Supplementary Material for "Quantile rank maps: a new tool for understanding individual brain development" by Chen, Kelly, Castellanos, He, Zuo and Reiss

Appendix A. Critical value for the simulations

Asymptotically, under the null hypothesis of no group difference, $\sqrt{n}D_n$ has the same distribution as $\sup_{\tau\in[0,1]} |B(\tau)|$ where $B(\cdot)$ is the Brownian bridge (Bhattacharya and Waymire, 1990). But as noted in the text, it is necessary to obtain a recalibrated K-S critical value. To do this for simulated data based on the ABIDE data, we followed a modified version of the resampling procedure described in the "Recalibrated Kolmogorov-Smirnov test for group differences" section of the main text:

1. We randomly chose $M = 100$ of the 6216 edges (pairs of ROIs) for the ABIDE data set.
2. For $m = 1, \ldots, M$, for $p = 1, \ldots, P = 100$:
   (a) We chose quantiles $\tau_1^{(mp)}, \ldots, \tau_{735}^{(mp)}$ by independent draws from the Uniform$(0, 1)$ distribution.
   (b) A simulated "null" data set (i.e., with no group difference) $y_1^{(mp)}, \ldots, y_{735}^{(mp)}$ was generated by substituting the observed ages $t_1, \ldots, t_{735}$, the random quantiles $\tau_1^{(mp)}, \ldots, \tau_{735}^{(mp)}$, and $\hat{\mu}_m(\cdot), \hat{\sigma}_m(\cdot), \hat{\nu}_m(\cdot)$ (i.e., the estimated age-varying distribution of functional connectivity for the mth edge, previously obtained from the real-data control group) into equation (6).

(c) Treating $y_1^{(mp)}, \ldots, y_{391}^{(mp)}$ as the "typical group" data and $y_{392}^{(mp)}, \ldots, y_{735}^{(mp)}$ as the "atypical group" data, we fitted the LMS model (2) to the former data and used the result to compute the K-S statistic $D^{(mp)}$ for the latter.

3. The 95th percentile of $\{D^{(mp)} : m = 1, \ldots, M, p = 1, \ldots, P\}$ was used as the critical value for rejection at level .05.

Appendix B. Generating simulated data

To evaluate the performance of the recalibrated K-S test in the "linear" case, we randomly chose 20 edges for which the real-data control group quantile curves (see Figure A1) were approximately linear. For each edge we generated 250 replicates for $\delta = 0$ (5000 replicates in total), and 50 replicates for each $\delta > 0$ (1000 in total). For each replicate we drew a set of random quantiles as in step 2a of Appendix A; then simulated data as in step 2b, but with random deviations added to the "atypical" data (the last 344 responses), as described in the main text; and finally computed the K-S statistic as in step 2c. Type-I error rates and power estimates were based on comparing these K-S statistics with the critical value. This entire procedure was repeated with 20 randomly chosen edges for which the real-data control group quantile curves were markedly nonlinear (see Figure A2).

