

(*In all ensuing code, A is a constant used for simplicity given by A = (a*omega)/(2D)*)

(*Basic solution (no assumptions)*)

t = DSolve[{p'[r] == A*(r - (rc^2)/r) - ((1)*Log[(rc - r)^2 + 1]/1)/r + (2*Sqrt[1]*rc*ArcTan[(rc - r)/Sqrt[1]]/r), p[rc] == 0}, p[r], r]

{p[r] →

$$\frac{1}{12} \left(6Ar^2 + iA\sqrt{1}\pi^2rc - 6Arc^2 - 12iA\sqrt{1}\pi rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + 24iA\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]^2 + 24iA\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] + 12A\sqrt{1}\pi rc \operatorname{Log}[2] - 12A\sqrt{1}\pi rc \operatorname{Log}\left[1 + e^{-2i\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]}\right] + 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] \operatorname{Log}\left[1 + e^{-2i\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]}\right] + 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \operatorname{Log}\left[1 - e^{2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]}\right] - 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] \operatorname{Log}\left[1 - e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] - 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \operatorname{Log}\left[1 - e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] - 12Arc^2 \operatorname{Log}[r] + 12A\sqrt{1}\pi rc \operatorname{Log}\left[\frac{1}{\sqrt{1+(r-rc)^2}}\right] - 12A \operatorname{Log}[r] \operatorname{Log}\left[1 + \frac{(r-rc)^2}{1}\right] + 12A \operatorname{Log}[r] \operatorname{Log}\left[-i\sqrt{1} + r - rc\right] + 12A \operatorname{Log}[r] \operatorname{Log}\left[i\sqrt{1} + r - rc\right] - 12A \operatorname{Log}\left[-i\sqrt{1} + r - rc\right] \operatorname{Log}\left[-\frac{ir}{\sqrt{1}-irc}\right] - 12A \operatorname{Log}\left[i\sqrt{1} + r - rc\right] \operatorname{Log}\left[\frac{ir}{\sqrt{1}+irc}\right] + 12Arc^2 \operatorname{Log}[rc] - 12A \operatorname{Log}\left[-i\sqrt{1}\right] \operatorname{Log}[rc] - 12A \operatorname{Log}\left[i\sqrt{1}\right] \operatorname{Log}[rc] + 12A \operatorname{Log}\left[-i\sqrt{1}\right] \operatorname{Log}\left[-\frac{irc}{\sqrt{1}-irc}\right] + 12A \operatorname{Log}\left[i\sqrt{1}\right] \operatorname{Log}\left[\frac{irc}{\sqrt{1}+irc}\right] - 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \operatorname{Log}\left[\frac{rc}{\sqrt{1}\sqrt{1+\frac{rc^2}{1}}}\right] + 24A\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right]\right] + 12iA\sqrt{1}rc \operatorname{PolyLog}\left[2, -e^{-2i\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]}\right] - 12iA\sqrt{1}rc \operatorname{PolyLog}\left[2, e^{2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]}\right] + 12iA\sqrt{1}rc \operatorname{PolyLog}\left[2, e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] + 12A \operatorname{PolyLog}\left[2, \frac{\sqrt{1}}{\sqrt{1}-irc}\right] - 12A \operatorname{PolyLog}\left[2, \frac{\sqrt{1}+i(r-rc)}{\sqrt{1}-irc}\right] + 12A \operatorname{PolyLog}\left[2, \frac{\sqrt{1}}{\sqrt{1}+irc}\right] - 12A \operatorname{PolyLog}\left[2, \frac{\sqrt{1}-ir+irc}{\sqrt{1}+irc}\right] \right)$$

(*Simplifying assumptions (domains of validity) *)

u = FullSimplify[t, rc > 0 && r > 0 && rc >= r && 1 > 0 && A > 0 && p > 0]

