Supplementary Material to *A Two-Sample Test for Equality of Means in High Dimension*

S.1 Complete Proof of Lemma 1 of the Main Paper

Lemma 1 *Let $X_{1j},\dots,X_{nj}$ and $Y_{1j},\dots,Y_{mj}$ be independent identically distributed random samples with $Var(X_{1j})=\sigma_{1j}^2$ and $Var(Y_{1j})=\sigma_{2j}^2$ and $EX_{1j}=EY_{1j}$ for all $j=1,\dots,p$. Assume that $\max\{E|X_{1j}|^{16},E|Y_{1j}|^{16},j=1,\dots,p\}=O(1)$ and that $\min\{\sigma_{1j}^2,\sigma_{2j}^2\}>c>0$. Let*

$$t_{nj}^2=n(\overline{X}_{nj}-\overline{Y}_{mj})^2\{s_{nj}^2+(n/m)\vartheta_{mj}^2\}^{-1}$$

where s_n^2 and ϑ_m^2 are the two sample variances and let $m\sim n$ as $n\to\infty$. Then $E(t_{nj}^2)=1+n^{-1}c_{nj}+n^{-2}d_{nj}+O(n^{-3})$ for

$$c_{nj} = \tau_{nj}^{-2}\{\sigma_{1j}^2+(n/m)^2\sigma_{2j}^2\}+2\tau_{nj}^{-6}\{\mu_{3j}'+(n/m)^2\eta_{3j}'\}^2 \tag{S.1}$$

and

$$\begin{aligned} d_{nj} = {} & \tau_{nj}^{-4}[\{\sigma_{1j}^2+(n/m)^2\sigma_{2j}^2\}-\{(\mu_{4j}'-3\sigma_{1j}^4)+(n/m)^4(\eta_{4j}'-3\sigma_{2j}^4)\}] \\ & +\tau_{nj}^{-6}\{\sigma_{1j}^2+(n/m)^2\sigma_{2j}^2\}\{(\mu_{4j}'-\sigma_{1j}^4)+(n/m)^3(\eta_{4j}'-\sigma_{2j}^4)\} \\ & -4\tau_{nj}^{-6}\{\mu_{3j}'+(n/m)^2\eta_{3j}'\}\{\mu_{3j}'+(n/m)^3\eta_{3j}'\} \\ & -2\tau_{nj}^{-6}\{(\mu_{3j}')^2+(n/m)^5(\eta_{3j}')^2\} \\ & -6\tau_{nj}^{-8}\{\mu_{3j}'+(n/m)^2\eta_{3j}'\}\{\mu_{5j}'-2\mu_{3j}'\sigma_{1j}^2+(n/m)^4(\eta_{5j}'-2\eta_{3j}'\sigma_{2j}^2)\} \\ & -3\tau_{nj}^{-8}\{(\mu_{4j}'-\sigma_{1j}^4)+(n/m)^3(\eta_{4j}'-\sigma_{2j}^4)\}^2 \\ & +6\tau_{nj}^{-8}\{\sigma_{1j}^2+(n/m)^2\sigma_{2j}^2\}\{\mu_{3j}'+(n/m)^2\eta_{3j}'\}^2 \\ & +3\tau_{nj}^{-10}\{\sigma_{1j}^2+(n/m)\sigma_{2j}^2\}\{(\mu_{4j}'-\sigma_{1j}^4)+(n/m)^3(\eta_{4j}'-\sigma_{2j}^4)\}^2 \\ & +12\tau_{nj}^{-10}\{\mu_{3j}'+(n/m)^2\eta_{3j}'\}^2\{(\mu_{4j}'-\sigma_{1j}^4)+(n/m)^3(\eta_{4j}'-\sigma_{2j}^4)\}, \end{aligned} \tag{S.2}$$

where $\tau_{nj}^2=\{\sigma_{1j}^2+(n/m)\sigma_{2j}^2\}$ and μ_{kj}' and η_{kj}' are the kth central moments of X_{1j} and Y_{1j}, respectively.

Proof 1 (Expanded Proof of Lemma 1) *For ease of syntax, ignore the subscript j, and, without loss of generality, assume that $EX_{1j}=EY_{1j}=0$. Let $\Delta_n=s_n^2-\sigma_1^2+(n/m)(\vartheta_m^2-\sigma_2^2)$*

and let t_n^2 be approximated by the expansion

$$\tilde{t}_n^2 = n(\overline{X}_n - \overline{Y}_m)^2(\tau_n^{-2} - \tau_n^{-4}\Delta_n + \tau_n^{-6}\Delta_n^2 - \tau_n^{-8}\Delta_n^3 + \tau_n^{-10}\Delta_n^4) \tag{S.3}$$

so that $t_n^2 = \tilde{t}_n^2 + O_p(n^{-3})$. An expression for the expected value $E(\tilde{t}_{nj}^2)$ would thus involve the quantities $n\tau_n^{-2k}E(\overline{X}_n - \overline{Y}_m)^2\Delta_n^{k-1}$ for $k = 1, \ldots, 5$. These expectations must be computed such that they retain terms out to the order of $O(n^{-3})$.

Let $\chi_{|B|}(\{X_j : j \in B\})$ represent the joint cumulant of the random variables in the set $\{X_j : j \in B\}$, where $|B|$ is the cardinality of B. Then the formula

$$E(X_1 \ldots X_k) = \Sigma_\pi \Pi_{B\in\pi}\chi_{|B|}(\{X_j : j \in B\}) \tag{S.4}$$

from Leonov & Shiryaev (1959) gives the expected value of a product of random variables in terms of joint cumulants, where Σ_π denotes summation over all possible partitions of $\{X_1, \ldots, X_k\}$, and $\Pi_{B\in\pi}$ denotes the product over all cells of the partition π. Using (S.4) to compute $E(\overline{X}_n - \overline{Y}_m)^2\Delta_n^{k-1}$ to within $O(n^{-4})$of their true values for $k = 1, \ldots, 5$ involves the joint cumulants tabulated below, where $\Delta \equiv \Delta_n$, $\overline{X} \equiv \overline{X}_n$, and $\overline{Y} \equiv \overline{Y}_m$.

	0	*1*	*2*
0		$\chi_1(\overline{X}-\overline{Y})$	$\chi_2(\overline{X}-\overline{Y},\overline{X}-\overline{Y})$
1	$\chi_1(\Delta)$	$\chi_2(\Delta,\overline{X}-\overline{Y})$	$\chi_3(\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y})$
2	$\chi_2(\Delta,\Delta)$	$\chi_3(\Delta,\Delta,\overline{X}-\overline{Y})$	$\chi_4(\Delta,\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y})$
3	$\chi_3(\Delta,\Delta,\Delta)$	$\chi_4(\Delta,\Delta,\Delta,\overline{X}-\overline{Y})$	$\chi_5(\Delta,\Delta,\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y})$

