\omega^2\theta_{rms}^2, \; W_V = \frac{1}{2}\kappa\theta_{rms}^2; \quad \text{hence,} \quad \frac{W_V}{W_T} = \frac{\omega_0^2}{\omega^2}. \qquad [5]$$

Since at 60 Hz, for systems of interest, $\omega_0^2 \gg \omega^2$ and $W_V \gg W_T$, we need consider only the potential energy to establish energy-transfer limits. The mean noise potential energy, $\frac{1}{2}\kappa\theta_{kT}^2 = kT$; hence, the condition that the potential energy is less than kT is the same as the condition that the amplitude, θ_{rms}, induced by the 60-Hz field, must be less than the mean excursion, θ_{kT}, induced by thermal agitation.

Neglecting ω^2 in the term $\omega_0^2 - \omega^2$, of Eq. 3,

$$W_V < \frac{(B_{rms}\mu)^2}{2}\frac{\kappa}{\kappa^2 + \beta^2\kappa^2}, \qquad [6]$$

where $B_{rms}^2 = B_0^2/2$. Since these inequalities pertain to nearly free individual magnetosomes resident in cells and the resistive effects of the biological interaction are neglected, the inequalities are strong.

From Eq. 6, the potential energy limit will be a maximum when the natural time constant, β/κ, is equal to $1/\omega$. Then,

$$W_V < \frac{(B_{rms}\mu)^2}{4\beta\omega} = B_{rms}^2\mu\cdot\frac{\mu}{4\beta\omega}. \qquad [7]$$

The form at the right exploits the proportionality between μ and β for single domains of magnetite. The magnetic moment of a magnetosome will be $\mu = H'\nu$, where $H' \approx 4.8 \times 10^5$ A/m is the magnetic field characteristic of magnetite domains (4). As the domains are roughly spherical, we take $\beta = 6\eta\nu$, where η is the viscosity of the medium about the domain and $\mu/\beta = H'/6\eta$ is independent of the size of the domain. For a value of the viscosity μ' measured in units of the viscosity of water, $\mu/4\beta\omega \approx 7.5 \times 10^4/\mu'\ T^{-1}$. The value will be smaller for magnetite imbedded in a matrix of organic material such as the magnetosome membrane, even as the volume of the whole rotating system will be greater than the volume of the magnetite.

From Eq. 7, for a 60-Hz field, $B_{rms} = 5\ \mu$T, the maximum energy transfer to a very large single-domain magnetosome, with a magnetic moment such that $B_e\mu = 25\ kT$, in a cellular environment with a viscosity of 0.005, will be $W_V = 0.14\ kT$. I take this as the maximum plausible transfer to a single magnetosome. But other limits are interesting. If the viscosity is as large as 100 times that of water (1), $W_V > kT$ only if $B_e\mu > 5700\ kT$. For a viscosity of zero, taking $\kappa = B_e\mu$, from Eq. 6, $W_V > kT$ only if $B_e\mu > 200\ kT$.

According to Butler and Banerjee (5), the maximum possible size of such single-domain crystals stable with respect to transitions to a specific two-domain configuration varies strongly with the length/width ratio. Though their results

admit very large domains for highly elongated crystals, other transitions may be favored for such crystals—and very large single-domain systems have not been found in nature. Biologically formed magnetite crystals have a variety of shapes and sizes (6) but single magnetosomes have not been identified with moments μ greater than that for which $B_e\mu = 25\ kT$—and most have moments such that $B_e\mu \approx kT$. Indeed, magnetosomes found in magnetotactic bacteria are of that size (4), which is near the maximum theoretical size of stable cubic single domains of magnetite. Hence, excepting the existence of single-domain magnetosomes much larger than have been observed, even for very small viscosities, 60-Hz 5-μT magnetic fields acting on single magnetosomes cannot be expected to affect biology—or public health.

The energy of multidomain crystals will be more strongly constrained by a factor, μ/μ', where μ' is the moment if the domain moments were perfectly aligned—i.e., if the magnetite assembly is saturated. Multidomain magnetite grains with diameters as large as 0.6 μm have been found in the human brain (6) with orientation energies, $B_e\mu$, that could be as great as 150 kT. For such a crystal, $W_V < 0.18\ kT$ for 60-Hz 5-μT magnetic fields. For composite systems such as cells, for a given magnetic moment, the energy limit of Eq. 7 should be multiplied by a factor $v/v' \ll 1$, where v is the volume of the aligned magnetite and v' is the total volume of the system. Hence, the energy induced in such systems will be small.

