Supplementary Material for "The Cusp Catastrophe Model as Cross-Sectional and Longitudinal Mixture Structural Equation Models"

Sy-Miin Chow
Pennsylvania State University
Katie Witkiewitz
University of New Mexico
Raoul Grasman
University of Amsterdam
Stephen A. Maisto
Syracuse University

This document features supplementary material from the Chow et al.'s the paper entitled "The Cusp Catastrophe Model as Cross-Sectional and Longitudinal Mixture Structural Equation Models."

0.1 M*plus* Scripts for Fitting MSEM Model 5

```
TITLE: MSEM Model 5, cusp-inspired mixture model, unknown beta group
DATA:     FILE = 'thedat01.csv';
VARIABLE: NAMES = id alpha beta alphabeta y

ybeta1 ybeta2 ybeta3 betaCat;
          MISSING = ALL (-999);
          USEV = alpha ybeta1-ybeta3 y;
          classes = c0(2) c1(3);

ANALYSIS:
          TYPE = MIXTURE;
          STARTS = 50 5;STSCALE=1.5;
          STITERATIONS = 100;

MODEL:
```

```
!-------------------------------------------------
%OVERALL%
!-------------------------------------------------
c1 on alpha@0;
c1 on beta@0;
ly by y@1;
[y@0];
y@0;

[c1#1@-10]; ! (lowBtoH);
[c1#2@-10]; ! (lowBtoL);
c1#1 ON c0#1*15 (hiBtoH);
c1#2 ON c0#1*15 (hiBtoL);

beta by ybeta1@1;
beta by ybeta2*1 (l1);
beta by ybeta3*1 (l2);
[ybeta1-ybeta3@0];

!-------------------------------------------------
Model c0:
!-------------------------------------------------
%c0#1%
[beta*2] (mbetah);
beta*.5 (vb1);
c1#1 ON alpha*3 (inita);
c1#2 ON alpha*-3 (inita2);

%c0#2%
[beta*-2] (mbetal);
beta*.5 (vb2);
c1#1 ON alpha@0;
c1#2 ON alpha@0;

!-------------------------------------------------
Model c1:
!-------------------------------------------------
%c1#1% !High beta, high DV
ly on alpha*.5 (balpha1);
[ly*2] (inth);
ly*.1 (vqHighB);
ly ON beta*.1 (bbeta2);

%c1#2%  !High beta, low DV
ly on alpha*.5 (balpha1);
```

```
[ly*-2] (intl);
ly*.1 (vqHighB);
ly ON beta*-.1 (bbeta3);

%c1#3% !Low beta, med DV
ly on alpha*.3 (balpha2);
[ly*.01] (intm);
ly*1 (vqLowB);
ly ON beta@0;

!-------------------------------------------------
MODEL CONSTRAINT:
!-------------------------------------------------
New (lowBtoH lowBtoL
PHIBTOH PHIBTOL PHIBTOM
PLOWBTOH PLOWBTOL PLOWBTOM);
lowBtoH = -10;
lowBtoL = -10;

PHIBTOH = exp(lowBtoH + hiBtoH)/
(exp(lowBtoH + hiBtoH) +
exp(lowBtoL +hiBtoL) + exp(0));

PHIBTOL = exp(lowBtoL +hiBtoL)/
(exp(lowBtoH + hiBtoH) +
exp(lowBtoL +hiBtoL) + exp(0));

PHIBTOM = exp(0)/
(exp(lowBtoH + hiBtoH) +
exp(lowBtoL +hiBtoL) + exp(0));

PLOWBTOH = exp(lowBtoH)/
(exp(lowBtoH) + exp(lowBtoL) + exp(0));

PLOWBTOL = exp(lowBtoL)/
(exp(lowBtoH) + exp(lowBtoL) + exp(0));

PLOWBTOM = exp(0)/
(exp(lowBtoH) + exp(lowBtoL) + exp(0));

inth > intm;
intm > intl;

!-------------------------------------------------
OUTPUT:        TECH1 TECH4 CINTERVAL;
```

```
SAVEDATA:
RESULTS = '01outPar94.txt';
FILE = '01Cross94.class';
RECORDLENGTH IS 5000;
SAVE = CPROBABILITIES;
```

