Dynamical phase transition in the open Dicke model

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Parameters of Bose-Einstein condensate. A cigar-shaped Bose-Einstein condensate (BEC) with Thomas-Fermi radii $(3.1, 3.3, 26.8)\,\mu$m and $N_a \approx 10^5$ ^{87}Rb-atoms, prepared in the upper hyperfine component of the ground state $|F = 2, m_F = 2\rangle$, is confined by three centimeter-sized solenoids [1, 2] arranged in a quadrupole Ioffe configuration [3], thus providing a magnetic trap with a nonzero bias field parallel to the z-axis with trap frequencies $\omega/2\pi = (215.6 \times 202.2 \times 25.2)$ Hz. The particle number in the atomic sample is measured by absorption imaging and by recording the cavity resonance shift due to forward scattering of a probe beam, coupled through one of the cavity mirrors. We thus find less than 10 % shot to shot fluctuations.

Cavity parameters. The high finesse of the standing wave cavity ($\mathcal{F} = 3.44 \pm 0.05 \times 10^5$) together with the narrow beam waist ($w_0 \approx 31.2 \pm 0.1\,\mu$m) yield a Purcell factor $\eta_c \equiv \frac{24\,\mathcal{F}}{\pi\,k^2 w_0^2} \approx 44 \pm 0.7$ ($k \equiv 2\pi/\lambda$, and $\lambda =$ wavelength of the pump light) [4, 5]. Due to the mirror separation of 48.93 ± 0.002 mm, the cavity exhibits an extremely low bandwidth of $\kappa = 2\pi \times 4.45 \pm 0.05$ kHz, which is smaller than $2\,\omega_{\text{rec}}$, with $\omega_{\text{rec}} = \hbar k^2/2m = 2\pi \times 3.55$ kHz denoting the recoil frequency. The cavity is oriented parallelly to the z-axis, such that the BEC is well matched to the mode volume of its TEM_{00}-modes with its elongated axis aligned parallel to the cavity axis. Note that the BEC extends across approximately 130 lattice sites of the intra-cavity standing wave and thus position fluctuations in the BEC preparation process yield only small population fluctuations between adjacent sites. For a uniform atomic sample the resonance frequency for right (+) and left (−) circular photons is shifted due to the dispersion of a single atom by an amount $\Delta_\pm/2$ with $\Delta_\pm = \frac{1}{2}\eta_c\kappa\,\Gamma\left(\frac{f_{1,\pm}}{\delta_1} + \frac{f_{2,\pm}}{\delta_2}\right)$ and $\delta_{1,2}$ denoting the pump frequency detunings with respect to the relevant atomic $\text{D}_{1,2}$ lines at 795.0 nm and 780.2 nm [5]. $\Gamma = 2\pi \times 6$ MHz is the intra-cavity field decay rate and the decay rate of the 5P state of ^{87}Rb, respectively. The prefactors $f_{1,\pm}$ and $f_{2,\pm}$ account for the effective line strengths of the D_1- and D_2-line components connecting to the $|F = 2, m_F = 2\rangle$ ground state. The values of these factors are $(f_{1,-}, f_{2,-}) = (\frac{2}{3}, \frac{1}{3})$ and $(f_{1,+}, f_{2,+}) = (0, 1)$.

The quoted expressions for $\Delta_\pm$ use the rotating wave approximation and assume that the contributions from different transition components may be added. Finite size effects of the atomic sample and deviations of the intra-cavity field geometry from a plane wave are neglected. A more realistic value, used in our work, is obtained experimentally: The dispersive resonance shift for N_a atoms and left polarized light $\delta_- = \frac{1}{2}N_a\Delta_-$ is measured by coupling a weak left polarized probe beam through one of the cavity mirrors to the TEM_{00}-mode. Its frequency is tuned across the resonance with and without atoms. At sufficiently low power levels of the probe the resonance is not affected by spatial structuring of the atoms due to back-action of the cavity field and hence merely results from the dispersion of the homogeneous sample. Accounting for the particle number N_a, known from absorption imaging with about 10 % precision, we find $\Delta_- \approx -2\pi \times 0.36 \pm 0.04$ Hz and $\Delta_+ \approx -2\pi \times 0.16 \pm 0.02$ Hz. Hence with $N_a = 10^5$ atoms $\delta_- = 2\pi \times 18$ kHz, which amounts to $4\,\kappa$, i.e., the cavity operates well within the regime of strong cooperative coupling.