Again from Eq. 6, for the very large values of κ relevant to elements held rigidly, $W_V \propto 1/\kappa$ and will be much smaller than the maximum values defined in Eq. 7. Assemblies of aligned magnetosomes with moments such that $B_e\mu \approx 5000\ kT$ have been observed in certain microorganisms (7). Such large assemblies are known to be made up of thousands of small domains (magnetosomes) attached rigidly by cytoskeleton elements to the cell as a whole. Then whole cells will oscillate under an external 60-Hz field. From Eq. 7, the maximum energy transfer to a spherical cell with a radius of 10 μm, holding a magnetic moment μ such that $B_e\mu = 5000\ kT$, will be less than 0.04 kT. But that maximum energy transfer supposes a binding torque very much larger than that from the earth's field, $\kappa \gg B_e\mu$. If the restoring force is taken as $B_e\mu$, from Eq. 6, $W_V < 2.5 \times 10^{-4}\ kT$.

With the value of $\kappa = \beta\omega$ set for maximum energy transfer and $B_{rms} = 5\ \mu$T,

$$\theta_{rms}^2 < \frac{(B_{rms}\mu)^2}{4\beta^2\omega^2},\ \theta_{rms} < B_{rms}\cdot\frac{\mu}{2\beta\omega} \rightarrow 0.106, \qquad [8]$$

and the small-angle condition used in Eq. 3 is nominally satisfied for any pure magnetite domain whatever the size. The angle of excursion will be smaller by the factor v/v' for multidomain composites. However, $\theta_{kT} \approx \sqrt{2kT/\kappa}$, the mean amplitude of thermal agitation will usually be larger (e.g., for $\kappa/kT = 10$, $\theta_{kT} \approx 0.45$ radians) and, since $\sin\theta < \theta$, the linearity of Eq. 2 will be impaired and the effective value of the restraining force parameter, κ, will be reduced. Since the maximum energy transfer is largely constrained by the resistive factor, β, and not by κ, the conclusions drawn from the linear equation are not much changed. Indeed, from Eq. 6, if $\kappa < \beta\omega$, which is the case for most systems, a smaller value of κ leads to a smaller energy transfer.

Time Dependencies

For the earth's field, $\omega = 0$ and $B_{rms} = B_e \approx 50\ \mu$T, $W_V = B_e\mu \gg kT$ for elements with large net magnetic moments. Hence, the rotational excursions of such magnetic elements must far exceed that induced by thermal fluctuations or by a 60-Hz 5-μT field. Then the smaller 60-Hz fields can affect biology in a manner different than the ubiquitous earth's field only through mechanisms insensitive to frequencies much lower than 60 Hz. Kirschvink (8) has pointed out that cellular processes with the perquisite long time constants are known. For example, the opening and closing of membrane channels take place over times of the order of 0.1 s (9). Hence, there may be cellular systems that accommodate to configurational changes that take place over periods as long as 0.1 s but are affected significantly by more rapid excursions.

Such mechanisms would not be affected by noise if the changes in orientation due to noise were slow. We estimate the characteristic rate of change from noise by equating the time, t, required for characteristic Brownian movement angular excursions, θ_{Br}, to equal the angular amplitudes, θ_{rms}, induced by the 60-Hz fields and comparing that time to the characteristic time, $1/\omega$.

From the relations describing Brownian rotation,

$$\theta_{Br}^2 = \frac{2kT}{6\eta v}t = \theta_{rms}^2 = \frac{(B_{rms}\mu)^2}{4(6\eta v\omega)^2} \quad \text{and} \quad t = \frac{(B_{rms}\mu)^2}{48\eta v\omega^2\ kT}. \qquad [9]$$

Then for the very large magnetosome with a moment such that $B_e\mu = 25kT$, for a 60-Hz magnetic field, $B_{rms} = 5\ \mu$T, and using a small value of viscosity, $\eta = 0.005$ N·s/m^2, we find the interaction time, $t = 1.8 \times 10^{-4}$ s $\ll 1/\omega$. Since the characteristic time for single-domain magnetosomes varies as the volume of the domain, smaller elements will have even shorter characteristic noise fluctuation times. For large composites, such as cells, for a given magnetic moment, the characteristic time varies as $1/v$, where v is the volume of the cell, and the characteristic noise times are always small. Hence, a low-bandwidth filter could exist that excludes effects from the earth's field but admits 60-Hz effects—and almost all noise.