0.2 M*plus* Scripts for Fitting MSEM-RS Model 5

```
TITLE: T > 1 cusp-inspired mixture model, unknown beta group
!#-----------------------------------------------------------
!#       c1
!#-----------------------------------------------------------
!# c0#1 #2
!#-----------------------------------------------------------
!#   Hi           Low
!#High beta  | lowBtoM + hiBtoH*+inita*alpha         0
!#  Med          Med
!#Low beta   |lowBtoM = [c1#1@10]        0
!#-------
!#Note: inita = c1#1 on alpha1 in c0#1
!@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
!#Conditional transition log-odds
!#-----------------------------------------------------
!#c0 = 2 (Low beta): reference group
!#-----------------------------------------------------
!#     c2
!#     -------------------------------------------------
!# c1 Med           Med
!#-----------------------------------------------------
!#Med  | MtoOth + lMtoM (c2#1 ON c1#1@20)     0
!#Med  | MtoOthM = [c2#1@-10]     0
!#-----------------------------------------------------
!#
!#c0 = 1 (High beta)
!#-----------------------------------------------------
!#      c2
!#       -------------------------------------------------
!# c1 High        Low
!#-----------------------------------------------------
!#High  | MtoOthM + lowtoH + lHtoH + lot11a*alpha       0
!#Low   | MtoOthM + lowtoH + lot21a*alpha   0
!#-----------------------------------------------------
!#Note: lot11a e.g., = c2#1 ON alpha2 in c0#1.c1#1
!#lot21a e.g., = c2#1 ON alpha2 in c0#1.c1#2
```

```
DATA:     FILE = 'thedatLong1.csv';
VARIABLE: NAMES = id alpha1 alpha2 alpha3 alpha4 alpha5
  alpha6 beta0 ab1 ab2 ab3 ab4 ab5 ab6
  y1 y2 y3 y4 y5 y6 ybeta1 ybeta2 ybeta3 betaCat;
MISSING = ALL (-999);
USEV = y1-y6 alpha1-alpha6 ybeta1-ybeta3;
classes = c0(2) c1(2) c2(2) c3(2) c4(2) c5(2) c6(2);

ANALYSIS:
TYPE = MIXTURE;
STARTS = 80 5; STSCALE= 2; !STITERATIONS = 100;

MODEL:
!----------------------------------------
%OVERALL%
!----------------------------------------
c1 on alpha1@0;
c2 on alpha2@0;
c3 on alpha3@0;
c4 on alpha4@0;
c5 on alpha5@0;
c6 on alpha6@0;

c1 on beta@0;
c2 on beta@0;
c3 on beta@0;
c4 on beta@0;
c5 on beta@0;
c6 on beta@0;

ly1 by y1@1;
ly2 by y2@1;
ly3 by y3@1;
ly4 by y4@1;
ly5 by y5@1;
ly6 by y6@1;

[y1-y6@0];
y1-y6@0;

ly1 with ly2@0;
ly1 with ly3@0;
ly1 with ly4@0;
ly1 with ly5@0;
ly1 with ly6@0;
```

```
ly2 with ly3@0;
ly2 with ly4@0;
ly2 with ly5@0;
ly2 with ly6@0;

ly3 with ly4@0;
ly3 with ly5@0;
ly3 with ly6@0;

ly4 with ly5@0;
ly4 with ly6@0;

ly5 with ly6@0;

[c1#1@10]; ! (lowBtoM);

[c2#1@-10]; !(MtoOthM)
[c3#1@-10]; !(MtoOthM)
[c4#1@-10]; !(MtoOthM)
[c5#1@-10]; !(MtoOthM)
[c6#1@-10]; !(MtoOthM)

! Notice that LO(low to high|high beta) = (MtoOthM + lowtoH)

c1#1 ON c0#1*-10 (hiBtoH);
!Expecting to be around 10 LO(low to high|high beta)
!as P(lo to high|high beta) when alpha = 0 should be close to .5.
c2#1 ON c0#1*11 (lowtoH); !Effect of c0#1 on [c2#1]
c3#1 ON c0#1*11 (lowtoH);
c4#1 ON c0#1*11 (lowtoH);
c5#1 ON c0#1*11 (lowtoH);
c6#1 ON c0#1*11 (lowtoH);

beta by ybeta1@1;