Pump lattice parameters. The pump lattice with $w_p = 80 \pm 0.4\,\mu$m radius is oriented along the y-axis, i.e., perpendicularly with respect to its weakly confined z-axis. Its linear polarization is oriented parallelly to the x-axis and it operates at a wavelength $\lambda = 803$ nm, i.e., with 8 nm detuning to the red side of the D_1-transition of ^{87}Rb. The pump strength is specified in terms of the magnitude of the antinode light shift $\varepsilon_p \geq 0$ induced by the pump lattice in units of the recoil energy $E_{\text{rec}} = \hbar\omega_{\text{rec}}$. In order to calibrate the pump strength, the BEC is adiabatically loaded into the pump lattice and the excitation spectrum is recorded and compared to a numerical band calculation. This yields a relative (absolute) uncertainty of ε_p of 1% (10%).

Our experiments require to tune the pump frequency with sub-kilohertz resolution across the resonance frequency of the TEM_{00}-mode interacting with the BEC. This is accomplished as follows (see also Ref. [6]): A reference laser operating at 803 nm is locked on resonance with a TEM_{11}-mode, which provides a cloverleaf-shaped transverse

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profile. This mode exhibits a nodal line at the cavity axis such that the interaction with the BEC, which is positioned well in the center of the TEM_{00}-mode, is suppressed with respect to the TEM_{00}-mode by a geometrical factor 9×10^{-5}. Adjusting right circular polarization for the reference beam and hence σ^+-coupling of the BEC yields another suppression factor ≈ 0.43. The pump laser, matched to couple the TEM_{00}-mode, is locked with an offset frequency of about 2.5 GHz to the reference laser. This offset is tunable over several MHz such that the vicinity of the resonance frequency of the TEM_{00}-mode can be accessed.

Detection of cavity photons. The light leaking out of the cavity is split into orthogonal circular polarization components and the photons of each component are counted with 56% quantum efficiency. The right circular photons, predominantly belong to the TEM_{11}-mode used to operate the stabilization of the pump beam frequency with respect to the cavity resonance (for details see Ref. [6]). Only a small fraction of these photons arises in the TEM_{00}-mode and results from the scattering of pump photons. In our experiments the ratio between left and right circularly polarized photons found in the TEM_{00}-mode was about 4. The photon counting signal is binned within a time-window of 4μs and a variable number of data sets is averaged.

Mean field model. We consider a BEC of two-level atoms scattering light from an external standing wave mode with the scalar electric field amplitude $\alpha_p(t)\cos(ky)$ (pump mode) into a cavity mode with the scalar electric field $\alpha(t)\cos(kz)$. Neglecting collisional interaction the system is described by the set of mean field equations [7]

$$\begin{aligned} i\,\frac{\partial}{\partial t}\psi(y,z,t) &= \left(-\frac{\hbar}{2m}\left[\frac{\partial^2}{\partial^2 y}+\frac{\partial^2}{\partial^2 z}\right]+\Delta_0|\alpha(t)\cos(kz)+\alpha_p(t)\cos(ky)|^2\right)\psi(y,z,t) \\ i\,\frac{\partial}{\partial t}\alpha(t) &= \left(-\delta_c+\Delta_0\langle\cos^2(kz)\rangle_\psi - i\kappa\right)\alpha(t)+\Delta_0\langle\cos(kz)\cos(ky)\rangle_\psi\,\alpha_p(t)\,, \end{aligned} \tag{1}$$