If $\kappa(i,j)$ denotes the ijth member of the table of joint cumulants, then (A.6) gives

$$E(\overline{X}-\overline{Y})^2 = \kappa(0,2) + O(n^{-4}) \tag{S.5}$$

$$E(\overline{X}-\overline{Y})^2\Delta = \kappa(1,2) + \kappa(0,2)\kappa(1,0) + O(n^{-4}) \tag{S.6}$$

$$\begin{aligned} E(\overline{X}-\overline{Y})^2\Delta^2 &= \kappa(2,2) + 2\kappa(1,0)\kappa(1,2) + \kappa(0,2)\kappa(2,0) \\ &\quad + 2\kappa^2(1,1) + \kappa(0,2)\kappa^2(1,0) + O(n^{-4}) \end{aligned} \tag{S.7}$$

$$\begin{aligned} E(\overline{X}-\overline{Y})^2\Delta^3 &= \kappa(0,2)\kappa(3,0) + 6\kappa(1,1)\kappa(2,1) \\ &\quad + 3\kappa(2,0)\kappa(1,2) + 3\kappa(1,0)\kappa(2,0)\kappa(0,2) \\ &\quad + 6\kappa(1,0)\kappa^2(1,1) + O(n^{-4}) \end{aligned} \tag{S.8}$$

$$E(\overline{X}-\overline{Y})^2\Delta^4 = 3\kappa(0,2)\kappa^2(2,0) + 12\kappa^2(1,1)\kappa(2,0) + O(n^{-4}) \tag{S.9}$$

after removing cumulant products of order smaller than $O(n^{-4})$ *and noting that* $\kappa(0,1) = 0$.

Each cumulant is simplified using rules found in Brillinger (1981), and the formula

$$\chi_k(X_1, \ldots, X_k) = \Sigma_\pi (-1)^{(|\pi|-1)}(|\pi|-1)! \, \Pi_{B\in\pi} E(\Pi_{i\in B} X_i) \tag{S.10}$$

from Leonov & Shiryaev (1959) provides expressions for the simplified cumulants in terms of moments. The cumulants are computed below, where each cumulant is either given exactly, or is approximated to the order necessary for the cumulant products in (S.5)–(S.9) to lie within $O(n^{-4})$ *of their true values.*

$$
\begin{aligned}
\kappa(0,1) &= \chi_1(\overline{X}-\overline{Y}) = E(\overline{X}-\overline{Y}) = 0 \\
\kappa(0,2) &= \chi_2(\overline{X}-\overline{Y},\overline{X}-\overline{Y}) = E(\overline{X}-\overline{Y})^2 - \{E(\overline{X}-\overline{Y})\}^2 \\
&= n^{-1}\{\sigma_1^2 + (n/m)\sigma_2^2\} \\
\kappa(1,0) &= \chi_1(\Delta) = E\{(s^2-\sigma_1^2) + (n/m)(\vartheta^2-\sigma_2^2)\} = -\{\sigma_1^2/n - (n/m)\sigma_2^2/m\} \\
&= -n^{-1}\{\sigma_1^2 + (n/m)^2\sigma_2^2\} \\
\kappa(1,1) &= \chi_2(\Delta,\overline{X}-\overline{Y}) = \chi_2(\overline{X^2}-\overline{X}^2,\overline{X}) + (n/m)\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= \chi_2(\overline{X^2},\overline{X}) - \chi_2(\overline{X}^2,\overline{X}) + (n/m)\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= n^{-1}\chi_2(X_1^2,X_1) - n^{-3}\chi_2(\Sigma_i X_i^2 + \Sigma_{i\neq j}X_iX_j, \Sigma_i X_i) + (n/m)\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= n^{-1}\mu_3' - n^{-2}\chi_2(X_1^2 + X_1\Sigma_{j=2}^n X_j, X_1) + (n/m)\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= (n^{-1}-n^{-2})\mu_3' + n^{-2}(n-1)\chi_2(X_1X_2,X_1) + (n/m)\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= (n^{-1}-n^{-2})\mu_3' + (n/m)(m^{-1}-m^{-2})\eta_3' \\
\kappa(1,2) &= \chi_3(\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y}) = \chi_3(\overline{X^2}-\overline{X}^2,\overline{X},\overline{X}) + (n/m)\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y},\overline{Y}) \\
&= n^{-2}\chi_3(X_1^2,X_1,X_1) - n^{-3}\chi_3(X_1^2,X_1,X_1) - n^{-4}\chi_3(\Sigma_i X_iX_j,\Sigma_i X_i,\Sigma_i X_i) \\
&\quad + (n/m)\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y},\overline{Y}) \\
&= (n^{-2}-n^{-3})(\mu_4'-\sigma_1^4) + (n/m)(m^{-2}-m^{-3})(\eta_4'-\sigma_2^4) \\
\kappa(2,0) &= \chi_2(\Delta,\Delta) = \chi_2(\overline{X^2}-\overline{X}^2,\overline{X^2}-\overline{X}^2) + (n/m)^2\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= \chi_2(\overline{X^2},\overline{X^2}) - 2\chi_2(\overline{X^2},\overline{X}^2) + (\overline{X}^2,\overline{X}^2) + (n/m)^2\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= n^{-1}\chi_2(X_1^2,X_1^2) - 2n^{-3}\{\chi_2(\Sigma_i X_i^2,\Sigma_i X_i^2) + \chi_2(\Sigma_{i\neq j}X_iX_j,\Sigma_i X_i^2)\} \\
&\quad + n^{-4}\{\chi_2(\Sigma_i X_i^2,\Sigma_i X_i^2) - 2\chi_2(\Sigma_{i\neq j}X_iX_j,\Sigma_i X_i^2) + \chi_2(\Sigma_{i\neq j}X_iX_j,\Sigma_{i\neq j}X_iX_j)\} \\
&\quad + (n/m)^2\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= (n^{-1}-2n^{-2}+n^{-3})(\mu_4'-\sigma_1^4) + n^{-4}\chi_2(\Sigma_{i\neq j}X_iX_j,\Sigma_{i\neq j}X_iX_j) \\
&\quad + (n/m)^2\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= (n^{-1}-2n^{-2}+n^{-3})(\mu_4'-\sigma_1^4) + \frac{2(n-1)}{n^3}\sigma_1^4 + (n/m)^2\chi_2(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= n^{-3}(n-1)^2\mu_4' - n^{-3}(n-1)(n-3)\sigma_1^4 \\
&\quad + (n/m)^2\{m^{-3}(m-1)^2\eta_4' - m^{-3}(m-1)(m-3)\sigma_2^4\} \\
&= (n^{-1}-2n^{-2})\mu_4' - (n^{-1}-4n^{-2})\sigma_1^4 \\
&\quad + (n/m)^2\{(m^{-1}-2m^{-2})\eta_4' - (m^{-1}-4m^{-2})\sigma_2^4\} + O(n^{-3})
\end{aligned}
$$

