10.1148/radiol.15141850

Appendix E1

Biexponential Analysis

The porosity index derived by using only two echoes was compared with the pore water fraction estimated from biexponential fitting of the 23-echo UTE MR imaging data that used a two-compartment model of bound and pore water involving two T2* time constants:

$$S(t) = A_{short}\, e^{-t/T2^*(short)} + A_{long}\, e^{-t/T2^*(long)} \quad (2)$$

where S (t) is the UTE signal at time (t) following the excitation pulse, and A_{short} and A_{long} coefficients represent the fraction of bound and pore water, respectively (45). The pore water fraction was defined as PWF = $A_{long}/(A_{short} + A_{long})$.

Effect of Pore Water T_2* Simulation

To investigate the impact of bound water T_2*, porosity index was analytically calculated by using the following equation, which was derived by combining Equations 1 and 2:

$$\mathrm{PI}\ (\%) = \frac{(1-PWF)\cdot E\left(TE_{long}, T2^*_{(short)}\right) + PWEF\cdot E\left(TE_{long}, T2^*_{(long)}\right)}{(1-PWF)\cdot E\left(TE_{long}, T2^*_{(short)}\right) + PWEF\cdot E\left(TE_{long}, T2^*_{(long)}\right)}\cdot 100$$

(3)

where E (TE,T2*) = $e^{-TE/T2^*}$. Pore water fraction (PWF) was varied in the range 10%–50% to simulate young and old bone. Bound water T2* (ie, $T2^*_{[short]}$) was fixed at 300 μs because its variation is small compared with pore water T2* (ie, $T2^*_{[long]}$), which was varied in the range of 1–10 msec to simulate values that corresponded to various pore sizes found in young and old cortical bone. The first echo in the dual-echo sequence was set to the minimum TE allowed by the imager (ie, TE_{short} = 50 μs). The optimum timing of the second echo was determined by simulating TEs in the range of 1–10 msec so that the dynamic range of porosity index was maximized for a wide range of T2*(long) values while keeping the TEs as short as possible to minimize the signal-to-noise ratio loss in the second echo. The dynamic range of porosity index was defined as the percentage difference in porosity index, which corresponded to 10% and 50% Pore water fraction values for each T2*(long) simulated in the range of 1–10 msec. Because TE_{short} is smaller than TE_{long} and T_2*(long), Equation 3 can be simplified (although not required for the work presented in our study) as follows:

$$\mathrm{PI} \approx \mathrm{PWF}\cdot \mathrm{E}\left(\mathrm{TE}_{long}, \mathrm{T2}^*_{(long)}\right) \quad (4).$$