## Additional file 3 - Associated equations of the model

The ODE system is given below. The notation $\dot{x}$ denotes the derivative with respect to time such that $\dot{x} = \frac{dx}{dt}$. The proportion of females at birth is $\delta$ and the proportion of males is $(1-\delta)$. Functions of horizontal transmission $(f_i)$ are defined in the following equations. The meaning of the parameters is given in Tables 1 and 2 of the article.

### Newborn ♀

(1a) $\dot{S^0_{B♀}} = -(\mu_{Juv}(t)+\alpha).S^0_{B♀} + \delta.\left(\eta_{Sa}(t).R_{Sa♀} + \eta_A.R_{A♀}\right)$

(1b) $\dot{S_{B♀}} = -(\mu_{Juv}(t)+f_1).S_{B♀} + \alpha.S^0_{B♀} + \delta.\left(\eta_{Sa}(t).S_{Sa♀} + \eta_A.S_{A♀}\right)$

(1c) $\dot{T_{B♀}} = -(\mu_{Juv}(t)+\mu^T+\gamma).T_{B♀} + f_1.S_{B♀}$

(1d) $\dot{R_{B♀}} = -\mu_{Juv}(t).R_{B♀} + \gamma.T_{B♀} + \delta.\left(\eta_{Sa}(t).T_{Sa♀} + \eta_A.T_{A♀} + \eta_{Sa}(t).(1-p_{Sa♀}).R^g_{Sa♀} + \eta_A.(1-p_{A♀}).R^g_{A♀}\right)$

(1e) $\dot{P_{B♀}} = -(\mu_{Juv}(t)+\mu^P).P_{B♀} + \delta.(1-\rho).\left(\eta_{SaP}(t).P_{Sa♀} + \eta_{AP}.P_{A♀} + \eta_{Sa}(t).p_{Sa♀}.R^g_{Sa♀} + \eta_A.p_{A♀}.R^g_{A♀}\right)$

### Newborn ♂

(2a) $\dot{S^0_{B♂}} = -(\mu_{Juv}(t)+\alpha).S^0_{B♂} + (1-\delta).\left(\eta_{Sa}(t).R_{Sa♀} + \eta_A.R_{A♀}\right)$

(2b) $\dot{S_{B♂}} = -(\mu_{Juv}(t)+f_1).S_{B♂} + \alpha.S^0_{B♂} + (1-\delta).\left(\eta_{Sa}(t).S_{Sa♂} + \eta_A.S_{A♀}\right)$

(2c) $\dot{T_{B♂}} = -(\mu_{Juv}(t)+\mu^T+\gamma).T_{B♂} + f_1.S_{B♂}$

(2d) $\dot{R_{B♂}} = -\mu_{Juv}(t).R_{B♂} + \gamma.T_{B♂} + (1-\delta).\left(\eta_{Sa}(t).T_{Sa♀} + \eta_A.T_{A♀} + \eta_{Sa}(t).(1-p_{Sa♀}).R^g_{Sa♀} + \eta_A.(1-p_{A♀}).R^g_{A♀}\right)$

(2e) $\dot{P_{B♂}} = -(\mu_{Juv}(t)+\mu^P).P_{B♂} + (1-\delta).(1-\rho).\left(\eta_{SaP}(t).P_{Sa♀} + \eta_{AP}.P_{A♀} + \eta_{Sa}(t).p_{Sa♀}.R^g_{Sa♀} + \eta_A.p_{A♀}.R^g_{A♀}\right)$

