Supporting Information of

Excess Relative Risk as an Effect Measure in Case-Control Studies of Rare Diseases

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S6 Exhibit. A proof that if the exposure under study is only associated with a specific disease entity, excess relative risk (ERR) for the exposure and this disease entity will be greater than that for the exposure and the disease as a whole.

Assumed that the disease under study is composed of two disease entities (D_{I} and D_{II}) and that the exposure under study (E) has no effect whatsoever on the occurrence of D_{II}, that is, $\Pr(D_{\mathrm{II}}=1 \mid E=1)=\Pr(D_{\mathrm{II}}=1 \mid E=2)$. We see that the excess risk ratio quantifying the relation between E and D (the D_{I} and D_{II} combined) is less than that between E and D_{I}:

$$
\begin{aligned}
\mathrm{ERR}_{\mathrm{I+II}} &= \frac{\Pr(D=1 \mid E=1)-\Pr(D=1 \mid E=2)}{\Pr(D=1 \mid E=2)} \\
&= \frac{\left[\Pr(D_{\mathrm{I}}=1 \mid E=1)+\Pr(D_{\mathrm{II}}=1 \mid E=1)\right]-\left[\Pr(D_{\mathrm{I}}=1 \mid E=2)+\Pr(D_{\mathrm{II}}=1 \mid E=2)\right]}{\Pr(D_{\mathrm{I}}=1 \mid E=2)+\Pr(D_{\mathrm{II}}=1 \mid E=2)} \\
&= \frac{\Pr(D_{\mathrm{I}}=1 \mid E=1)-\Pr(D_{\mathrm{I}}=1 \mid E=2)}{\Pr(D_{\mathrm{I}}=1 \mid E=2)+\Pr(D_{\mathrm{II}}=1 \mid E=2)} \\
&< \frac{\Pr(D_{\mathrm{I}}=1 \mid E=1)-\Pr(D_{\mathrm{I}}=1 \mid E=2)}{\Pr(D_{\mathrm{I}}=1 \mid E=2)} \\
&= \mathrm{ERR}_{\mathrm{I}}.
\end{aligned}
$$