

Appendix A. Definitions of the Law of Likelihood, and estimated, profile, and modified profile likelihoods

Law of Likelihood: If hypothesis A implies that the probability that a random variable X takes the value x is $p_A(x)$, while hypothesis B implies that the probability is $p_B(x)$, then the observation $X = x$ is evidence supporting A over B if and only if $p_A(x) > p_B(x)$, and the likelihood ratio, $p_A(x)/p_B(x)$, measures the strength of that evidence [1].

Estimated likelihood: Given the joint likelihood $L(\psi, \lambda)$, the estimated likelihood of ψ is:

$$L_E(\psi) = L(\psi, \hat{\lambda}),$$

where $\hat{\lambda}$ is the maximum likelihood estimates of λ . The estimated likelihood is also called a ‘pseudo-likelihood’ in [2].

Profile likelihood: Given the joint likelihood $L(\psi, \lambda)$, the profile likelihood of ψ is:

$$L_P(\psi) = \sup_{\lambda} L(\psi, \lambda),$$

where the supremum is with respect to λ and is performed at each value of ψ , which is held fixed.

Modified profile likelihood: Given the joint likelihood $L(\psi, \lambda)$ and the profile likelihood $L_P(\psi)$, the modified profile likelihood of ψ is:

$$L_M(\psi) = L_P(\psi) \left| j_{\lambda\lambda}(\psi, \hat{\lambda}_{\psi}) \right|^{-\frac{1}{2}} \left| \frac{\partial \hat{\lambda}_{\psi}}{\partial \hat{\lambda}} \right|^{-1} = L_P(\psi) \frac{\left| j_{\lambda\lambda}(\psi, \hat{\lambda}_{\psi}) \right|^{\frac{1}{2}}}{\left| l_{\lambda; \hat{\lambda}}(\psi, \hat{\lambda}_{\psi}) \right|},$$

where $\hat{\lambda}_{\psi}$ are the maximum likelihood estimates of λ at a fixed value of ψ ,

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$\hat{\psi}$ is the maximum likelihood estimate of ψ ,

$$j_{\lambda\lambda}(\psi, \hat{\lambda}_{\psi}) = -\frac{\partial^2}{\partial \lambda^2} l(\psi, \hat{\lambda}_{\psi}; \hat{\psi}, \hat{\lambda}) \Big|_{\lambda=\hat{\lambda}_{\psi}} \quad \text{and}$$

$$l_{\lambda; \hat{\lambda}}(\psi, \hat{\lambda}_{\psi}) = \frac{\partial^2}{\partial \lambda \hat{\lambda}} l(\psi, \hat{\lambda}_{\psi}; \hat{\psi}, \hat{\lambda}) \Big|_{\lambda=\hat{\lambda}_{\psi}}.$$

References

1. Hacking I. Logic of statistical inference. New York: Cambridge University Press; 1965.
2. Gong G, Samaniego FJ. Pseudo maximum likelihood estimation: theory and applications. The Annals of Statistics. 1981;9(4):861–869.