APPENDIX

The disease free equilibrium points of the model are as follow
$S_H = \dfrac{\lambda_h N_H}{\mu_h}$, $S_E = \dfrac{\mu_m K_E(a-(s+\mu_e))}{as}$, $S_M = \dfrac{K_E(a-(s+\mu_e))}{a}$.
The reproduction number R_0 can be obtained using the method introduced by van den Driessche (2002). We write the system of differential equations as $\psi = f - v$ where

$$\psi = \begin{bmatrix} I_H \\ I_E \\ L_M \\ I_M \end{bmatrix} \quad ,f = \begin{bmatrix} \dfrac{x_1 b b_h I_M S_H}{N_H} \\ a(1-\dfrac{S_E+I_E}{K_E})\gamma I_M \\ \dfrac{x_2 b b_m S_M I_H}{N_H} \\ 0 \end{bmatrix}, v = \begin{bmatrix} (\mu_h + r)I_H \\ (\mu_e + s)I_E \\ (\mu_m + c)L_M \\ -cL_M - sI_E + \mu_m I_M \end{bmatrix} \tag{1}$$

The jacobian matrices F and V, associated with f and v, respectively, at the disease free equilibrium are:

$$F = \begin{bmatrix} 0 & 0 & 0 & x_1 b b_h \\ 0 & 0 & 0 & a\gamma(1-\dfrac{S_E}{K_E}) \\ x_2 b b_m \frac{N_M}{N_H} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} \mu_h + r & 0 & 0 & 0 \\ 0 & \mu_e + s & 0 & 0 \\ 0 & 0 & \mu_m + c & 0 \\ 0 & -s & -c & \mu_m \end{bmatrix}, \tag{2}$$

and the next generation matrix $G = FV^{-1}$ is:

$$G = \begin{bmatrix} 0 & 0 & \dfrac{x_2 b b_m c}{(\mu_m + c)\mu_m} & \dfrac{x_1 b b_h}{\mu_m} \\ 0 & 0 & \dfrac{a\gamma(K_E - S_E)c}{K_E(\mu_m + c)\mu_m} & \dfrac{a\gamma(K_E - S_E)}{K_E \mu_m} \\ \dfrac{\mu_m + c}{\mu_h + r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

R_0 is then the spectral radius of the next generation matrix, $R_0 = \rho(G)$, the largest eigenvalue of G.
Thus

$$R_0 = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \beta}, \tag{4}$$

where

$$\alpha = a\left(1 - \frac{N_E}{K_E}\right)\frac{sr}{(\mu_e + s)\mu_m}$$
$$\beta = \frac{x_1 x_2 b^2 b_h b_m N_m c}{\mu_m N_h (\mu_m + e)(\mu_h + r)}.$$

Note that β is the basic reproduction number proposed by MacDonald (1952).

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