with the matter wave function ψ normalized to N_a particles, and the electric fields normalized such that $|\alpha_p|^2$ and $|\alpha|^2$ denote the number of photons in the pump mode and the cavity mode, respectively. The light shift per intra-cavity photon is denoted by Δ_0. $\langle\ldots\rangle_\psi$ indicates integration over the BEC volume weighted with $|\psi|^2$. A plane wave expansion of $\psi(y,z,t)$ with respect to the relevant (y,z)-plane yields the corresponding scaled momentum space equations

$$\begin{aligned} i\,\frac{\partial}{\partial t}\phi_{n,m} &= \omega_{\mathrm{rec}}\left(n^2+m^2-\frac{1}{2}|\beta|^2-\frac{1}{2}\epsilon_p\right)\phi_{n,m} \\ &- \frac{1}{4}\omega_{\mathrm{rec}}\,|\beta|^2\left(\phi_{n,m-2}+\phi_{n,m+2}\right)-\frac{1}{4}\omega_{\mathrm{rec}}\,\epsilon_p\left(\phi_{n-2,m}+\phi_{n+2,m}\right) \\ &+ \frac{1}{2}\omega_{\mathrm{rec}}\sqrt{\epsilon_p}\,\mathrm{Im}(\beta)\left(\phi_{n-1,m-1}+\phi_{n+1,m-1}+\phi_{n-1,m+1}+\phi_{n+1,m+1}\right) \\ i\,\frac{\partial}{\partial t}\beta &= \left[-\delta_{\mathrm{eff}}+\frac{1}{2}N_a\Delta_0\sum_{n,m}\mathrm{Re}[\phi_{n,m}\phi^*_{n,m+2}]-i\kappa\right]\beta \\ &- i\,\frac{1}{8}N_a\Delta_0\sqrt{\epsilon_p}\sum_{n,m}\phi_{n,m}(\phi^*_{n+1,m+1}+\phi^*_{n+1,m-1})+\phi^*_{n,m}(\phi_{n+1,m+1}+\phi_{n+1,m-1})\,, \end{aligned} \tag{2}$$

with $\phi_{n,m}$ denoting the normalized ($\sum_{n,m}|\phi_{n,m}|^2=1$) amplitude of the momentum state $(n,m)\,\hbar k$. Upon the assumption of negative Δ_0 the intra-cavity field β is scaled such that $|\beta|^2=-|\alpha|^2\Delta_0/\omega_{\mathrm{rec}}$ denotes the magnitude of the induced anti-node light-shift in units of the recoil energy. The pump strength parameter $\epsilon_p \equiv -|\alpha_p|^2\Delta_0/\omega_{\mathrm{rec}}$ is defined as the antinode light-shift induced by the pump wave in units of the recoil energy. The effective detuning is $\delta_{\mathrm{eff}} \equiv \delta_c - \frac{1}{2}N_a\Delta_0$ with the detuning δ_c between the pump frequency and the empty cavity resonance. Eq. (2) is the mean-field approximation to the Heisenberg equation for a Dicke Hamiltonian generalized to the case of a collection of N_a identical multi-level systems each consisting of the momentum states $\phi_{n,m}$. The additional term $i\kappa$ accounts for damping of the intra-cavity light field. The conventional two-level Dicke-Hamiltonian [8] arises, if only the two most relevant matter modes $\phi_{0,0}$ and $\phi \equiv \frac{1}{2}\left(\phi_{1,1}+\phi_{1,-1}+\phi_{-1,1}+\phi_{-1,-1}\right)$ are accounted for. In our recoil selective cavity set-up, this approximation is well justified since initially the atoms populate the BEC mode $\phi_{0,0}$ and near resonant coupling via the cavity is practically limited to ϕ, which corresponds to the motional state excited, if a single photon from the standing pump wave is scattered into the cavity. Note that within the sub-space spanned by the states $\phi_{\pm1,\pm1}$ the superposition ϕ represents the minimal energy state, because the associated density grating localizes the particles in the minima of the intra-cavity lattice potential induced by photon scattering into the cavity.

