Supplemental material:
Transition to chaos in random networks with cell-type-specific connectivity

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S1 Largest Lyapunov exponent

We used the standard method [1] to compute the largest Lyapunov exponent for the system parameterized as follows: $D = 2$, $\alpha_1 = \alpha_2 = \frac{1}{2}$ and

$$\mathbf{g} = \begin{pmatrix} \kappa/2 & 1/2 \\ 2 & \kappa/2 \end{pmatrix}. \tag{S1}$$

When κ is varied between 0.3 and 3, Λ_1 varies between 0.71 and 1.27. For this parametrization, the average synaptic gain $\bar{g} = (\sum_{c,d=1}^{D} \alpha_c \alpha_d g_{cd}^2)^{\frac{1}{2}}$ is greater than 1 for all values of κ in that range (see Fig. S1). Since N is finite the transition does not occur exactly at $\Lambda_1 = 1$. It is however in excellent agreement with the Lyapunov exponent for the single cell-type ($D = 1$) system (when N is the same for the two networks and g is matched to $\sqrt{\Lambda_1}$).

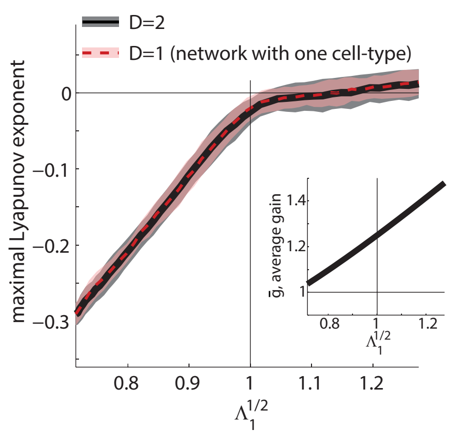

Figure S1: The maximal Lyapunov exponent for 25 realizations in each value of Λ_1 for a family of $D = 2$ networks (parameters in text) compared with a network with a single cell-type ($D = 1$). Inset: the value of the average synaptic gain $\bar{g}$ is always greater than 1 for this parametrization.

S2 Universality and sparsity

In the main text we have discussed connectivity matrices with elements drawn from Gaussian distributions. However, Girko's circular law is universal, meaning that the

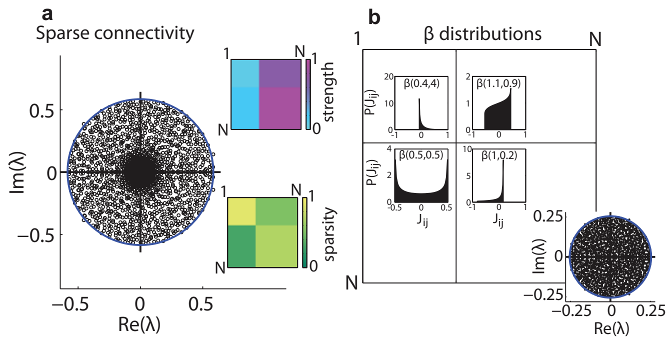

Figure S2: Universality and sparse connectivity. (a) Our results extend to sparse connectivity matrices, an example matrix with non-Gaussian element distributions. The formula for the radius (blue circle) is in agreement with the numerical results. Insets shows average synaptic strengths, $\mathbf{G}$ (top) and the sparsity levels (bottom). (b) In each block the elements of $\mathbf{J}$ were drawn from a centered β distribution with different parameters leading to skewed and bimodal distributions.

spectral density of connectivity matrices describing single cell-type networks depends only on the second moment of the matrix entry distribution [2] (as long as the mean remains zero). This suggests that our results for matrices with block structured variances, extend to non-Gaussian distributions, provided that $\langle J_{ij} \rangle = 0$ and $N\langle J_{ij}^2 \rangle = g_{c_i d_j}^2 < \infty$.

Using numerical simulations, we have verified that the formula for the spectral radius (Eq. (8) in the main text) holds for a number of non-Gaussian matrix element distributions, including networks where connection strengths were taken from sparse and β distributions (Fig. S2). In the sparse example, s_{cd} is the fraction of nonzero elements, randomly drawn from a Gaussian distribution with variance g_{cd}^2/N. The block-wise variance is therefore $s_{cd} g_{cd}^2/N$, and eigenvalues are bounded by a circle with radius calculated using $M_{cd} = \alpha_d s_{cd} g_{cd}^2$.

S3 Effective and average synaptic gain

We now demonstrate that the network with cell-type dependent connectivity, and specifically our simplified model of adult neurogenesis, can have an increased computational capacity when compared to a network with a single cell type. In the main text we show that the effective gain, $\sqrt{\Lambda_1}$, serves as a good coordinate to describe the network's capacity to perform computational tasks. We have also shown that $\sqrt{\Lambda_1}$ is in general different from the average synaptic gain in the network, $\bar{g}$. The average gain is the gain one would get by starting with a network with cell-type dependent connectivity and randomly permuting all the entries of the connectivity matrix.

A simple calculation shows that for the parametrization $(\alpha_1, \gamma, \epsilon)$, for any $\alpha_1 \in (0,1)$ and for any $\gamma > (1-\epsilon) > 0$ the effective gain is larger than the average gain. We plot

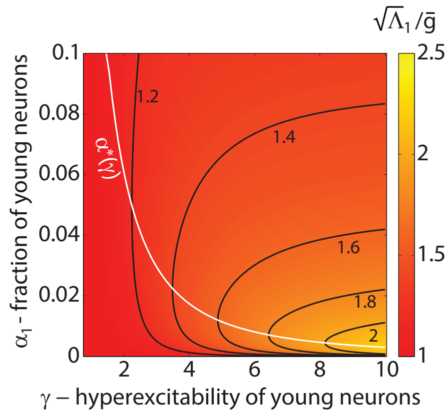

Figure S3: Ratio of effective and average synaptic gain in a simplified model of adult neurogenesis. In the entire $\gamma - \alpha_1$ plane $\sqrt{\Lambda_1}/\bar{g} > 1$. This ratio is equal to 1 by definition for $\alpha_1 = 0$ and $\alpha_1 = 1$ (networks with one group of neurons). In black we plot the contour lines of the ratio and in white we plot $\alpha^\star(\gamma)$, the value of α_1 such that the ratio is maximal for a given γ.

the ratio $\sqrt{\Lambda_1}/\bar{g}$ for fixed $\epsilon = 0.2$ and $\alpha_1, \ \gamma$ in the same range used in the main text (α_1 is extended to 0).

References

[1] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, Physica D: Nonlinear Phenomena **16**, 285 (1985).

[2] T. Tao, V. Vu, and M. Krishnapur, The Annals of Probability **38**, 2023 (2010).