Multiple Signals

Coherence. If many detectors act cooperatively, or "coherently," the signal-to-noise ratio of the set may be greater than the signal-to-noise ratio of an individual detector.

We address such coherence by examining the energy transfers from the field to a system of M magnetosome detectors associated as if they were tied together mechanically—and are, therefore, completely coherent—so the sum of their torques act on some biologically sensitive element. The moment of the set will then be $M\mu$, where μ is the moment of an individual magnetosome.

From Eqs. 6 and 7, in the limit of zero viscosity, $\beta = 0$, the energy transfer to the system will be M^2/κ times the energy transfer to one magnetosome acting alone. However, if the system responds coherently to the perturbing field, it must respond coherently to the earth's field. Hence, the binding torque $\kappa = MB_e\mu$ and the energy transfer to the system will be increased only by M. Since the noise of the set will be M times the noise from one detector, the signal-to-noise ratio of the M coherent detectors, $(S/N)_M$, will be the same as S/N for one detector.

If the viscosity is large, $\beta\omega \gg B_e\mu$. But since β is proportional to the volume of the magnetosome set and then proportional to M, $(S/N)_M$ of the set will still be equal to S/N for the individual detector.

However, if the magnetosome is strongly bound by other mechanisms, and $\kappa \gg B_e\mu$, then $(S/N)_M = M(S/N)$. (And the ratio of signal amplitudes to noise amplitudes for the set of M detectors will be greater by a factor $\sqrt{M}$ than for one detector.) However, from Eq. 6, $S/N \propto 1/\kappa$ and will be small for large values of κ, and the ratio of signal energy to noise energy for M coherent magnetosome detectors will never be greater than the S/N for one detector where $\kappa = B_e\mu$ and $\beta = 0$. Hence, even if many magnetosomes act coherently,

their interaction with the canonical fields cannot be expected to affect biology.

Incoherent Sources. But the summation of signals from incoherent sources, acted upon by the same 60-Hz signal, can be more important. If, from a set of K detectors, a subset of M elements can activate a binary (yes–no) system, such as opening and closing a gate, with a signal $S \ll kT$, only for $K = (K/M)^2(kT/S)^2$ elements, will $(S/N)_K = 1$ (*Appendix A*). In other terms, for M active elements, $(S/N)_M = \sqrt{M}(S/N)$. In particular, under the canonical 5-μT 60-Hz field, if appropriate summation processes are in effect, if $S/N = S/kT$ for one element has the maximum value of 0.14 for magnetosomes that are of the maximum observed size and have moments such that $B_e\mu = 25kT$, and if (e.g.) 50% of these magnetosomes are aligned so as to trigger a transition on a binary system through torques generated by the field, then $(S/N)_K = 1$ for $K = 200$ elements. However, the total magnetite is then such that $B_e\mu = 5000kT$ for the whole cell, which is near the maximum observed in any cell.

Multiple Signals Over Time. In general, the maximum possible signal-to-noise ratio is increased by a factor $\sqrt{Q}$ if the detector responds coherently to a train of Q pulses (the paradigm is resonance action). Hence, in principle, the signal-to-noise ratio of a detector can be increased by a factor of 60 by responding coherently to the 3600 60-Hz pulses observed in a minute. However, this requires a detector frequency selectivity or bandwidth of about 1/3600 Hz—at exactly 60 Hz—which is not possible (10).

However, "rectification" mechanisms, such as suggested by Weaver and Astumian (11) in their consideration of biological effects of weak electrical fields and by Kirschvink (8) for magnetic fields, allow a significant periodic flow of product across the membrane by actions initiated by fields that induce energy transfers smaller than kT. In these models, the perturbing fields, which need not be precisely periodic and thus define a narrow bandwidth, change the transmission of the cell membrane by opening gates for specific ions or neutral molecules.

For weak signals where the signal energy, $dw \ll kT$, the induced current can generally be described by the Nernst equation (*Appendix B*),

$$I = I_0\left[-\frac{dw}{kT} + \frac{1}{2}\left(\frac{dw}{kT}\right)^2 \cdots\right], \qquad [10]$$

where I_0 is a "dark current" and $dw \approx B_e\mu\theta_{kT}\theta_0 \cos \omega t = dw_0 \times \cos \omega t$.