$$\left\{ \left\{ p[r] \rightarrow \frac{1}{12} A \left(6\pi^2 + i\sqrt{1}\pi^2rc + 6(r-rc)(r+rc) + 6 \left(4i\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]^2 + 2\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] \left(-i\pi + 2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] + \operatorname{Log}[4] - 2\operatorname{Log}\left[1 - e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] + \operatorname{Log}[1] - 2\operatorname{Log}\left[\sqrt{1} + i(r-rc)\right] \right) + 2\sqrt{1}\pi rc \operatorname{Log}\left[\sqrt{1} + i(r-rc)\right] - \sqrt{1}\pi rc \operatorname{Log}\left[1 + (r-rc)^2\right] - 2\operatorname{Log}[r] \operatorname{Log}\left[1 + \frac{(r-rc)^2}{1}\right] + 2rc^2 \operatorname{Log}\left[\frac{rc}{r}\right] - \operatorname{Log}[1] \operatorname{Log}\left[-i\sqrt{1} + rc\right] + 2\operatorname{Log}\left[i\sqrt{1} + r - rc\right] \operatorname{Log}\left[-i\sqrt{1} + rc\right] - i\pi \operatorname{Log}\left[1(-i\sqrt{1} + rc)\right] + i\pi \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] + \operatorname{Log}[1] \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] + 2i \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] \operatorname{Log}\left[i\sqrt{1} - r + rc\right] + 4\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \left(-i\pi + \operatorname{Log}\left[\frac{2irc}{\sqrt{1}-irc}\right] - \operatorname{Log}\left[-\frac{2irc\sqrt{\frac{1}{1+rc^2}}}{1-i\sqrt{1}r-rc+rc^2}\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right]\right) \right) \right\} + iA\sqrt{1}rc \left(\operatorname{PolyLog}\left[2, -e^{-2i\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]}\right] - \operatorname{PolyLog}\left[2, e^{2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]}\right] + \operatorname{PolyLog}\left[2, e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] \right) + A \operatorname{PolyLog}\left[2, \frac{1}{1-\frac{irc}{\sqrt{1}}}\right] + A \operatorname{PolyLog}\left[2, \frac{1}{1+\frac{irc}{\sqrt{1}}}\right] - A \operatorname{PolyLog}\left[2, 1 - \frac{r}{-i\sqrt{1} + rc}\right] - A \operatorname{PolyLog}\left[2, 1 - \frac{r}{i\sqrt{1} + rc}\right] \right\}$$

(*Further refinement*)

SolutionForm = refine[u]

refine[

$$\left\{ \left\{ p[r] \rightarrow \frac{1}{12} A \left(6\pi^2 + i\sqrt{1}\pi^2rc + 6(r-rc)(r+rc) + 6 \left(4i\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]^2 + 2\sqrt{1}rc \operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] \left(-i\pi + 2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] + \operatorname{Log}[4] - 2\operatorname{Log}\left[1 - e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] + \operatorname{Log}[1] - 2\operatorname{Log}\left[\sqrt{1} + i(r-rc)\right] \right) + 2\sqrt{1}\pi rc \operatorname{Log}\left[\sqrt{1} + i(r-rc)\right] - \sqrt{1}\pi rc \operatorname{Log}\left[1 + (r-rc)^2\right] - 2\operatorname{Log}[r] \operatorname{Log}\left[1 + \frac{(r-rc)^2}{1}\right] + 2rc^2 \operatorname{Log}\left[\frac{rc}{r}\right] - \operatorname{Log}[1] \operatorname{Log}\left[-i\sqrt{1} + rc\right] + 2\operatorname{Log}\left[i\sqrt{1} + r - rc\right] \operatorname{Log}\left[-i\sqrt{1} + rc\right] - i\pi \operatorname{Log}\left[1(-i\sqrt{1} + rc)\right] + i\pi \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] + \operatorname{Log}[1] \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] + 2i \operatorname{Log}\left[-\frac{1}{i\sqrt{1} + rc}\right] \operatorname{Log}\left[i\sqrt{1} - r + rc\right] + 4\sqrt{1}rc \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right] \left(-i\pi + \operatorname{Log}\left[\frac{2irc}{\sqrt{1}-irc}\right] - \operatorname{Log}\left[-\frac{2irc\sqrt{\frac{1}{1+rc^2}}}{1-i\sqrt{1}r-rc+rc^2}\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right]\right) \right) \right\} + iA\sqrt{1}rc \left(\operatorname{PolyLog}\left[2, -e^{-2i\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right]}\right] - \operatorname{PolyLog}\left[2, e^{2i\operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]}\right] + \operatorname{PolyLog}\left[2, e^{2i\left(\operatorname{ArcTan}\left[\frac{r-rc}{\sqrt{1}}\right] + \operatorname{ArcTan}\left[\frac{rc}{\sqrt{1}}\right]\right)}\right] \right) + A \operatorname{PolyLog}\left[2, \frac{1}{1-\frac{irc}{\sqrt{1}}}\right] + A \operatorname{PolyLog}\left[2, \frac{1}{1+\frac{irc}{\sqrt{1}}}\right] - A \operatorname{PolyLog}\left[2, 1 - \frac{r}{-i\sqrt{1} + rc}\right] - A \operatorname{PolyLog}\left[2, 1 - \frac{r}{i\sqrt{1} + rc}\right] \right\}$$