### Young ♀

(3a) $\dot{S^0_{Y♀}} = -(\mu_{Juv}(t)+\alpha).S^0_{Y♀}$

(3b) $\dot{S_{Y♀}} = -(\mu_{Juv}(t)+f_1).S_{Y♀} + \alpha.S^0_{Y♀}$

(3c) $\dot{T_{Y♀}} = -(\mu_{Juv}(t)+\mu^T+\gamma).T_{Y♀} + f_1.S_{Y♀}$

(3d) $\dot{R_{Y♀}} = -\mu_{Juv}(t).R_{Y♀} + \gamma.T_{Y♀}$

(3e) $\dot{P_{Y♀}} = -(\mu_{Juv}(t)+\mu^P).P_{Y♀}$

### Young ♂

(4a) $\dot{S^0_{Y♂}} = -(\mu_{Juv}(t)+\alpha).S^0_{Y♂}$

(4b) $\dot{S_{Y♂}} = -(\mu_{Juv}(t)+f_1).S_{Y♂} + \alpha.S^0_{Y♂}$

(4c) $\dot{T_{Y♂}} = -(\mu_{Juv}(t)+\mu^T+\gamma).T_{Y♂} + f_1.S_{Y♂}$

(4d) $\dot{R_{Y♂}} = -\mu_{Juv}(t).R_{Y♂} + \gamma.T_{Y♂}$

(4e) $\dot{P_{Y♂}} = -(\mu_{Juv}(t)+\mu^P).P_{Y♂}$

**Sub-adult ♀**

$$\dot{S_{Sa♀}} = -(\mu_{Sa♀} + f_1).S_{Sa♀} \tag{5a}$$

$$\dot{T_{Sa♀}} = -(\mu_{Sa♀} + \mu^T + \gamma).T_{Sa♀} + f_1.S_{Sa♀} \tag{5b}$$

$$\dot{R^g_{Sa♀}} = -\mu_{Sa♀}.R^g_{Sa♀} + \nu(t).\gamma.T_{Sa♀} \tag{5c}$$

$$\dot{R_{Sa♀}} = -\mu_{Sa♀}.R_{Sa♀} + (1-\nu(t)).\gamma.T_{Sa♀} \tag{5d}$$

$$\dot{P_{Sa♀}} = -(\mu_{Sa♀} + \mu^P).P_{Sa♀} \tag{5e}$$

**Sub-adult ♂**

$$\dot{S_{Sa♂}} = -(\mu_{Sa♂} + f_2).S_{Sa♂} \tag{6a}$$

$$\dot{T_{Sa♂}} = -(\mu_{Sa♂} + \mu^T + \gamma).T_{Sa♂} + f_2.S_{Sa♂} \tag{6b}$$

$$\dot{R_{Sa♂}} = -\mu_{Sa♂}.R_{Sa♂} + \gamma.T_{Sa♂} \tag{6c}$$

$$\dot{P_{Sa♂}} = -(\mu_{Sa♂} + \mu^P).P_{Sa♂} \tag{6d}$$

**Adult ♀**

$$\dot{S_{A♀}} = -(\mu_{A♀} + f_1).S_{A♀} + \omega.R_{A♀} \tag{7a}$$

$$\dot{T_{A♀}} = -(\mu_{A♀} + \mu^T + \gamma).T_{A♀} + f_1.S_{A♀} \tag{7b}$$

$$\dot{R^g_{A♀}} = -\mu_{A♀}.R^g_{A♀} + \nu(t).\gamma.T_{A♀} \tag{7c}$$

$$\dot{R_{A♀}} = -(\mu_{A♀} + \omega).R_{A♀} + (1-\nu(t)).\gamma.T_{A♀} \tag{7d}$$

$$\dot{P_{A♀}} = -(\mu_{A♀} + \mu^P).P_{A♀} \tag{7e}$$

**Adult ♂**

$$\dot{S_{A♂}} = -(\mu_{A♂} + f_3).S_{A♂} + \omega.R_{A♂} \tag{8a}$$

$$\dot{T_{A♂}} = -(\mu_{A♂} + \mu^T + \gamma).T_{A♂} + f_3.S_{A♂} \tag{8b}$$

$$\dot{R_{A♂}} = -(\mu_{A♂} + \omega).R_{A♂} + \gamma.T_{A♂} \tag{8c}$$

$$\dot{P_{A♂}} = -(\mu_{A♂} + \mu^P).P_{A♂} \tag{8d}$$

$p_{X♀}$ is the proportion of females in the class $X$ (sub-adults or adults) infected during the first half of gestation. $T_{N,X♀}(t)$ is the number of new cases of transiently infected at time $t$ in females who will be part of the class $X$ (sub-adult or adult) at the time of mating.

$$p_{X♀}(t) = \frac{\sum\limits_{k=t-170}^{k=t-86} T_{N,X♀}(k)}{\sum\limits_{k=t-170}^{k=t-\frac{1}{\gamma}} T_{N,X♀}(k)} \tag{9}$$

Function of horizontal transmission designated by $f_i$ depends on the sex and age of animals. Thus, according to the contact matrix above, it is considered three forces of infection : $f_1$ for juvenile females and males, sub-adults females and adults females, $f_2$ for sub-adult males and $f_3$ for adults males. $N$ is the total population, $T$ and $P$ denote the set of transiently infected and persistently infected. A frequency-dependent function is used because it is considered that the contact structure is heterogeneous.

(10a) $$f_1 = \left(\beta_T.\frac{T-\tau(t).T_{A♂}}{N-\tau(t).A_♂} + \beta_P.\frac{P-\tau(t).P_{A♂}}{N-\tau(t).A_♂}\right)$$

(10b) $$f_2 = \left(\beta_T.\frac{T}{N} + \beta_P.\frac{P}{N}\right)$$

(10c) $$f_3 = \left(\beta_T.\frac{T-\tau(t).(T_B+T_Y+T_{Sa♀}+T_{A♀})}{N-\tau(t).(B+Y+Sa_♀+A_♀)} + \beta_P.\frac{P-\tau(t)(P_B+P_Y+P_{Sa♀}+P_{A♀})}{N-\tau(t).(B+Y+Sa_♀+A_♀)}\right)$$

**Density dependence**

Sigmoig function use for density dependence on the mortality rate of juveniles and the fertility rate of females subadults. I the case of fertility rate, the considered population size is at birth, i.e. two years before.

$$\mu_{Juv}(t) = \mu_{Juv}^{min} + \frac{\mu_{Juv}^{max} - \mu_{Juv}^{min}}{1 + exp(K \times d - d \times N(t))}$$

$$\eta_{Sa}(t) = \frac{\eta_{Sa}^{max}}{1 + exp(d \times N(t-2yrs) - K \times d)}$$

$$\eta_{SaP}(t) = \frac{\eta_{SaP}^{max}}{1 + exp(d \times N(t-2yrs) - K \times d)}$$