Over a time $t \gg 1/\omega$, the mean number of molecules transmitted through the membrane as a consequence of the 60-Hz field will be $Q_s = (I_0/4)(dw_0/kT)^2 t$. There will also be a "noise" transfer across the membrane equal to the square root of the number of molecules passing through the membrane (in plus out). At equilibrium—with no external field—the probable drift will be $Q_{kT} = [(I_{in} + I_{out})t]^{1/2} \approx (2I_0t)^{1/2}$ molecules in a time t. For K gates,

$$\left(\frac{S}{N}\right)_K = \frac{Q_s}{Q_{kT}} = \sqrt{\frac{I_0 t}{32}}\left(\frac{dw_0}{kT}\right)^2. \qquad [11]$$

If the number of gates were increased by A^2, $I_0 \rightarrow A^2I_0$, and if the magnetic signal were reduced by a factor of A, $\theta_0 \rightarrow \theta_0/A$ and $dw_0 \rightarrow dw_0/A$, the signal-to-noise ratio will be reduced by the factor $1/A$.

The incremental transfer with time from rectification, which varies as the $\sqrt{t}$, may be important (11). By setting $S/N = 1$, in Eq. 11, the rectified current will equal the mean thermal noise drift in a time, $t = (32/I_0)(kT/dw_0)^4$. Taking, for example, a maximal value of the dark current through one channel, in the absence of the field with the channel nominally closed, as $I_0 = 1\ s^{-1}$, and the value, $dw_0/kT = 0.14$, calculated for a large magnetosome, such that $B_e\mu = 25kT$ in a 5-μT field, we find a time of $8 \times 10^4/P$ seconds for P activated channels. Homeostatic effects that bring cells to equilibrium in the face of normal thermal drift will ensure that no biological effects follow from so small a rate of change. If there were very many channels, the transfer could be significant, but the amount of magnetite required to generate substantial signals is much greater than that which is found, or can be expected to be found, in a cell.

Summation of Signals from Different Cells. The effects of weak fields on cells through the conjunction of signals from many magnetosomes are limited by the amount of magnetite the cells hold. If mechanisms exist that can sum signals from the magnetite in different cells of an organism, that limit is impeached and bees (8, 12), and probably other species, detect small static magnetic fields that induce changes in individual detectors smaller than that from thermal noise through complex sensory mechanisms that sum the signals from many individual receptors thus increasing the signal-to-noise ratio substantially.

These mechanisms have been developed and honed by evolution through the advantages in survival and propagation accorded to those individual members of the species who are able to use this facility. However, the signal from 60-Hz fields is much reduced by the viscous impedance to the rapid cycling. And, since the ability to detect 60-Hz fields, not found in nature, could not confer any survival advantage in the past, we should not expect that the complex signal processing mechanisms that allow animals such as the honeybee to detect small DC magnetic field anomalies would be developed specially to process 60-Hz signals. Indeed, while honeybees seem to detect DC magnetic field anomalies smaller than 1 μT quite reliably, they do not seem to be as good at detecting 2.2-mT 60-Hz fields, fields several thousand times greater (13).

Summary

(*i*) For a 60-Hz 5-μT field, acting on biological magnetite, biologically deleterious effects can be expected only through mechanisms that are insensitive to the 10-times larger DC field of the earth. (*ii*) For any magnetite system, the kinetic energy induced by the field is negligible. (*iii*) For any cell-size composite system, the potential energy induced by the field is much less than the noise energy. Hence, any biological effects must stem from energies transmitted to quasi-free single magnetosomes in cells. But there is no histological evidence for the rotation of single magnetosomes inside of the cell. (*iv*) For such magnetosome, free to rotate inside of the cell, $W_V \ll kT$ for magnetosomes of that size, $B_e\mu \approx kT$, most generally found in biological systems, and the interaction of magnetic fields on such systems cannot affect biology. (*v*) If the cytoplasm viscosity is 100 times that of water, as indicated by some measurements (1), $W_V \ll kT$ for any magnetosome and the fields cannot affect biology. (*vi*) Though theoretical signal-to-noise ratios increase with the square root of the number M of independent free magnetosome detectors in a cell, $(S/N)_M \geq 1$ only for very large numbers, M, of very large domains, of which a high portion are properly aligned to exercise a biological function insensitive to low-frequency perturbations; moreover, a special summation mechanism must operate.

