SUPPLEMENTARY DATA

S1. Derivation of the logistic function-based sub-model Spatial Heterogeneity, in *ChaMRoots*.

(Eq. 1) gives the definition of production efficiency of the finest roots at a given point due to the surrounding trees:

$$E = \frac{\mathrm{d}RID_{t,0-1}}{\mathrm{d}p} \qquad \text{(Eq. 1)}$$

(Eq. 3) gives the generic form the logistic function-based sub-model spatial heterogeneity (SH):

$$E = rRID_{t,x-y}(1 - \frac{RID_{t,0-1}}{K} - \theta\frac{RID_{u,0-1}}{K}) \qquad \text{(Eq. 3)}$$

(Eq. 5) indicates that $RID_{t,x-y}$ refers to two sources of roots:

$$RID_{t,x-y} = RID_{t,0-5} = RID_{t,0-1} + RID_{t,1-5} \qquad \text{(Eq. 5)}$$

(Eq. 6) gives the linear relationship between $RID_{t,1-5}$ and $RID_{t,0-1}$:

$$RID_{t,1-5} = aRID_{t,0-1} + b \qquad \text{(Eq. 6)}$$

Applying (Eq. 1), (Eq. 5) and (Eq. 6) to (Eq. 3), we obtain (Eq. S1):

$$\frac{\mathrm{d}RID_{t,0-1}}{\mathrm{d}p} = r[(1+a)RID_{t,0-1} + b](1 - \frac{RID_{t,0-1}}{K} - \theta\frac{RID_{u,0-1}}{K}) \qquad \text{(Eq. S1)}$$

To simplify the equation expression, let:

$$A = 1 + a \qquad \text{(Eq. S2)}$$

$$L = 1/K \qquad \text{(Eq. S3)}$$

$$Q = 1 - \theta\frac{RID_{u,0-1}}{K} \qquad \text{(Eq. S4)}$$

Therefore, (Eq. S1) becomes:

$$\frac{\mathrm{d}RID_{t,0-1}}{\mathrm{d}p} = r(A \cdot RID_{t,0-1} + b)(Q - L \cdot RID_{t,0-1})$$

$$\Rightarrow \frac{\mathrm{d}RID_{t,0-1}}{(A \cdot RID_{t,0-1} + b)(Q - L \cdot RID_{t,0-1})} = r\mathrm{d}p$$

To facilitate formula integration, the above formula should be first transformed into the following form:

$$(\frac{M}{A \cdot RID_{t,0-1} + b} + \frac{N}{Q - L \cdot RID_{t,0-1}})\mathrm{d}RID_{t,0-1} = r\mathrm{d}p \qquad \text{(Eq. S5a)}$$

where *M* and *N* are two intermediary variables that meet the following relationship:

$$M(Q - L \cdot RID_{t,0-1}) + N(A \cdot RID_{t,0-1} + b) = 1$$

$$\Rightarrow (AN - LM)RID_{t,0-1} + (bN + QM) = 1 \qquad \text{(Eq. S5b)}$$

It is sufficient to find one pair of solutions of *M* and *N* for (Eq. S5b). Hence, let:

$$\begin{cases} AN - LM = 0 \\ bN + QM = 1 \end{cases}$$

Solving the above simultaneous linear equations, we obtain a pair of M and N:

$$\begin{cases} M = \frac{A}{bL+AQ} & \text{(Eq. 6a)} \\ N = \frac{L}{bL+AQ} & \text{(Eq. 6b)} \end{cases}$$

(Eq. S5a) can be written as:

$$\frac{M}{A \cdot RID_{t,0-1}+b}\, \mathrm{d}RID_{t,0-1} + \frac{N}{Q-L \cdot RID_{t,0-1}}\, \mathrm{d}RID_{t,0-1} = r\mathrm{d}p \qquad \text{(Eq. S7)}$$