$$\begin{aligned}
\kappa(2,1) &= \chi_3(\Delta,\Delta,\overline{X}-\overline{Y}) \\
&= \chi_3(\overline{X^2}-\overline{X}^2,\overline{X^2}-\overline{X}^2,\overline{X}) + (n/m)^2\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2,\overline{Y}) \\
&= \chi_3(\overline{X^2},\overline{X^2},\overline{X}) + (n/m)^2\chi_3(\overline{Y^2},\overline{Y^2},\overline{Y}) + O(n^{-3}) \\
&= n^{-2}\chi_3(X_1^2,X_1^2,X_1) + (n/m)^2 m^{-2}\chi_3(Y_1^2,Y_1^2,Y_1) + O(n^{-3}) \\
&= n^{-2}(\mu_5' - 2\mu_3'\sigma_1^2) + (n/m)^2 m^{-2}(\eta_5' - 2\eta_3'\sigma_2^2) + O(n^{-3}) \\
&= n^{-2}\{(\mu_5' - 2\mu_3'\sigma_1^2) + (n/m)^4(\eta_5' - 2\eta_3'\sigma_2^2)\} + O(n^{-3}) \\
\kappa(2,2) &= \chi_4(\Delta,\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y}) \\
&= \chi_4(\overline{X^2}-\overline{X}^2,\overline{X^2}-\overline{X}^2,\overline{X},\overline{X}) + (n/m)^2\chi_4(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2,\overline{Y},\overline{Y}) \\
&= n^{-3}\chi_4(X_1^2,X_1^2,X_1,X_1) + (n/m)^2 m^{-3}\chi_4(Y_1^2,Y_1^2,Y_1,Y_1) + O(n^{-4}) \\
&= n^{-3}[\mu_6' - 3\sigma_1^2\mu_4' - 2(\mu_3')^2 + 2\sigma_1^6 + (n/m)^5\{\eta_6' - 3\sigma_2^2\eta_4' - 2(\eta_3')^2 + 2\sigma_2^6\}] + O(n^{-3}) \\
\kappa(3,0) &= \chi_3(\Delta,\Delta,\Delta) = \chi_3(\overline{X^2}-\overline{X}^2,\overline{X^2}-\overline{X}^2,\overline{X^2}-\overline{X}^2) \\
&\quad + (n/m)^3\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= \chi_3(\overline{X^2},\overline{X^2},\overline{X^2}) - 3\chi_3(\overline{X^2},\overline{X^2},\overline{X}^2) \\
&\quad + 3\chi_3(\overline{X^2},\overline{X}^2,\overline{X}^2) + \chi_3(\overline{X}^2,\overline{X}^2,\overline{X}^2) \\
&\quad + (n/m)^3\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) \\
&= n^{-2}\chi_3(X_1^2,X_1^2,X_1^2) \\
&\quad + (n/m)^3\chi_3(\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2,\overline{Y^2}-\overline{Y}^2) + O(n^{-3}) \\
&= n^{-2}\{(\mu_6' - 3\sigma_1^2\mu_4' + 2\sigma_1^6) + (n/m)^5(\eta_6' - 3\sigma_2^2\eta_4' + 2\sigma_2^6)\} + O(n^{-3}) \\
\kappa(3,2) &= \chi_5(\Delta,\Delta,\Delta,\overline{X}-\overline{Y},\overline{X}-\overline{Y}) = O(n^{-4})
\end{aligned}$$

Plugging the above expressions into (S.5)–(S.9) and dropping terms of smaller order than $O(n^{-3})$ yields

$$
\begin{aligned}
n\tau_n^{-2}E(\overline{X}_n - \overline{Y}_m)^2 &= 1 \\
n\tau_n^{-4}E(\overline{X}_n - \overline{Y}_m)^2\Delta_n &= n^{-1}\tau_n^{-4}[(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)] \\
&\quad - n^{-1}\tau_n^{-2}[\sigma_1^2 + (n/m)^2\sigma_2^2] \\
&\quad - n^{-2}\tau_n^{-4}[(\mu_4' - \sigma_1^4) + (n/m)^4(\eta_4' - \sigma_2^4)] \\
n\tau_{m,n}^{-6}E(\overline{X}_n - \overline{Y}_m)^2\Delta_n^2 &= n^{-1}\tau_n^{-4}[(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)] \\
&\quad + n^{-2}\tau_n^{-4}[\sigma_1^2 + (n/m)^2\sigma_2^2]^2 \\
&\quad - n^{-2}\tau_n^{-4}[(2\mu_4' - 4\sigma_1^4) + (n/m)^4(2\eta_4' - 4\sigma_1^4)] \\
&\quad + 2n^{-1}\tau_n^{-6}[\mu_3' + (n/m)^2\eta_3']^2 \\
&\quad - 4n^{-2}\tau_n^{-6}[\mu_3' + (n/m)^2\eta_3'][\mu_3' + (n/m)^3\eta_3'] \\
&\quad - 2n^{-2}\tau^{-6}[\sigma_1^2 + (n/m)^2\sigma_2^2][(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)] \\
&\quad + n^{-2}\tau_n^{-6}[\mu_6' - 3\sigma_1^2\mu_4' - 2(\mu_3')^2 + 2\sigma_1^6] \\
&\quad + n^{-2}\tau_n^{-6}(n/m)^5[\eta_6' - 3\sigma_2^2\eta_4' - 2(\eta_3')^2 + 2\sigma_2^6] \\
&\quad + O(n^{-3}) \\
n\tau_n^{-8}E(\overline{X}_n - \overline{Y}_m)^2\Delta_n^3 &= n^{-2}\tau_n^{-6}[\mu_6' - 3\sigma_1^2\mu_4' + 2\sigma_1^6 + (n/m)^5(\eta_6' - 3\sigma_2^2\eta_4' + 2\sigma_2^6)] \\
&\quad - 3n^{-2}\tau_n^{-6}[\sigma_1^2 + (n/m)^2\sigma_2^2][(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)] \\
&\quad + 6n^{-2}\tau_n^{-8}[\mu_3' + (n/m)^2\eta_3'][(\mu_5' - 2\mu_3'\sigma_1^2) + (n/m)^4(\eta_5' - 2\eta_3'\sigma_2^2)] \\
&\quad + 3n^{-2}\tau_n^{-8}[(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)]^2 \\
&\quad - 6n^{-2}\tau_n^{-8}[\sigma_1^2 + (n/m)^2\sigma_2^2][\mu_3' + (n/m)^2\eta_3']^2 \\
&\quad + O(n^{-3}) \\
n\tau_n^{-10}E(\overline{X}_n - \overline{Y}_m)^2\Delta_n^4 &= 3n^{-2}\tau_n^{-10}[\sigma_1^2 + (n/m)\sigma_2^2][(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)]^2 \\
&\quad + 12n^{-2}\tau_n^{-10}[\mu_3' + (n/m)^2\eta_3']^2[(\mu_4' - \sigma_1^4) + (n/m)^3(\eta_4' - \sigma_2^4)] \\
&\quad + O(n^{-3})
\end{aligned}
$$

Adding and subtracting these quantities according to the expansion in (S.3) and gathering terms out of which n^{-1} and n^{-2} can be factored yields c_n from (S.1) and d_n from (S.2), respectively, thus completing the proof.

S.2 Additional Simulation Output

$p = 300$, $\Sigma_1 = \Sigma_2$			Normal(0, 1) Innovations								
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.07	0.04	0.09	0.06	0.07	0.07	0.06	0.08	0.07
		ARMA	0.06	0.04	0.08	0.06	0.07	0.07	0.07	0.08	0.07
		LR	0.05	0.04	0.10	0.06	0.06	0.07	0.08	0.09	0.07
90	120	IND	0.05	0.04	0.07	0.06	0.06	0.06	0.07	0.08	0.06
		ARMA	0.05	0.04	0.06	0.07	0.07	0.08	0.08	0.09	0.08
		LR	0.03	0.03	0.07	0.05	0.05	0.07	0.06	0.08	0.07
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.07	0.04	0.09	0.07	0.07	0.07	0.07	0.08	0.07
		ARMA	0.06	0.04	0.08	0.07	0.07	0.07	0.07	0.08	0.07
		LR	0.05	0.04	0.10	0.07	0.07	0.08	0.08	0.09	0.08
90	120	IND	0.05	0.04	0.07	0.06	0.06	0.07	0.07	0.08	0.07
		ARMA	0.05	0.04	0.06	0.08	0.08	0.08	0.08	0.09	0.08
		LR	0.03	0.03	0.07	0.06	0.06	0.06	0.07	0.09	0.06

Table S.1: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under Normal$(0, 1)$ innovations with $\Sigma_1 = \Sigma_2$.