beta by ybeta2*1 (l1);
beta by ybeta3*1 (l2);
[ybeta1-ybeta3@0];
!----------------------------------------
Model c0:
!----------------------------------------
%c0#1%
[beta*2] (mbetah);
beta*.5 (vb1);
ly1 on alpha1 (balpha1);
```

```
ly2 on alpha2 (balpha1);
ly3 on alpha3 (balpha1);
ly4 on alpha4 (balpha1);
ly5 on alpha5 (balpha1);
ly6 on alpha6 (balpha1);
c1#1 on alpha1 (inita);
! Notice that LO(high to high|high beta) = (MtoOthM + lowtoH + lHtoH)
c2#1 ON c1#1*.1 (lHtoH);
c3#1 ON c2#1*.1 (lHtoH);
c4#1 ON c3#1*.1 (lHtoH);
c5#1 ON c4#1*.1 (lHtoH);
c6#1 ON c5#1*.1 (lHtoH);

%c0#2%
[beta*-2] (mbetal);
beta*.5 (vb2);
ly1 on alpha1 (balpha2);
ly2 on alpha2 (balpha2);
ly3 on alpha3 (balpha2);
ly4 on alpha4 (balpha2);
ly5 on alpha5 (balpha2);
ly6 on alpha6 (balpha2);

c2#1 ON c1#1@20; ! (lMtoM);
c3#1 ON c2#1@20; ! (lMtoM);
c4#1 ON c3#1@20; ! (lMtoM);
c5#1 ON c4#1@20; ! (lMtoM);
c6#1 ON c5#1@20; ! (lMtoM);

!-----------------------------------------------------
MODEL c0.c1: !Specify model for P(Ct| C_t-1, C1)
!-----------------------------------------------------
!c0 = 1 (high beta); c0 = 2 (low beta)

%c0#1.c1#1% !High DV
[ly1*2] (inth);
ly1*.1 (vqHighB);
ly1 ON beta*.1 (bbeta2);
c2#1 ON alpha2 (lot11a);

%c0#1.c1#2% !Low DV
[ly1*2] (intl);
ly1*.1 (vqHighB);
ly1 ON beta*-.1 (bbeta3);
c2#1 ON alpha2 (lot21a);
```

```
%c0#2.c1#1%
[ly1*.01] (intm);
ly1*1 (vqLowB);
ly1 ON beta@0;

%c0#2.c1#2%
[ly1*.01] (intm);
ly1*1 (vqLowB);
ly1 ON beta@0;

!-------------------------------------------------------------
MODEL c0.c2: !Specify model for P(Ct| C_t-1, C1)
!-------------------------------------------------------------

%c0#1.c2#1%
[ly2*2] (inth);
ly2*.1 (vqHighB);
ly2 ON beta*.1 (bbeta2);
!ly2 ON ab2*.5 (bab2);
c3#1 ON alpha3 (lot11a);

%c0#1.c2#2%
[ly2*2] (intl);
ly2*.1 (vqHighB);
ly2 ON beta*-.1 (bbeta3);
c3#1 ON alpha3 (lot21a);

%c0#2.c2#1%
[ly2*.01] (intm);
ly2*1 (vqLowB);
ly2 ON beta@0;

%c0#2.c2#2%
[ly2*.01] (intm);
ly2*1 (vqLowB);
ly2 ON beta@0;

!-------------------------------------------------
MODEL c0.c3: !Specify model for P(Ct| C_t-1, C1)
!-------------------------------------------------
! c0 = 1 (high beta); c0 = 2 (low beta);
%c0#1.c3#1%
[ly3*2] (inth);
ly3*.1 (vqHighB);
```

```
ly3 ON beta*.1 (bbeta2);
c4#1 ON alpha4 (lot11a);

%c0#1.c3#2%
[ly3*2] (intl);
ly3*.1 (vqHighB);
ly3 ON beta*-.1 (bbeta3);
c4#1 ON alpha4 (lot21a);

%c0#2.c3#1%
[ly3*.01] (intm);
ly3*1 (vqLowB);
ly3 ON beta@0;

%c0#2.c3#2%
[ly3*.01] (intm);
ly3*1 (vqLowB);
ly3 ON beta@0;

!-----------------------------------------------------
MODEL c0.c4: !Specify model for P(Ct| C_t-1, C1)
!-----------------------------------------------------
! c0 = 1 (high beta); c0 = 2 (low beta);

%c0#1.c4#1%
[ly4*2] (inth);
ly4*.1 (vqHighB);
ly4 ON beta*.1 (bbeta2);
c5#1 ON alpha5 (lot11a);

%c0#1.c4#2%
[ly4*2] (intl);
ly4*.1 (vqHighB);
ly4 ON beta*-.1 (bbeta3);
c5#1 ON alpha5 (lot21a);

%c0#2.c4#1%
[ly4*.01] (intm);
ly4*1 (vqLowB);
ly4 ON beta@0;

%c0#2.c4#2%
[ly4*.01] (intm);
ly4*1 (vqLowB);
ly4 ON beta@0;
```

```
!-----------------------------------------------------
