

The analytical derivation of the characteristic resistance, R_1

At the inlet of the root vessel of a truncated arterial crown, the impedance with the Womersley velocity profile is

$$Z(0, \omega) = \frac{\lambda}{i\omega C} \frac{i\omega C Z(L, \omega) \cosh(\lambda L) + \lambda \sinh(\lambda L)}{\lambda \cosh(\lambda L) + i\omega C \sinh(\lambda L) Z(L, \omega)}, \quad (1)$$

where $Z(L, \omega)$ is a known impedance at the outlet of the root vessel and $\lambda = \sqrt{\frac{-\rho C \omega^2}{A_0(1-F_J)}}$ is the wave number as shown in the main text. According to the relation between the trigonometric functions and the hyperbolic functions, equation (1) can be rewritten as

$$Z(0, \omega) = \frac{\lambda}{\omega C} \frac{\frac{\omega C}{\lambda} Z(L, \omega) - \tan(i\lambda L)}{1 + \frac{\omega C}{\lambda} Z(L, \omega) \tan(i\lambda L)} = \frac{\lambda}{\omega C} \tan(\phi - i\lambda L), \quad (2)$$

where ϕ is defined by $\tan \phi = \frac{\omega C Z(L, \omega)}{\lambda}$. Let $a = \text{Re}(\phi - i\lambda L)$, $b = \text{Im}(\phi - i\lambda L)$, then we have

$$Z(0, \omega) = \frac{\lambda}{\omega C} \tan(a + ib) = \frac{\lambda}{\omega C} \frac{\tan a + i \tanh b}{1 - i \tan a \tanh b}. \quad (3)$$

When the frequency ω is large, the Womersley number, $W = r_0 \sqrt{\omega \rho / \mu}$, is also large, therefore, we have the following approximation:

$$F_J(W) \sim \frac{2}{W i^{0.5}} \left(1 + \frac{1}{2W}\right), \quad \lambda \sim \sqrt{\frac{\rho C}{A_0}} \left(-i\omega + \sqrt{\frac{\omega \gamma}{\rho}} \frac{1+i}{4}\right), \quad (4)$$

where $\gamma = \frac{8\pi\mu}{A_0}$. Then we have $\text{Im}(i\lambda L) \rightarrow \infty$ and $|b| \rightarrow \infty$ as $\omega \rightarrow \infty$. Thus, $\tanh b \rightarrow \frac{|b|}{b}$ and $\frac{\tan(a) + i \tanh(b)}{1 - i \tan(a) \tanh(b)} \rightarrow i$ as $\omega \rightarrow \infty$. Finally, we can obtain

$$R_1 = \lim_{\omega \rightarrow \infty} Z(0, \omega) = \sqrt{\frac{\rho}{A_0 C}}. \quad (5)$$

With the expression of the pulse wave speed, $c = \sqrt{\frac{2Eh}{3\rho r_0(x)}}$, and the area compliance, $C = \frac{3A_0 r_0}{2Eh}$, in the main text, we can also arrive at

$$R_1 = \frac{\rho c}{A_0}. \quad (6)$$