$p = 300$, $\Sigma_2 = 2\Sigma_1$			Normal(0, 1) Innovations								
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.07	0.08	0.05	0.05	0.06	0.05	0.08	0.06
		ARMA	0.04	0.03	0.04	0.06	0.07	0.07	0.07	0.08	0.07
		LR	0.05	0.05	0.04	0.06	0.06	0.07	0.08	0.09	0.07
90	120	IND	0.07	0.12	0.07	0.07	0.07	0.08	0.08	0.10	0.08
		ARMA	0.05	0.05	0.04	0.07	0.07	0.08	0.08	0.09	0.08
		LR	0.06	0.09	0.06	0.05	0.06	0.06	0.07	0.08	0.06
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.07	0.08	0.05	0.06	0.06	0.07	0.08	0.06
		ARMA	0.04	0.03	0.04	0.09	0.09	0.09	0.08	0.09	0.09
		LR	0.05	0.05	0.04	0.06	0.06	0.07	0.08	0.08	0.07
90	120	IND	0.07	0.12	0.07	0.08	0.08	0.08	0.08	0.11	0.08
		ARMA	0.05	0.05	0.04	0.07	0.08	0.08	0.08	0.09	0.08
		LR	0.06	0.09	0.06	0.06	0.06	0.07	0.08	0.09	0.07

Table S.2: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under Normal$(0, 1)$ innovations with $\Sigma_2 = 2\Sigma_1$.

$p = 300$, $\Sigma_1 = \Sigma_2$						Centered Gamma$(4, 2)$ Innovations					
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.05	0.03	0.11	0.05	0.05	0.06	0.07	0.07	0.06
		ARMA	0.04	0.03	0.06	0.07	0.07	0.08	0.08	0.09	0.08
		LR	0.04	0.01	0.08	0.06	0.06	0.07	0.06	0.07	0.07
90	120	IND	0.04	0.03	0.09	0.06	0.07	0.07	0.08	0.09	0.07
		ARMA	0.04	0.03	0.07	0.06	0.06	0.06	0.06	0.07	0.06
		LR	0.04	0.03	0.06	0.05	0.05	0.05	0.06	0.07	0.05
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.05	0.03	0.11	0.07	0.08	0.09	0.09	0.11	0.09
		ARMA	0.04	0.03	0.06	0.08	0.09	0.09	0.09	0.10	0.09
		LR	0.04	0.01	0.08	0.08	0.08	0.09	0.09	0.11	0.09
90	120	IND	0.04	0.03	0.09	0.08	0.09	0.09	0.10	0.11	0.09
		ARMA	0.04	0.03	0.07	0.07	0.07	0.07	0.07	0.07	0.07
		LR	0.04	0.03	0.06	0.06	0.06	0.07	0.07	0.08	0.07

Table S.3: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under centered gamma$(4, 2)$ innovations with $\Sigma_1 = \Sigma_2$.

$p = 300$, $\Sigma_2 = 2\Sigma_1$						Centered Gamma$(4, 2)$ Innovations					
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.08	0.07
		ARMA	0.05	0.04	0.03	0.06	0.07	0.07	0.07	0.07	0.07
		LR	0.04	0.07	0.04	0.05	0.05	0.06	0.07	0.07	0.06
90	120	IND	0.05	0.08	0.09	0.05	0.06	0.06	0.07	0.07	0.06
		ARMA	0.05	0.05	0.04	0.06	0.06	0.07	0.07	0.08	0.07
		LR	0.04	0.08	0.05	0.05	0.05	0.06	0.06	0.07	0.06
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.05	0.06	0.06	0.08	0.09	0.09	0.09	0.11	0.09
		ARMA	0.05	0.04	0.03	0.07	0.08	0.08	0.09	0.09	0.08
		LR	0.04	0.07	0.04	0.09	0.09	0.10	0.10	0.11	0.10
90	120	IND	0.05	0.08	0.09	0.05	0.06	0.06	0.07	0.08	0.06
		ARMA	0.05	0.05	0.04	0.08	0.08	0.08	0.07	0.09	0.08
		LR	0.04	0.08	0.05	0.08	0.08	0.08	0.09	0.09	0.08

Table S.4: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under centered gamma$(4, 2)$ innovations with $\Sigma_2 = 2\Sigma_1$.

$p = 300$, $\Sigma_1 = \Sigma_2$						Double Pareto(16.5, 8) Innovations					
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.02	0.10	0.05	0.05	0.06	0.06	0.08	0.06
		ARMA	0.04	0.02	0.06	0.06	0.07	0.07	0.08	0.08	0.07
		LR	0.04	0.03	0.07	0.06	0.06	0.06	0.06	0.08	0.06
90	120	IND	0.03	0.03	0.09	0.05	0.06	0.06	0.06	0.07	0.06
		ARMA	0.05	0.04	0.03	0.08	0.08	0.08	0.08	0.09	0.08
		LR	0.05	0.04	0.05	0.06	0.06	0.06	0.07	0.08	0.06
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.02	0.10	0.05	0.04	0.04	0.05	0.05	0.04
		ARMA	0.04	0.02	0.06	0.09	0.08	0.08	0.08	0.10	0.08
		LR	0.04	0.03	0.07	0.08	0.08	0.08	0.08	0.08	0.08
90	120	IND	0.03	0.03	0.09	0.05	0.05	0.06	0.06	0.06	0.06
		ARMA	0.05	0.04	0.03	0.09	0.09	0.09	0.09	0.09	0.09
		LR	0.05	0.04	0.05	0.06	0.06	0.07	0.08	0.08	0.07

Table S.5: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under double Pareto$(16.5, 8)$ innovations with $\Sigma_1 = \Sigma_2$.

$p = 300$, $\Sigma_2 = 2\Sigma_1$						Double Pareto(16.5, 8) Innovations					
$\widehat{\xi}_n \equiv 1$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.06	0.07	0.06	0.06	0.06	0.07	0.07	0.06
		ARMA	0.04	0.03	0.03	0.05	0.05	0.05	0.06	0.07	0.05
		LR	0.05	0.04	0.06	0.06	0.06	0.05	0.06	0.07	0.05
90	120	IND	0.05	0.09	0.04	0.07	0.07	0.07	0.07	0.08	0.07
		ARMA	0.05	0.07	0.02	0.08	0.08	0.09	0.09	0.11	0.09
		LR	0.04	0.09	0.02	0.05	0.06	0.06	0.06	0.08	0.06
$\widehat{\xi}_n \equiv 1 + a_n/n + b_n/n^2$						Parzen Window			Trapezoid Window		
n	m	Cov	Ch-Q	SK	CLX	L = 10	L = 15	L = 20	L = 10	L = 15	L = 20
45	60	IND	0.04	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.08
		ARMA	0.04	0.03	0.03	0.07	0.06	0.07	0.07	0.08	0.07
		LR	0.05	0.04	0.06	0.06	0.06	0.07	0.06	0.08	0.07
90	120	IND	0.05	0.09	0.04	0.07	0.07	0.07	0.07	0.09	0.07
		ARMA	0.05	0.07	0.02	0.09	0.10	0.10	0.10	0.12	0.10
		LR	0.04	0.09	0.02	0.06	0.06	0.07	0.07	0.07	0.07

Table S.6: Type I error rates over $S = 500$ simulations with nominal size $\alpha = .05$ for the moderate- and large-p GCT under the Parzen and trapezoid lag windows at lengths $L = 10, 15, 20$ and for the Ch-Q, SK, CLX tests under double Pareto$(16.5, 8)$ innovations with $\Sigma_2 = 2\Sigma_1$.