MODEL c0.c5: !Specify model for P(Ct| C_t-1, C1)
!-----------------------------------------------------
! c0 = 1 (high beta); c0 = 2 (low beta);
%c0#1.c5#1%
[ly5*2] (inth);
ly5*.1 (vqHighB);
ly5 ON beta*.1 (bbeta2);
c6#1 ON alpha6 (lot11a);

%c0#1.c5#2%
[ly5*2] (intl);
ly5*.1 (vqHighB);
ly5 ON beta*-.1 (bbeta3);
c6#1 ON alpha6 (lot21a);

%c0#2.c5#1%
[ly5*.01] (intm);
ly5*1 (vqLowB);
ly5 ON beta@0;

%c0#2.c5#2%
[ly5*.01] (intm);
ly5*1 (vqLowB);
ly5 ON beta@0;

!-----------------------------------------------------
MODEL c0.c6: !Specify model for P(Ct| C_t-1, C1)
!-----------------------------------------------------
! c0 = 1 (high beta); c0 = 2 (low beta);

%c0#1.c6#1%
[ly6*2] (inth);
ly6*.1 (vqHighB);
ly6 ON beta*.1 (bbeta2);

%c0#1.c6#2%
[ly6*2] (intl);
ly6*.1 (vqHighB);
ly6 ON beta*-.1 (bbeta3);

%c0#2.c6#1%
[ly6*.01] (intm);
ly6*1 (vqLowB);
```

```
ly6 ON beta@0;

%c0#2.c6#2%
[ly6*.01] (intm);
ly6*1 (vqLowB);
ly6 ON beta@0;

!-----------------------------------------------------
MODEL CONSTRAINT:
!-----------------------------------------------------
New (
MtoOthM sda
lowBtoM lMtoM
hiLoAlp medAlp hiBeta loBeta
inth0 intl0 intm0 hiLoVy medVy
hiBtoH0 lowtoH0 lHtoH0 inita_0
lot11a0 lot21a0
HiBp110 HiBp120 HiBp210 HiBp220
HiBp111 HiBp121 HiBp211 HiBp221
HiBp11 HiBp12 HiBp21 HiBp22 dalpha
mbetah0 mbetal0 vb10 vb20 lambda1 lambda2);
sda = 2.31;

MtoOthM = -10;
lowBtoM = 10;
lMtoM = 20;

hiLoAlp = balpha1;
medAlp = balpha2;
hiBeta = bbeta2;
loBeta = bbeta3;
inth0 = inth;
intl0 = intl;
intm0 = intm;
hiLoVy = vqHighB;
medVy = vqLowB;

hiBtoH0 = hiBtoH;
lowtoH0 = lowtoH;
lHtoH0 = lHtoH;
inita_0 = inita;
lot11a0 = lot11a;
lot21a0 = lot21a;

mbetah0 = mbetah;
```

```
mbetal0 = mbetal;
vb10 = vb1;
vb20 = vb2;
lambda1 = l1;
lambda2 = l2;

!High beta high alpha days
HiBp111 = exp(MtoOthM + lowtoH + lHtoH + lot11a*sda)
/(exp(MtoOthM + lowtoH + lHtoH + lot11a*sda) + exp(0));

HiBp121 = exp(0)/
(exp(MtoOthM + lowtoH + lHtoH + lot11a*sda) + exp(0));

HiBp211 = exp(MtoOthM + lowtoH + lot21a*sda)
/(exp(MtoOthM + lowtoH + lot21a*sda) + exp(0));

HiBp221 = exp(0)/(exp(MtoOthM + lowtoH + lot21a*sda) + exp(0));

!High beta low alpha days
HiBp110 = exp(MtoOthM + lowtoH + lHtoH + lot11a*(-1*sda))
/(exp(MtoOthM + lowtoH + lHtoH + lot11a*(-1*sda)) + exp(0));

HiBp120 = exp(0)/
(exp(MtoOthM + lowtoH + lHtoH + lot11a*(-1*sda)) + exp(0));

HiBp210 = exp(MtoOthM + lowtoH + lot21a*(-1*sda))
/(exp(MtoOthM + lowtoH + lot21a*(-1*sda)) + exp(0));

HiBp220 = exp(0)/(exp(MtoOthM + lowtoH + lot21a*(-1*sda)) + exp(0));

!High beta average alpha days
HiBp11 = exp(MtoOthM + lowtoH + lHtoH)
/(exp(MtoOthM + lowtoH + lHtoH) + exp(0));

HiBp12 = exp(0)/
(exp(MtoOthM + lowtoH + lHtoH) + exp(0));

HiBp21 = exp(MtoOthM + lowtoH)
/(exp(MtoOthM + lowtoH) + exp(0));

HiBp22 = exp(0)/(exp(MtoOthM + lowtoH) + exp(0));

inth > intm;
intm > intl;
balpha2 = balpha1 + dalpha;
```

```
!-------------------------------------------------
OUTPUT:        TECH1 TECH4 CINTERVAL;

SAVEDATA:
RESULTS = ‘01outPar3.txt’;
FILE = ’01LTA3.class’;
RECORDLENGTH IS 5000;
SAVE = CPROBABILITIES;
```