A steady state solution of Eqs. (2) is the homogeneous phase $\beta = 0$ and $\phi_{n,m} = \delta_{n,0}\,\delta_{m,0}$, which describes the unperturbed condensate with no photons in the cavity. The stability properties of this solution may be studied by reducing Eqs. (2) to the two matter modes $\phi_{0,0}$ and ϕ. Switching to a basis such that the condensate has zero energy and neglecting its depletion, i.e., $\phi_{0,0} \approx 1$, one finds the system of linear equations

$$i\,\frac{\partial}{\partial t}\begin{pmatrix} \beta \\ \beta^* \\ \phi \\ \phi^* \end{pmatrix} = \begin{pmatrix} -\delta_{\mathrm{eff}} - i\kappa & 0 & i\,\lambda_1 & i\,\lambda_1 \\ 0 & \delta_{\mathrm{eff}} - i\kappa & i\,\lambda_1 & i\,\lambda_1 \\ -i\lambda_2 & i\lambda_2 & 2\omega_{\mathrm{rec}} & 0 \\ i\lambda_2 & -i\lambda_2 & 0 & -2\omega_{\mathrm{rec}} \end{pmatrix}\begin{pmatrix} \beta \\ \beta^* \\ \phi \\ \phi^* \end{pmatrix} \tag{3}$$

with the coupling parameters $\lambda_1 \equiv -\frac{1}{2}\,N_a\Delta_0\,\sqrt{2\epsilon_p}$ and $\lambda_2 \equiv \frac{1}{2}\,\omega_{\mathrm{rec}}\,\sqrt{\epsilon_p/2}$, which formally resembles a Schrödinger equation for a four-level system with a non-Hermitian Hamiltonian [9]. It is equivalent to the mean field approximation of the Heisenberg equation obtained from the Dicke Hamiltonian after applying the Holstein-Primakoff transformation, introducing the thermodynamic limit, and adding cavity dissipation [10]. If the imaginary part of one of the eigenvalues of the matrix on the right hand side of Eq. (3) is positive, an exponential instability arises and hence the system is rapidly driven away from the homogeneous phase. For negative detuning $\delta_{\mathrm{eff}} < 0$ the instability boundary is the known equilibrium Dicke phase boundary.

In order to compare experimental observations with the model in Eq. (2) and Eq. (3), the experimental parameters $\Delta_\pm, \varepsilon_p$ and the model parameters Δ_0, ϵ_p must be connected accounting for the fact that in the model two-level atoms are assumed and the vectorial character of the electric field is neglected. In the experiment, the strongest coupling to the atoms arises for left circular light with respect to the natural quantization axis fixed by the magnetic offset field along the z-axis. Hence, we identify $\Delta_0 = \Delta_-$. Inside the cavity, the linear $\hat{x}$-polarization of the pump beam may be decomposed into equally strong left and right circular components with respect to the z-axis. Only the left circular component can scatter into the left circularly polarized cavity mode. Hence, the light shift ε_p induced by the pump beam in the experiment is related to the number of pump photons $|\alpha_p|^2$ used in the model description by $\varepsilon_p = -|\alpha_p|^2(\Delta_+ + \Delta_-)/\omega_{\mathrm{rec}}$ and thus $\varepsilon_p/\epsilon_p = (\Delta_+ + \Delta_-)/\Delta_0 = 1.44$. A more involved description, which is deferred to forthcoming work, should account for two orthogonal polarization modes of the cavity operating with different effective detunings.

