

Additional file to:
Ranking treatments in frequentist network meta-analysis works without re-sampling methods

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Probability of being best and AUC

We have

$$P(\mu_A > \mu_B) = \int_{-\infty}^{\infty} P(\mu_A > y) dP_{\mu_B} = \int_{-\infty}^{\infty} (1 - F_A(y)) dP_{\mu_B}.$$

Writing f_B for the density of μ_B , we obtain

$$P(\mu_A > \mu_B) = \int_{-\infty}^{\infty} (1 - F_A(y)) f_B(y) dy.$$

Substituting y by t with $t = 1 - F_B(y)$, we obtain $y = F_B^{-1}(1 - t)$ and $dt = -f_B(y) dy$ and thus

$$P(\mu_A > \mu_B) = \int_0^1 1 - F_A(F_B^{-1}(1 - t)) dt.$$

Interpreting

$$R(t) = 1 - F_A(F_B^{-1}(1 - t))$$

as a receiver operating characteristic (ROC) function characterizing the distance of the distributions F_A and F_B , we see that

$$P(\mu_A > \mu_B) = \int_0^1 R(t) dt$$

corresponds to the area under the ROC curve (AUC).