0.3 Tables of Simulation Results

The tables included here show the summary statistics of the parameter estimates from all remaining MSEM and MSEM-RS models considered but not presented in Chow et al.'s the paper entitled "The Cusp Catastrophe Model as Cross-Sectional and Longitudinal Mixture Structural Equation Models."

Table 1: Summary Statistics of Parameter Estimates for MSEM Model 2 with $T = 1$ and $n = 500$ across 500 Monte Carlo Replications.

	Mean $\hat{\theta}$	SD	2.5 %tile	97.5%tile	$a\widehat{SE}$
τ_{high}	1.10	0.07	0.97	1.23	0.07
$b_{\alpha,high\&low}$	0.12	0.01	0.09	0.14	0.01
$b_{\beta,high}$	0.19	0.01	0.17	0.21	0.01
$\psi_{high\&low}$	0.11	0.01	0.09	0.14	0.01
τ_{low}	-1.10	0.07	-1.25	-0.97	0.07
$b_{\beta,low}$	-0.19	0.01	-0.21	-0.17	0.01
τ_{med}	0.01	0.12	-0.21	0.25	0.11
$b_{\alpha,med}$	0.18	0.07	0.02	0.27	0.05
ψ_{med}	0.18	0.06	0.05	0.30	0.05
a_{1_0}, logit intercept for $\mathrm{R}_{\text{high }\beta}$	-0.00	0.09	-0.17	0.16	0.09
$b_{11_1,\Delta}$, Δ LO(high $\beta \rightarrow$ high)	2.52	2.52	0.17	7.12	1.51
$b_{12_1,\Delta}$, Δ LO(high $\beta \rightarrow$ low)	2.37	2.10	0.08	6.54	1.36
a_{1_1}, logit intercept for LO(low $\beta \rightarrow$ high)	-1.59	2.49	-5.70	0.31	1.32
a_{2_1}, logit intercept for LO(low $\beta \rightarrow$ low)	-1.43	2.09	-5.56	0.58	1.17
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)	1.70	1.18	0.48	4.36	0.67
$b_{12_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ low)	-1.74	1.53	-3.91	-0.47	0.68
$b_{21_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ high)	0.89	0.84	0.17	3.15	0.53
$b_{22_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ low)	-0.88	0.73	-2.79	-0.19	0.48
Pr(high $\beta \rightarrow$ high)	0.41	0.10	0.20	0.60	0.10
Pr(high $\beta \rightarrow$ low)	0.41	0.10	0.22	0.59	0.10
Pr(high $\beta \rightarrow$ med)	0.18	0.08	0.03	0.36	0.08
Pr(low $\beta \rightarrow$ high)	0.18	0.11	0.00	0.42	0.10
Pr(low $\beta \rightarrow$ low)	0.19	0.12	0.00	0.45	0.10
Pr(low $\beta \rightarrow$ med)	0.63	0.17	0.26	0.91	0.14

[a] The class and transition probabilities were computed using Equations 8, 9 and 12 with α_i set to 0.
Proportion(cases free of SE estimation problems for logit parameters in Model 2) =.94

Table 2: Summary Statistics of Parameter Estimates for MSEM Model 2 with $T = 1$ and $n = 200$ across 500 Monte Carlo Replications.

	Mean $\hat{\theta}$	SD	2.5 %tile	97.5%tile	$a\widehat{SE}$
τ_{high}	1.10	0.10	0.90	1.29	0.09
$b_{\alpha,high\&low}$	0.11	0.02	0.07	0.16	0.02
$b_{\beta,high}$	0.19	0.02	0.17	0.23	0.01
$\psi_{high\&low}$	0.11	0.02	0.07	0.15	0.02
τ_{low}	-1.10	0.11	-1.32	-0.92	0.09
$b_{\beta,low}$	-0.19	0.02	-0.23	-0.16	0.02
τ_{med}	0.01	0.30	-0.63	0.95	0.13
$b_{\alpha,med}$	0.18	0.14	-0.06	0.39	0.05
ψ_{med}	0.13	0.08	0.00	0.30	0.05
a_{1_0}, logit intercept for $\mathrm{R}_{\mathrm{high}\ \beta}$	0.01	0.14	-0.30	0.26	0.14
$b_{11_1,\Delta}$, Δ LO(high $\beta \rightarrow$ high)	3.31	6.92	-3.55	26.31	2.59
$b_{12_1,\Delta}$, Δ LO(high $\beta \rightarrow$ low)	3.17	7.01	-3.36	24.60	2.48
a_{1_1}, logit intercept for LO(low $\beta \rightarrow$ high)	-1.17	5.42	-12.82	5.74	2.13
a_{2_1}, logit intercept for LO(low $\beta \rightarrow$ low)	-1.13	5.39	-12.05	4.97	2.02
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)	1.95	2.28	-0.39	6.28	0.95
$b_{12_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ low)	-2.29	2.57	-9.23	-0.04	0.93
$b_{21_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ high)	1.11	1.75	-0.41	5.53	0.85
$b_{22_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ low)	-1.01	1.59	-5.57	0.36	0.68
Pr(high $\beta \rightarrow$ high)[a]	0.42	0.17	0.12	0.80	0.14
Pr(high $\beta \rightarrow$ low)	0.40	0.17	0.03	0.74	0.14
Pr(high $\beta \rightarrow$ med)	0.18	0.13	0.00	0.48	0.10
Pr(low $\beta \rightarrow$ high)	0.23	0.18	0.00	0.69	0.13
Pr(low $\beta \rightarrow$ low)	0.24	0.18	0.00	0.65	0.13
Pr(low $\beta \rightarrow$ med)	0.52	0.26	0.00	0.95	0.17

[a] The class and transition probabilities were computed using Equations 8, 9 and 12 with α_i set to 0.

Proportion(cases free of SE estimation problems for logit parameters in Model 2) =.54

Table 3: Summary Statistics of Parameter Estimates for MSEM Model 6 with $T = 1$ and $n = 200$ across 500 Monte Carlo Replications.