In Fig. 2(c) of the main text the maximum of the imaginary parts of the four eigenvalues of the matrix on the right hand side of Eq. (3) is plotted versus δ_{eff} and ε_p. For $\delta_{\mathrm{eff}} = \pm 2\pi \times 20$ kHz the real and imaginary parts of all eigenvalues are plotted in Fig. 2(d) and (e), respectively. Figures 3(b) and (c) in the main text were obtained by solving Eqs. (2) including all modes with $-4 \leq n, m \leq 4$. A small initial deviation from $\phi_{0,0}(0) = 1$ is required in order to leave the unstable homogeneous phase. In the experiment, this deviation is naturally provided by thermal or quantum fluctuations. We assumed that the first excited modes $(\pm 1, \pm 1)\,\hbar k$ are populated according to a Boltzman factor with a temperature $T = 0.2\,T_c$ (T_c = critical temperature of the BEC). Hence, we set $\phi_{0,0}(0) = \cos(\theta)$, $\phi_{\pm 1,\pm 1}(0) = e^{i\xi}\sin(\theta)/2$ with $\theta = \arctan(2\,e^{-\hbar\omega_{\mathrm{rec}}/k_B T})$, and $\phi_{n,m}(0) = 0$ if $|n|, |m| > 1$. The choice of equal $\phi_{\pm 1,\pm 1}(0)$ corresponds to low energy excitations for which the atoms are localized in the intra-cavity light shift potential with a spatial phase determined by the positions of the cavity mirrors and the position of the pump wave. The choice of ξ determines the phase of the oscillations in the upper trace of Fig. 3(c), which are washed out, if an average over ξ is applied. For $T < T_c$, the exponents of the power law behavior in this figure does not show notable dependence on ξ or the value of T. Note that the higher orders $\phi_{n,m}$ with $|n|, |m| > 1$ remain small in the calculation of Fig. 3(b) and the hysteresis is also reproduced in the simplified case $-1 \leq n, m \leq 1$, which corresponds to a description in terms of the Dicke model for a collection of two-level systems.

Power law scaling, relation to Kibble Zurek model. We consider a quench across the equilibrium Dicke phase transition implemented by tuning the pump strength parameter $\varepsilon_p(t) = \varepsilon_{p,c} + (-1)^\mu \frac{\Delta\varepsilon}{\tau_Q}\,(t - t_c)$ across the critical value $\varepsilon_p(t_c) = \varepsilon_{p,c}$, with $\Delta\varepsilon$ denoting the interval of $\varepsilon_p(t)$ scanned during the quench time τ_Q. For $\mu \in \{1, 2\}$ we identify $\varepsilon_{p,c} = \varepsilon_{p,\mu}(\infty)$, where $\varepsilon_{p,\mu}(\infty)$ denote the threshold values found in a quench with negative ($\mu = 1$) or positive ($\mu = 2$) slope in the limit of infinite τ_Q. According to our experimental observations, the quantities $\Delta\varepsilon_{p,\mu}(\tau_Q) \equiv \varepsilon_{p,\mu}(\tau_Q) - \varepsilon_{p,\mu}(\infty)$ follow power laws, i.e., $\Delta\varepsilon_{p,\mu} \propto \tau_Q^{n_\mu}$ with $n_1 = -0.57$ and $n_2 = -0.85$. At this point we argue in the spirit of the Kibble Zurek model [11], that the time lag between the threshold value for the transition to occur in a quench of duration τ_Q and the equilibrium critical point, i.e. $\tau_Q\,\Delta\varepsilon_{p,\mu}(\tau_Q)$, equals the relaxation time τ of the system, and hence $\tau \propto \tau_Q^{n_\mu+1}$. As a second input from the Kibble Zurek scenario, we assume a power law scaling $\tau \propto \Delta\varepsilon_{p,\mu}^{-z_\mu\nu_\mu}$. This results in the relation $z_\mu\nu_\mu = -(1 + 1/n_\mu)$, and hence $z_1\nu_1 = 0.75$ and $z_2\nu_2 = 0.18$.

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