Integrating the above equation (with $RID_{t,0-1} \in [0, +\infty)$ and $p \in [0, +\infty)$), we obtain:

$$\frac{M}{A}\ln\left(A \cdot RID_{t,0-1} + b\right) - \frac{N}{L}\ln(Q - L \cdot RID_{t,0-1}) = rp + C_1 \qquad \text{(Eq. S8)}$$

where, C_1 is a constant; the terms within ln() are > 0. Applying the solved M, i.e. (Eq. S6a), and N, i.e. (Eq. S6b), to (Eq. S8), we obtain:

$$\frac{\ln\left(A \cdot RID_{t,0-1}+b\right)-\ln(Q-L \cdot RID_{t,0-1})}{bL+AQ} = rp + C_1$$

$$\Rightarrow \ln\left(\frac{A \cdot RID_{t,0-1}+b}{Q-L \cdot RID_{t,0-1}}\right) = (rp + C_1)(bL + AQ)$$

$$\Rightarrow \frac{A \cdot RID_{t,0-1}+b}{Q-L \cdot RID_{t,0-1}} = e^{(rp+C_1)(bL+AQ)}$$

$$= e^{rp(bL+AQ)+C_1(bL+AQ)}$$

$$= e^{rp(bL+AQ)} \cdot e^{C_1(bL+AQ)}$$

$$= C_2 \cdot e^{rp(bL+AQ)} \qquad \text{(Eq. S9)}$$

where, C_2 is a constant and $C_2 = C_1\,(bL + AQ)$.

Let:

$$U = e^{rp(bL+AQ)} \qquad \text{(Eq. S10)}$$

Applying (Eq. S10) to (Eq. S9), $RID_{t,0-1}$ can be solved:

$$RID_{t,0-1} = \frac{C_2UQ-b}{A+C_2LU} \qquad \text{(Eq. S11)}$$

In the present study, when potential of tree root provision (p) is equal to 0 (no tree), $RID_{t,0-1}$= 0 (no tree roots). Therefore, by linking (Eq. S10) and (Eq. S11) we obtain:

$$C_2 = \frac{b}{Q} \qquad \text{(Eq. S12)}$$

Applying (Eq. S12) to (Eq. S11), we obtain:

$$RID_{t,0-1} = \frac{b(U-1)}{A+bLU/Q} \qquad \text{(Eq. S13)}$$

Finally, applying (Eq. S2), (Eq. S3) and (Eq. S4) to (Eq. S13) and (Eq. S10),

we obtain the following generic equations to calculate $RID_{t,0-1}$:

$$RID_{t,0-1} = \frac{b(U-1)}{1+a+bU/(K-\theta RID_{u,0-1})} \quad \text{(Eq. S14a)}$$

$$U = e^{rp[b/K+(1+a)(1-\theta\frac{RID_{u,0-1}}{K})]} \quad \text{(Eq. S14b)}$$

If θ is equal to zero (i.e. ignoring the effect of competition between understorey roots and tree roots), (Eq. S14a) and (Eq. S14b) turn into

$$RID_{t,0-1} = \frac{b(U-1)}{1+a+bU/K} \quad \text{(Eq. S15a)}$$

$$U = e^{(1+a+b/K)rp} \quad \text{(Eq. S15b)}$$

Replacing K by K_t, (Eq. S15a) and (Eq. S15b) becomes identical with (Eq. 7a) and (Eq. 7b) in the manuscript.

The global tree root potential (p) at a given point is defined by the following general form: (Eq. 2):

$$p = \sum_{s=1}^{N_s}\sum_{e=1}^{N_e} p_{e,s} = \sum_{s=1}^{N_s}\left(\sum_{e=1}^{N_e}\frac{g_{e,s}^{\lambda_s}}{\beta+(D_{e,s}^{\alpha})^{\varphi_s}}O_{e,s}\right) \quad \text{(Eq. 2)}$$

Where, N_s is the number of tree species around the target point ($N_s \geq 0$, $s \in [0, N_s]$). N_e is the number of tree individuals of a given species s around the target point ($N_e \geq 0$, $e \in [0, N_e]$). $p_{e,s}$ is the potential of tree root provision contributed by the tree (e, s). $g_{e,s}$ is basal area at a height of 1.3 m of an individual tree e of species s (m²). $D_{e,s}$ is horizontal distance from the centre of the tree (e, s) to the target point (m) and $D_{e,s} \geq 0$. $O_{e,s}$ is the absence of emerged obstacles on the ground between the target point and the tree e of species s.

By setting $N_s = N_e = 1$ and associating (Eq. 2) with (Eq. S15a) and (Eq. S15b), we can perform sensitivity analyses on the contribution of a single tree to provide roots at a given distance from the trunk (See the figures below as examples).