	True θ	Mean $\hat{\theta}$	SD	2.5 %tile	97.5%tile	$\widehat{aSE}$
$\lambda_{\beta,1}$	1.20	1.20	0.01	1.17	1.23	0.01
$\lambda_{\beta,2}$	0.90	0.90	0.01	0.88	0.92	0.01
$\psi^2_{\epsilon,\beta_1}$	0.09	0.09	0.01	0.07	0.12	0.01
$\psi^2_{\epsilon,\beta_2}$	0.09	0.09	0.02	0.06	0.13	0.02
$\psi^2_{\epsilon,\beta_3}$	0.09	0.09	0.01	0.07	0.11	0.01
$\mu_{\beta,high}$	2.50	2.50	0.07	2.36	2.64	0.07
$\psi_{\beta,high}$	0.49	0.47	0.07	0.33	0.63	0.07
$\mu_{\beta,low}$	-2.50	-2.50	0.08	-2.64	-2.36	0.07
$\psi_{\beta,low}$	0.49	0.49	0.07	0.37	0.63	0.07
τ_{high}		1.05	0.15	0.69	1.32	0.09
$b_{\alpha,high\&low}$		0.12	0.02	0.08	0.17	0.02
$b_{\beta,high}$		0.20	0.05	0.12	0.30	0.02
$\psi_{high\&low}$		0.11	0.02	0.08	0.16	0.02
τ_{low}		-1.01	0.17	-1.29	-0.62	0.09
$b_{\beta,low}$		-0.21	0.06	-0.38	-0.13	0.02
τ_{med}		0.02	0.41	-0.77	0.88	0.13
$b_{\alpha,med}$		0.17	0.14	-0.15	0.39	0.04
ψ_{med}		0.12	0.08	0.00	0.28	0.04
a_{1_0}, logit intercept for $R_{\text{high }\beta}$	0.00	0.00	0.01	-0.01	0.02	0.14
$b_{11_1,\Delta}$, Δ LO(high $\beta \rightarrow$ high)		2.66	6.97	-8.94	25.84	5.32
$b_{12_1,\Delta}$, Δ LO(high $\beta \rightarrow$ low)		2.91	7.41	-8.82	25.46	2.70
a_{1_1}, logit intercept for LO(low $\beta \rightarrow$ high)		-1.23	5.64	-15.58	8.32	4.91
a_{2_1}, logit intercept for LO(low $\beta \rightarrow$ low)		-1.38	6.57	-23.28	9.17	2.40
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)		2.10	2.36	-0.00	7.18	0.97
$b_{12_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ low)		-1.95	1.89	-5.48	-0.03	0.90
$b_{21_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ high)		1.10	2.02	-2.45	6.85	1.39
$b_{22_1,\alpha}$, $\alpha \rightarrow$ LO(low $\beta \rightarrow$ low)		-0.70	2.47	-4.27	2.47	0.83
Pr(high $\beta \rightarrow$ high)[a]		0.38	0.17	0.07	0.71	0.15
Pr(high $\beta \rightarrow$ low)		0.41	0.17	0.08	0.74	0.15
Pr(high $\beta \rightarrow$ med)		0.20	0.14	0.00	0.53	0.11
Pr(low $\beta \rightarrow$ high)		0.23	0.18	0.00	0.61	0.13
Pr(low $\beta \rightarrow$ low)		0.30	0.24	0.00	0.81	0.13
Pr(low $\beta \rightarrow$ med)		0.46	0.30	0.00	0.96	0.16

[a] The class and transition probabilities were computed using Equations 8, 9 and 12 with α_i set to 0.
Proportion(cases free of SE estimation problems for logit parameters in Model 2) =.38

Table 4: Summary Statistics of Parameter Estimates for MSEM-RS Model 1 with $T = 6$ and $n = 500$ across 500 Monte Carlo Replications.