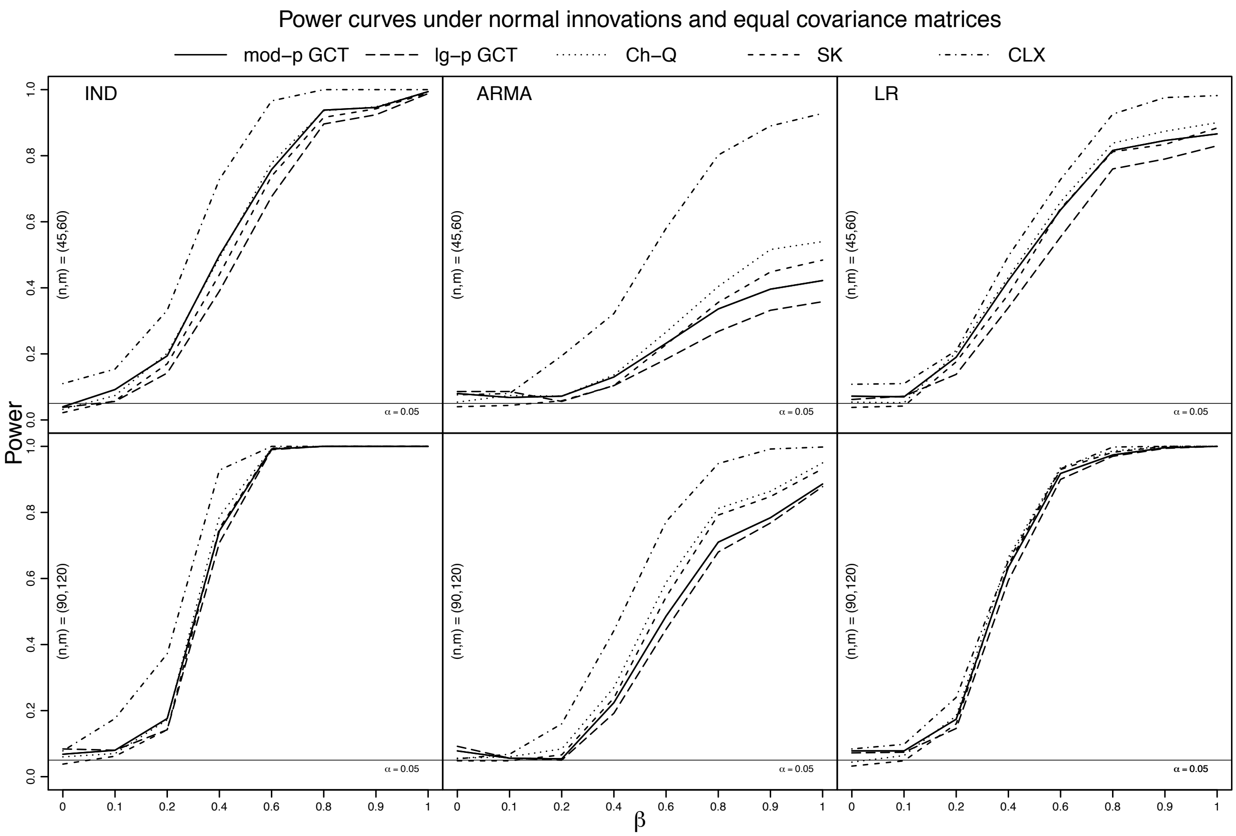

Figure S.1: Power curves at sample sizes $(n,m) = (45,60),(90,120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with Normal$(0,1)$ innovations and $\Sigma_1 = \Sigma_2$. Based on $S = 500$ simulations.

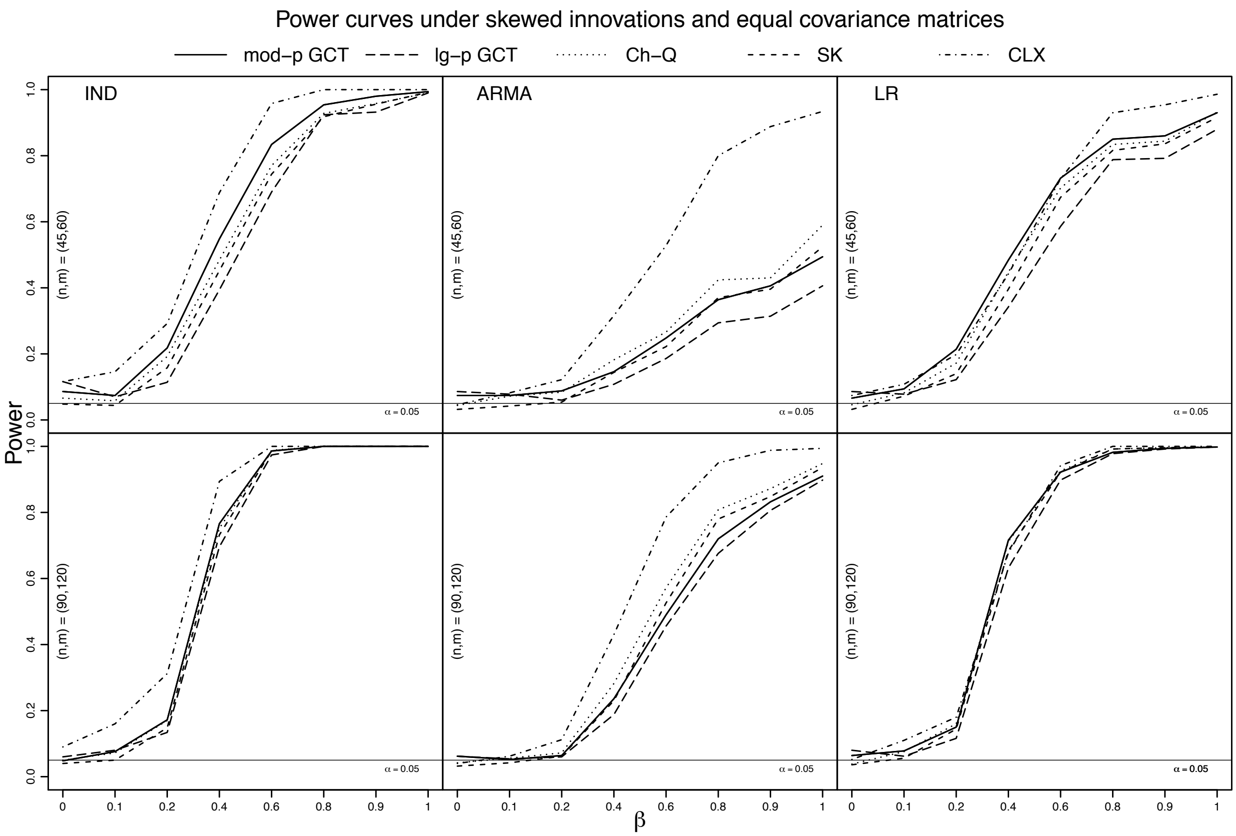

Figure S.2: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with centered gamma$(4, 2)$ innovations and $\Sigma_1 = \Sigma_2$. Based on $S = 500$ simulations.