In conclusion, 60-Hz magnetic fields weaker than 5 μT cannot be expected to generate biological effects through their interaction with biological magnetite because the small effects of the fields will always be masked by thermal agitation noise. However, the arguments presented here do not preclude effects from larger 60-Hz fields.

Appendix A: Signal-to-Noise Ratio

From standard information theory (3), the information transfer in bits, M, over a period, t, can be written, $M \geq (Wt)\log_2(1 + P/N) + \log_2\varepsilon$, where P/N is the signal-to-noise power, W is the channel bandwidth, and $\varepsilon < 1$ is the error probability. The error probability per bit in the presence of thermal noise can then be taken, setting $Wt = 1$, as $\varepsilon = 0.5/(1 + S/kT)$, where S is the signal energy.

Then, for detectors that select one of two conditions, the probability of the detector answer being right is $p = 1 - 0.5/(1 + S/kT)$. For a set of K detectors of which Q are sensitive to the signal and for $S \ll kT$, the number required so that the excess that is "right" will equal the standard error is $K = (K/Q)^2 \cdot (kT/S)^2$, and, for $K = Q$, $(S/N)_K = \sqrt{K}\,(S/kT)$.

Appendix B: The Nernst Equation

Taking the number of closed gates in the membrane as P_0 and the number of open gates as $P_1 = P - P_0$, from the partition,

$$P_0 = P\frac{1}{1 + e^{-w/kT}} \quad \text{and} \quad P_1 = P\frac{e^{-w/kT}}{1 + e^{-w/kT}}. \qquad [12]$$

Taking the current, I_{in}, proportional to the number of open gates,

$$I = I_{in} - I_{out} = N\left[\frac{e^{-w/kT}}{1 + e^{-w/kT}} - \frac{e^{-w_0/kT}}{1 + e^{-w_0/kT}}\right], \qquad [13]$$

where N is a proportionality constant and $w = w_0 + dw$, where dw is the energy added by the field. For $dw \ll kT$,

$$I = I_0\,[e^{-dw/kT} - 1] \rightarrow I_0\left[-\frac{dw}{kT} + \frac{1}{2}\left(\frac{dw}{kT}\right)^2\right], \qquad [14]$$

where $I_0 = N(e^{-w_0/kT})/(1 + e^{-w_0/kT})^2$.

The energy of interaction, w, between the magnetic field $\mathbf{B}$ and the magnetic moment, μ, is $w = (\mathbf{B}\cdot\mu)$ and $dw = (d\mathbf{B}\cdot\mu) - B\mu \sin\theta\, d\theta$, where $\mathbf{B} = \mathbf{B}_e$, $d\mathbf{B} = \mathbf{B}_0\cos\omega t$, $\sin\theta \approx \theta_{kT} \approx (2\,kT/B_e\mu)^{1/2}$, and $d\theta = \theta_0\cos\omega t$. For $\kappa = B_e\mu$, from Eq. 3, $\theta_0 < B_0/B_e$ and we write $\theta_0 = B_0/B_e$.

For the systems of most interest, $(\mathbf{B}_0\cdot\mu) = 0$, and

$$dw = dw_0\cos\omega t \approx \theta_{kt}\theta_0 B_0\cos\omega t$$

$$= \sqrt{\frac{2kT}{B_e}}\cdot\frac{B_0}{B_e}B_0\cos\omega t \qquad [15]$$

and $dw_0/kT \approx 0.1\cos\omega t$ for the canonical system such that $B_0 = 5\mu$T and $B_e\mu = 25kT$.

We estimate the dark current, per "closed" channel, as $I_0 = (n/\tau)(a/4\pi r^2)$, where n is the difference in the density of molecules across the membrane, $a = 25 \times 10^{-20}$ m^2 is the area of a passage through the membrane, and $\tau = r^2(6\pi\eta\alpha/2kT) = 0.08$ s, for is the time the molecule, with a radius $\alpha = 5 \times 10^{-10}$ m, takes to diffuse a distance equal to cellular radius, $r = 10$ μm, by Brownian motion. For $n = (dV/ed)4\pi\varepsilon_0 r^2 = 10^6$, from the concentration of singly charged ions responsible for the potential difference of about $dV = 0.1$ V across the cell membrane with a thickness $d = 10^{-8}$ m, we find $I_0 = 0.0026$ molecule per s. Though the uncertainties in the estimate are large, we consider our use of $I_0 = 1$ per s per channel conservative.

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