τ_{high}	0.59	0.06	0.48	0.71	0.06
$b_{\alpha,high\&low}$	0.24	0.00	0.23	0.25	0.01
$b_{\beta,high}$	0.27	0.02	0.22	0.31	0.02
$\psi_{high\&low}$	0.18	0.01	0.17	0.20	0.01
τ_{low}	-0.59	0.06	-0.71	-0.46	0.06
$b_{\beta,low}$	-0.27	0.02	-0.32	-0.22	0.02
τ_{med}	-0.00	0.01	-0.03	0.03	0.01
$b_{\alpha,med}$	0.24	0.00	0.23	0.25	0.01
ψ_{med}	0.26	0.01	0.24	0.28	0.01
$b_{11_1,\Delta}$, Δ LO(high $\beta \rightarrow$ high)	-10.01	0.31	-10.66	-9.39	0.30
$a_{1,1_t,\Delta}$, Δ LO(low$\rightarrow$high| high β)	10.00	0.20	9.61	10.39	0.19
$b_{1,11_t,\Delta}$, Δ LO(high $\rightarrow$ high|high β)	0.00	0.27	-0.57	0.55	0.27
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)	2.66	0.53	1.94	3.93	0.47
$b_{1,11_t,\alpha}$, $\alpha \rightarrow$ LO(high $\rightarrow$ high|high β)	2.57	0.32	2.06	3.32	0.29
$b_{1,21_t,\alpha}$, $\alpha \rightarrow$ LO(low $\rightarrow$ high|high β)	2.57	0.32	2.08	3.25	0.29
Pr(high $\rightarrow$ high|high β, low α)[a]	0.00	0.00	0.00	0.01	0.00
Pr(high $\rightarrow$ low|high β, low α)	1.00	0.00	0.99	1.00	0.00
Pr(low $\rightarrow$ high|high β, low α)	0.00	0.00	0.00	0.01	0.00
Pr(low $\rightarrow$ low|high β, low α)	1.00	0.00	0.99	1.00	0.00
Pr(high $\rightarrow$ high|high β, high α)	1.00	0.00	0.99	1.00	0.00
Pr(high $\rightarrow$ low|high β, high α)	0.00	0.00	0.00	0.01	0.00
Pr(low $\rightarrow$ high|high β, high α)	1.00	0.00	0.99	1.00	0.00
Pr(low $\rightarrow$ low|high β, high α)	0.00	0.00	0.00	0.01	0.00
Pr(high $\rightarrow$ high|high β, avg α)	0.50	0.05	0.41	0.59	0.05
Pr(high $\rightarrow$ low|high β, avg α)	0.50	0.05	0.41	0.59	0.05
Pr(low $\rightarrow$ high|high β, avg α)	0.50	0.05	0.40	0.60	0.05
Pr(low $\rightarrow$ low|high β, avg α)	0.50	0.05	0.40	0.60	0.05

[a] The transition probabilities were computed using Equations 13 and 14. Avg α = value of α was set to 0; high α = value of α was set to 1 SD above the mean of 0; low α = value of α was set to 1 SD below the mean of 0.
Proportion(cases free of SE estimation problems for logit parameters in Model 2) = 1.00

Table 5: Summary Statistics of Parameter Estimates for MSEM-RS Model 1 with $T = 6$ and $n = 200$ across 500 Monte Carlo Replications.

τ_{high}	0.59	0.10	0.38	0.80	0.04
$b_{\alpha,high\&low}$	0.24	0.01	0.23	0.26	0.00
$b_{\beta,high}$	0.27	0.04	0.19	0.34	0.01
$\psi_{high\&low}$	0.18	0.01	0.16	0.21	0.01
τ_{low}	-0.59	0.11	-0.79	-0.39	0.03
$b_{\beta,low}$	-0.27	0.04	-0.34	-0.19	0.00
τ_{med}	0.00	0.02	-0.04	0.04	0.02
$b_{\alpha,med}$	0.24	0.01	0.23	0.26	0.00
ψ_{med}	0.26	0.01	0.24	0.29	0.02
$b_{11_1,\Delta}$, ΔLO(high $\beta \rightarrow$ high)	-10.04	0.58	-11.34	-8.88	0.52
$a_{1,1_t,\Delta}$, ΔLO(low$\rightarrow$high| high β)	10.01	0.30	9.43	10.55	0.30
$b_{1,11_t,\Delta}$, ΔLO(high $\rightarrow$ high|high β)	0.01	0.43	-0.91	0.82	0.43
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)	2.94	1.23	1.61	6.12	0.79
$b_{1,11_t,\alpha}$, $\alpha \rightarrow$ LO(high $\rightarrow$ high|high β)	2.62	0.51	1.88	3.75	0.46
$b_{1,21_t,\alpha}$, $\alpha \rightarrow$ LO(low $\rightarrow$ high|high β)	2.62	0.52	1.87	3.95	0.46
Pr(high $\rightarrow$ high|high β, low α)	0.00	0.00	0.00	0.01	0.00
Pr(high $\rightarrow$ low|high β, low α)	1.00	0.00	0.99	1.00	0.00
Pr(low $\rightarrow$ high|high β, low α)	0.00	0.00	0.00	0.01	0.00
Pr(low $\rightarrow$ low|high β, low α)	1.00	0.00	0.99	1.00	0.00
Pr(high $\rightarrow$ high|high β, high α)	1.00	0.00	0.99	1.00	0.00
Pr(high $\rightarrow$ low|high β, high α)	0.00	0.00	0.00	0.01	0.00
Pr(low $\rightarrow$ high|high β, high α)	1.00	0.00	0.99	1.00	0.00
Pr(low $\rightarrow$ low|high β, high α)	0.00	0.00	0.00	0.01	0.00
Pr(high $\rightarrow$ high|high β, avg α)	0.50	0.07	0.35	0.64	0.07
Pr(high $\rightarrow$ low|high β, avg α)	0.50	0.07	0.36	0.65	0.07
Pr(low $\rightarrow$ high|high β, avg α)	0.50	0.07	0.36	0.63	0.07
Pr(low $\rightarrow$ low|high β, avg α)	0.50	0.07	0.37	0.64	0.07

[a] The transition probabilities were computed using Equations 13 and 14. Avg α = value of α was set to 0; high α = value of α was set to 1 SD above the mean of 0; low α = value of α was set to 1 SD below the mean of 0.
Proportion(cases free of SE estimation problems for logit parameters in Model 2) =.93

Table 6: Summary Statistics of Parameter Estimates for MSEM-RS Model 5 with $T = 6$ and $n = 200$ across 500 Monte Carlo Replications.