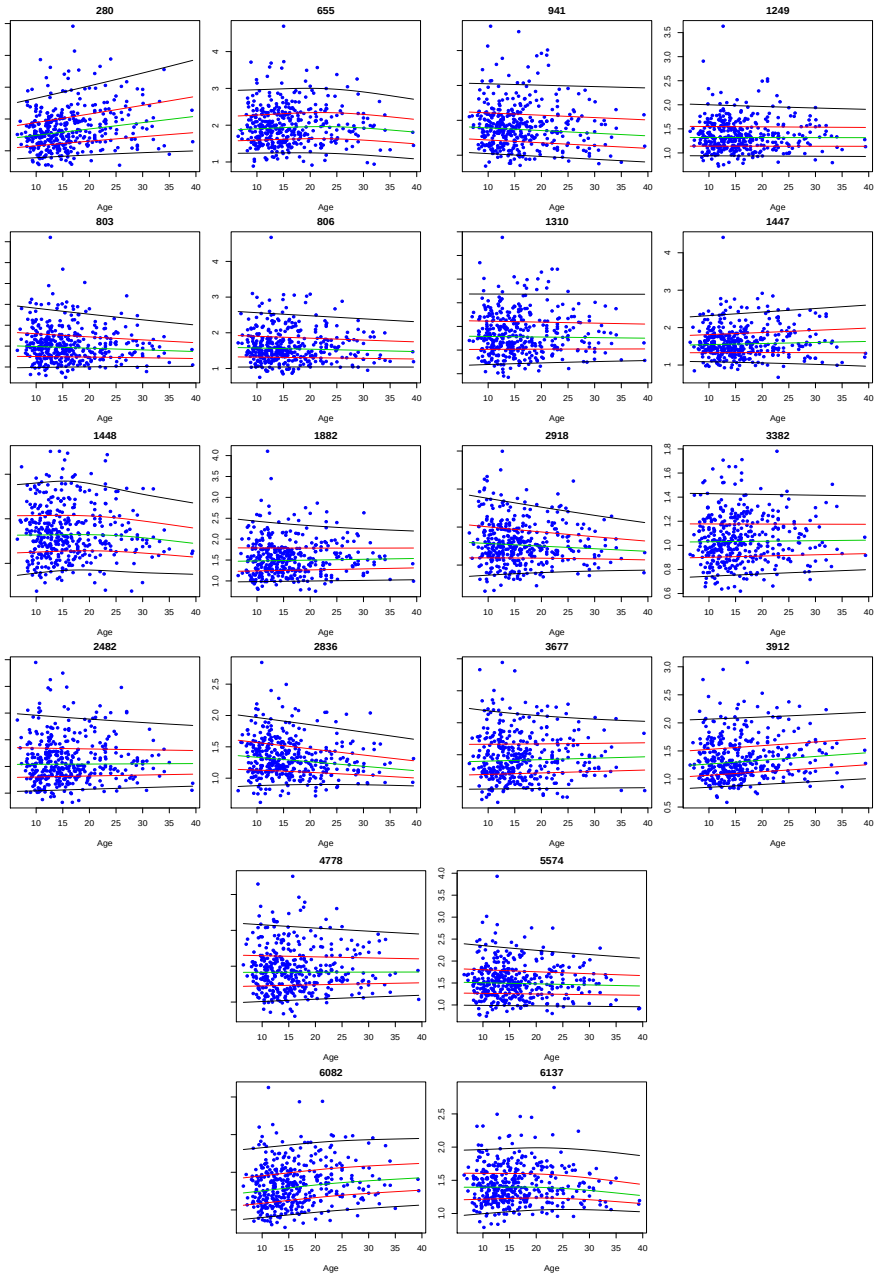

Figure A1: iFC data and estimated quantile curves for the ABIDE control group at 20 randomly selected edges for which the curves are approximately linear.

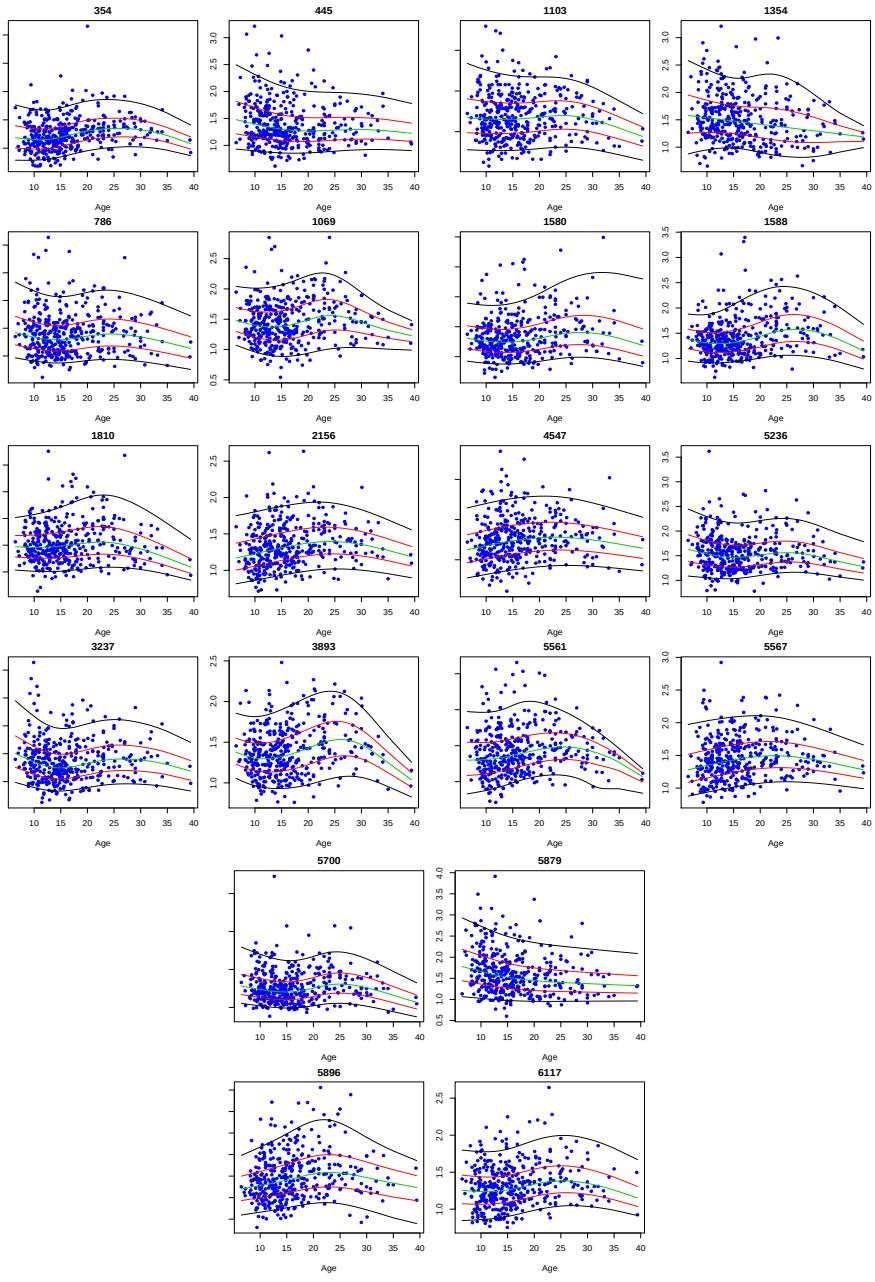

Figure A2: iFC data and estimated quantile curves for the ABIDE control group at 20 randomly selected edges for which the curves show noticeable nonlinearity.