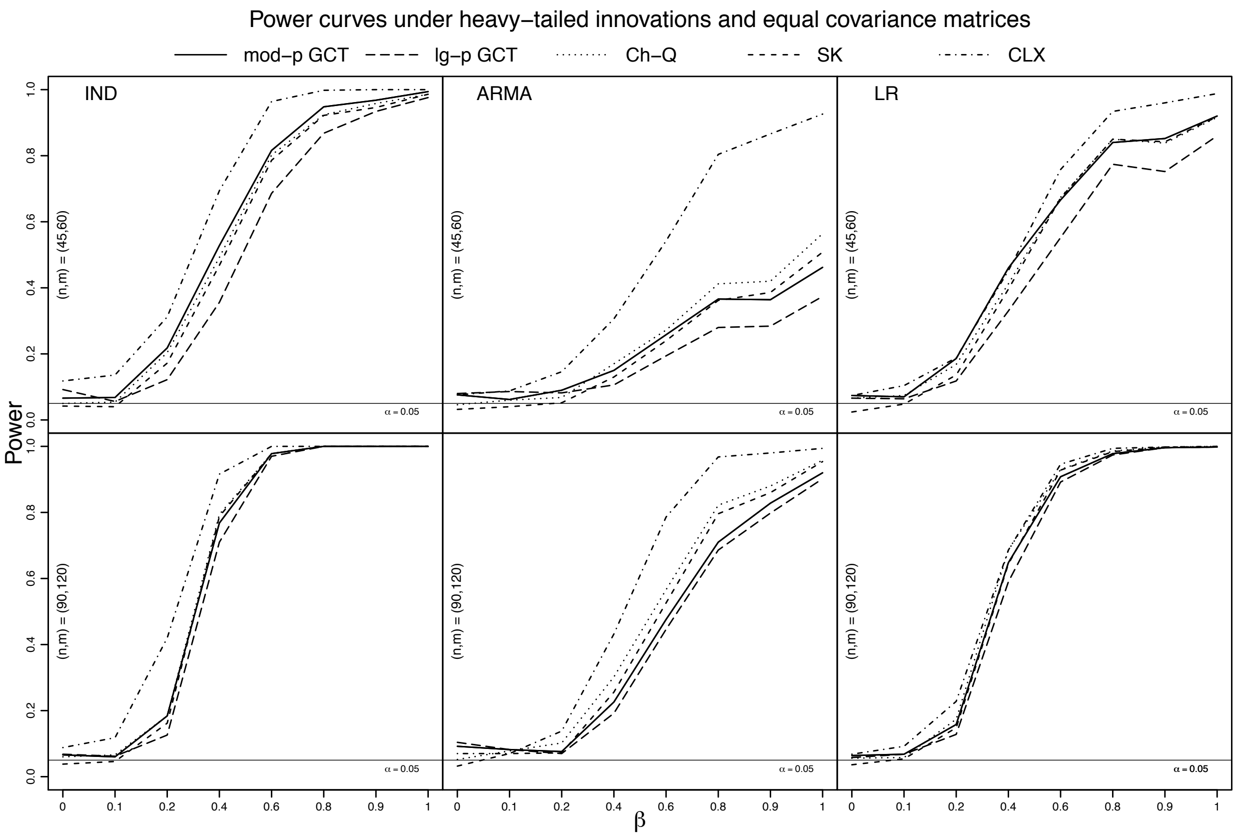

Figure S.3: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with double Pareto(16.5,8) innovations and $\Sigma_1 = \Sigma_2$. Based on $S = 500$ simulations.

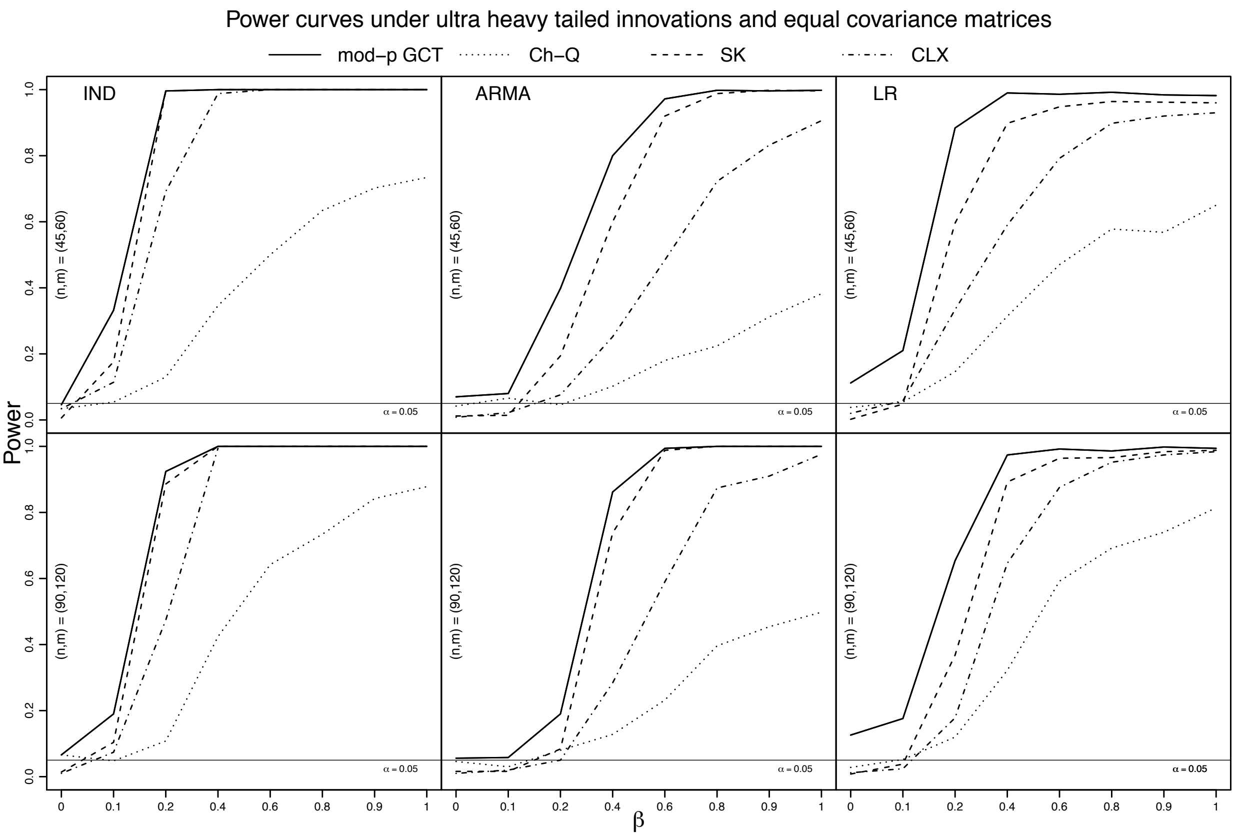

Figure S.4: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with double Pareto(1.5,1) innovations and $\Sigma_1 = \Sigma_2$. Based on $S = 500$ simulations.