	True θ	Mean $\hat{\theta}$	SD	2.5 %tile	97.5%tile	$a\widehat{SE}$
$\lambda_{\beta,1}$	1.20	1.20	0.01	1.17	1.22	0.01
$\lambda_{\beta,2}$	0.90	0.90	0.01	0.88	0.92	0.01
$\psi^2_{\epsilon,\beta_1}$	0.09	0.09	0.01	0.07	0.12	0.01
$\psi^2_{\epsilon,\beta_2}$	0.09	0.09	0.02	0.06	0.12	0.02
$\psi^2_{\epsilon,\beta_3}$	0.09	0.09	0.01	0.07	0.11	0.01
$\mu_{\beta,high}$	2.50	2.50	0.07	2.35	2.64	0.07
$\psi_{\beta,high}$	0.49	0.49	0.09	0.34	0.66	0.08
$\mu_{\beta,low}$	-2.50	-2.50	0.07	-2.64	-2.35	0.07
$\psi_{\beta,low}$	0.49	0.48	0.07	0.36	0.62	0.07
τ_{high}		0.54	0.20	0.00	0.78	0.14
$b_{\alpha,high\&low}$		0.25	0.01	0.23	0.26	0.03
$b_{\beta,high}$		0.28	0.09	0.20	0.49	0.04
$\psi_{high\&low}$		0.18	0.01	0.16	0.21	0.02
τ_{low}		-0.50	0.24	-0.79	0.00	0.12
$b_{\beta,low}$		-0.29	0.10	-0.49	-0.19	0.04
τ_{med}		-0.00	0.02	-0.05	0.04	0.02
$b_{\alpha,med}$		0.25	0.01	0.23	0.26	0.03
ψ_{med}		0.26	0.01	0.23	0.29	0.02
a_{1_0}, logit intercept for $R_{\text{high }\beta}$	0.00	0.00	0.01	-0.00	0.02	0.14
$b_{11_1,\Delta}$, Δ LO(high $\beta \rightarrow$ high)		-9.99	0.95	-11.56	-8.72	0.58
$a_{1,1_t,\Delta}$, ΔLO(low$\rightarrow$high| high β)		10.01	0.33	9.37	10.63	0.31
$b_{11_t,\Delta}$, Δ LO(high $\rightarrow$ high|high β)		-0.01	0.45	-0.84	0.82	0.45
$b_{11_1,\alpha}$, $\alpha \rightarrow \Delta$LO(high $\beta \rightarrow$ high)		3.18	2.14	1.60	7.37	0.95
$b_{11_t,\alpha}$, $\alpha \rightarrow$ LO(high $\rightarrow$ high)		2.64	0.61	1.93	3.81	0.50
$b_{21_t,\alpha}$, $\alpha \rightarrow$ LO(low $\rightarrow$ high)		2.69	0.65	1.88	4.00	0.50
Pr(high $\rightarrow$ high|high β, low α)[a]		0.01	0.06	0.00	0.01	0.00
Pr(high $\rightarrow$ low|high β, low α)		0.99	0.06	0.99	1.00	0.00
Pr(low $\rightarrow$ high|high β, low α)		0.01	0.06	0.00	0.01	0.00
Pr(low $\rightarrow$ low|high β, low α)		0.99	0.06	0.99	1.00	0.00
Pr(high $\rightarrow$ high|high β, high α)		0.99	0.06	0.99	1.00	0.00
Pr(high $\rightarrow$ low|high β, high α)		0.01	0.06	0.00	0.01	0.00
Pr(low $\rightarrow$ high|high β, high α)		0.99	0.06	0.99	1.00	0.00
Pr(low $\rightarrow$ low|high β, high α)		0.01	0.06	0.00	0.01	0.00
Pr(high $\rightarrow$ high|high β, avg α)		0.50	0.08	0.35	0.63	0.08
Pr(high $\rightarrow$ low|high β, avg α)		0.50	0.08	0.37	0.65	0.08
Pr(low $\rightarrow$ high|high β, avg α)		0.50	0.08	0.35	0.65	0.08
Pr(low $\rightarrow$ low|high β, avg α)		0.50	0.08	0.35	0.65	0.08

[a] The transition probabilities were computed using Equations 13 and 14. Avg α = value of α was set to 0; high α = value of α was set to 1 SD above the mean of 0; low α = value of α was set to 1 SD below the mean of 0. Proportion(cases free of SE estimation problems for logit parameters in Model 2) = .99