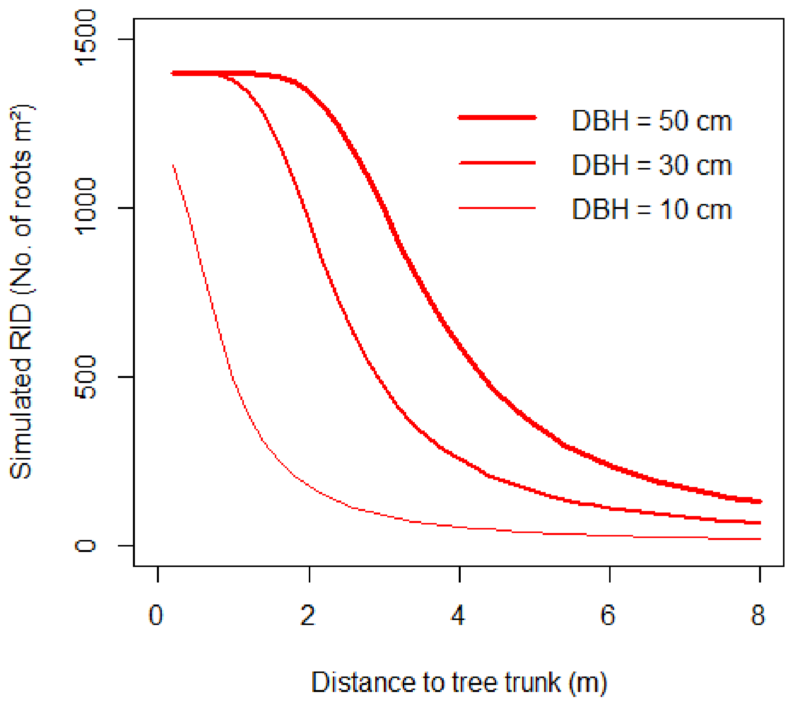

Figure S1 An example of sensitivity analyses on the effect of tree size (represented by diameter at breast height) and distance on the root density based on (Eq. S15a) and (Eq. S15b). In this case, the following coefficient values were used: α= 1.5; β = 1; λ_s=φ_s =1; r = 25; K_t = 1400; a = 1.2 and b = 200.

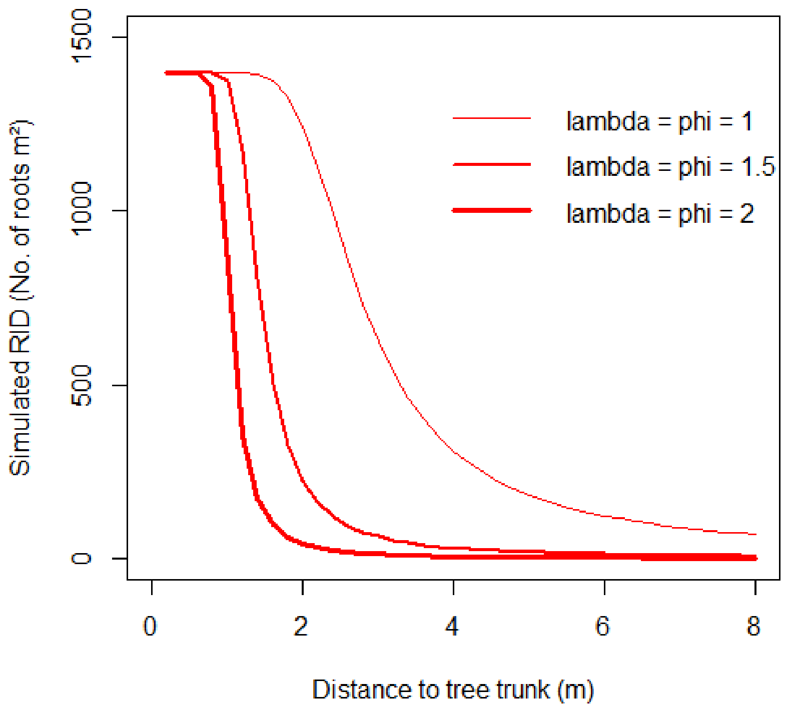

Figure S2 An example of sensitivity analysis on the effect of the factors controlling tree species (λ_s and φ_s) on the root density provision based on (Eq. S15a) and (Eq. S15b). In this case, following coefficient values were used: α= 1.5; β = 0; r = 25; K_t = 1400; a = 1.2; b = 200. The diameter at breast height of the tree is 30 cm.