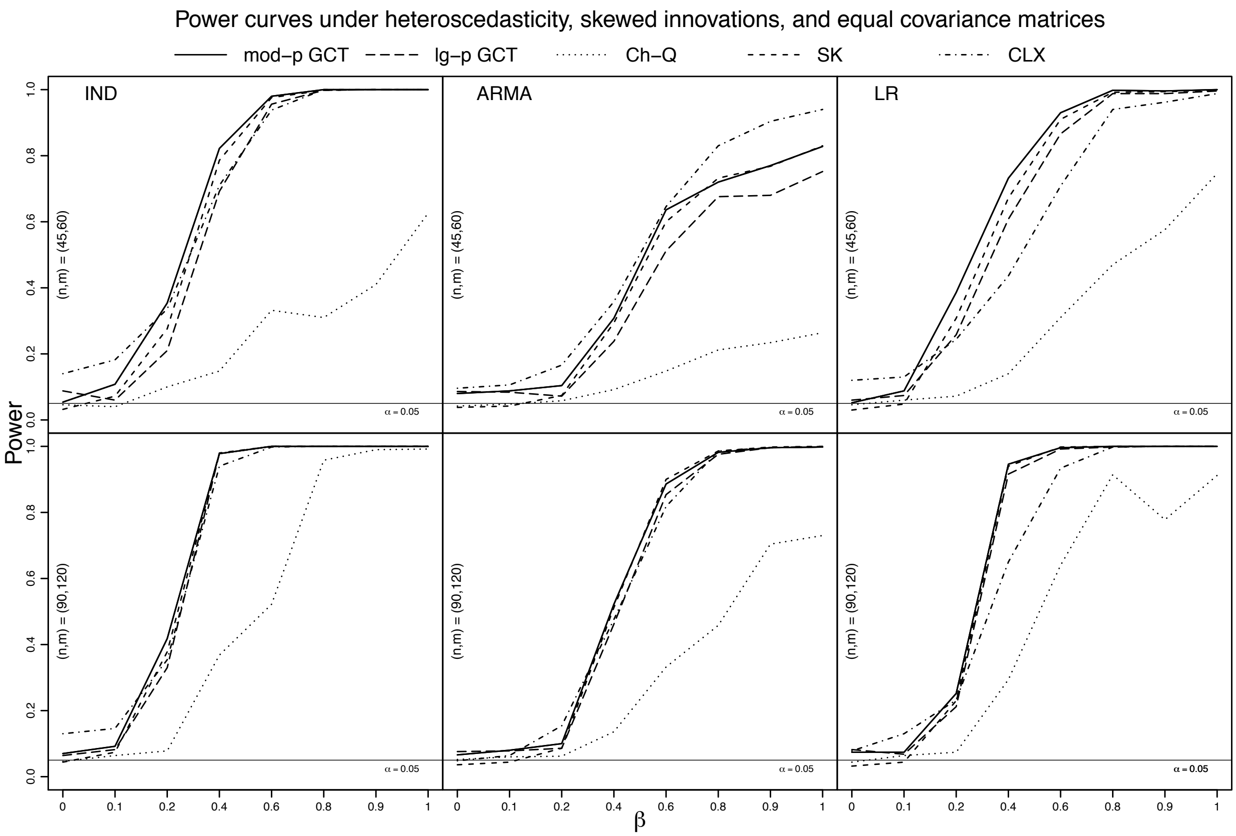

Figure S.5: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with heteroscedastic centered gamma$(4, 2)$ innovations and $\Sigma_1 = \Sigma_2$. Based on $S = 500$ simulations.

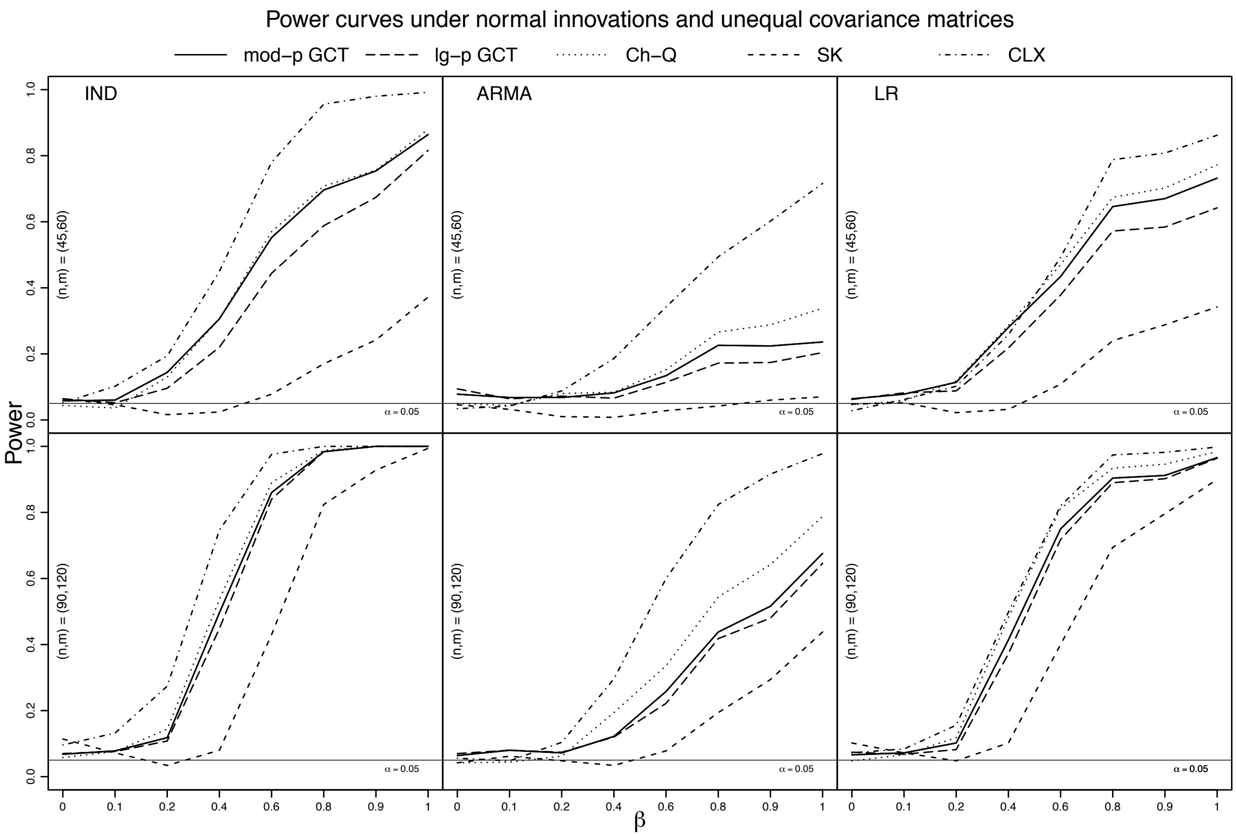

Figure S.6: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with Normal$(0, 1)$ innovations and $\Sigma_2 = 2\Sigma_1$. Based on $S = 500$ simulations.

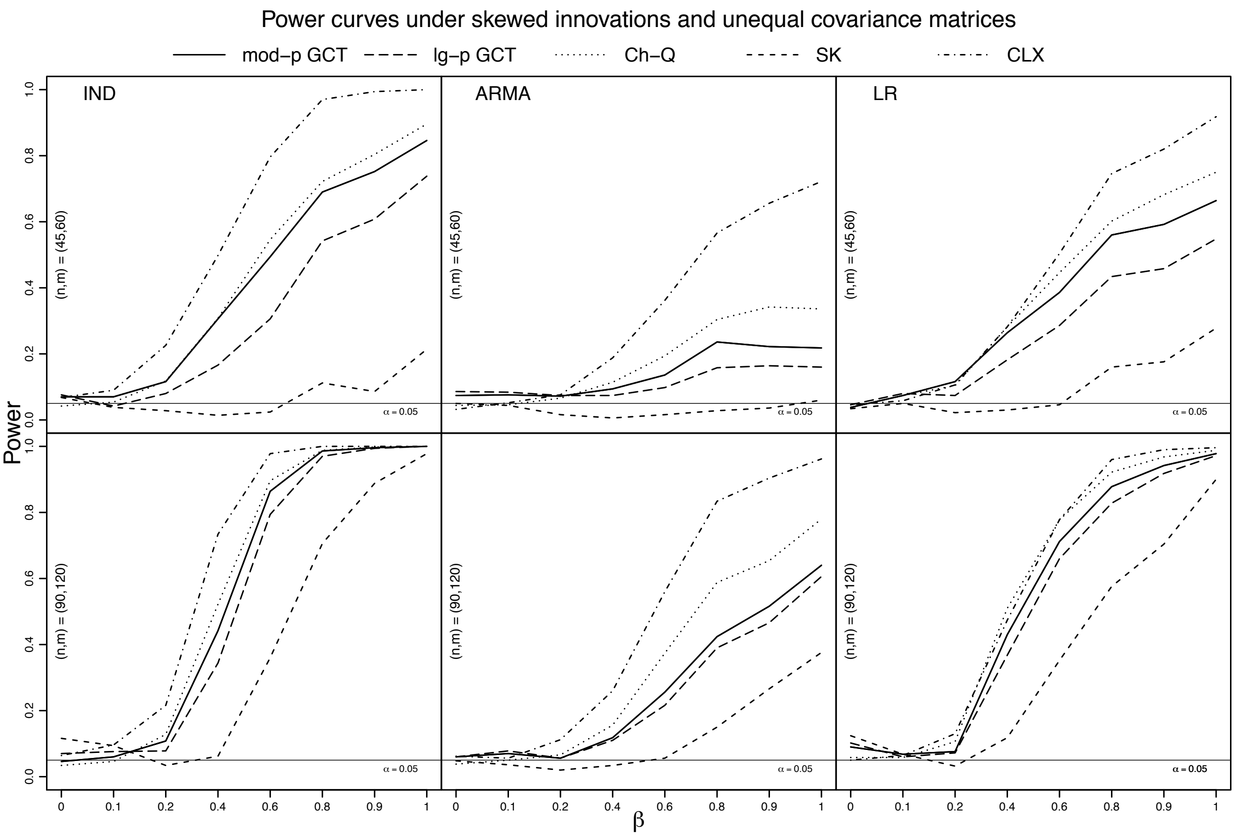

Figure S.7: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with centered gamma$(4, 2)$ innovations and $\Sigma_2 = 2\Sigma_1$. Based on $S = 500$ simulations.

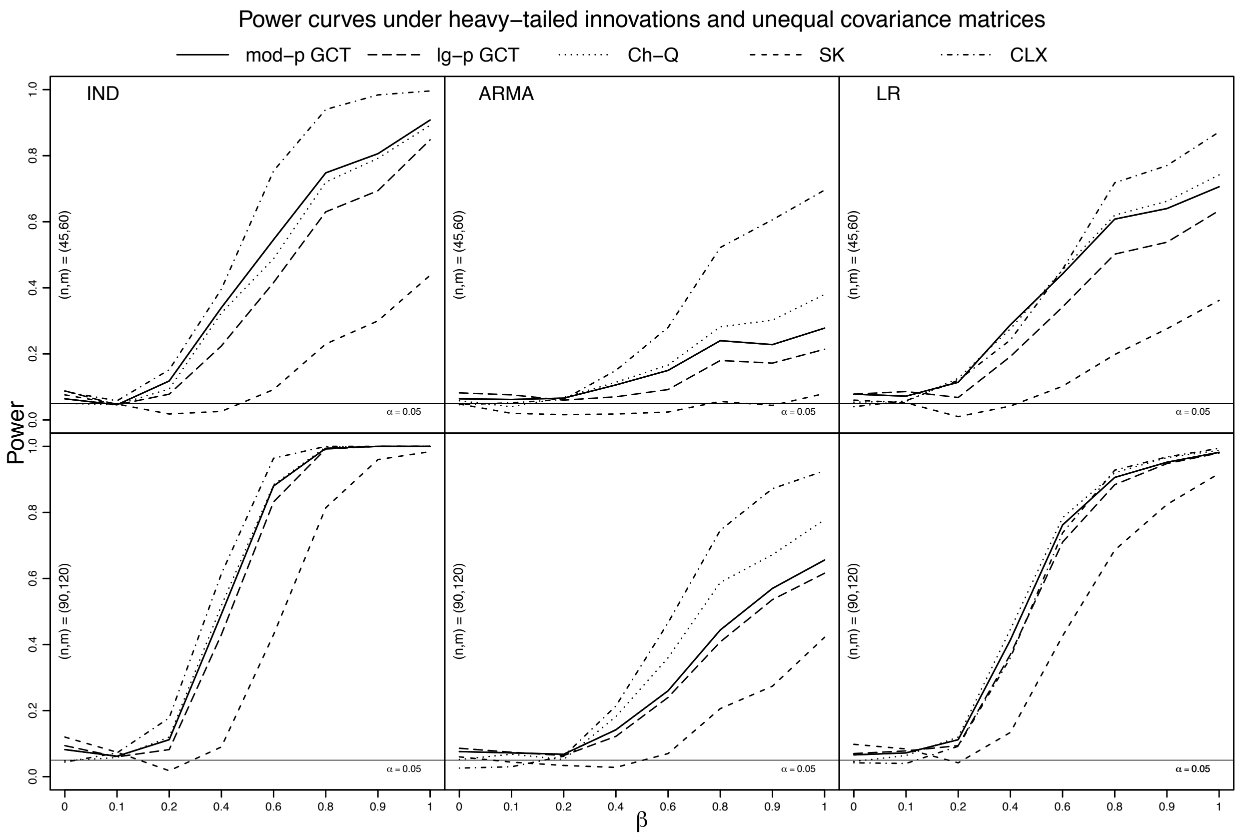

Figure S.8: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with double Pareto(16.5,8) innovations and $\Sigma_2 = 2\Sigma_1$. Based on $S = 500$ simulations.

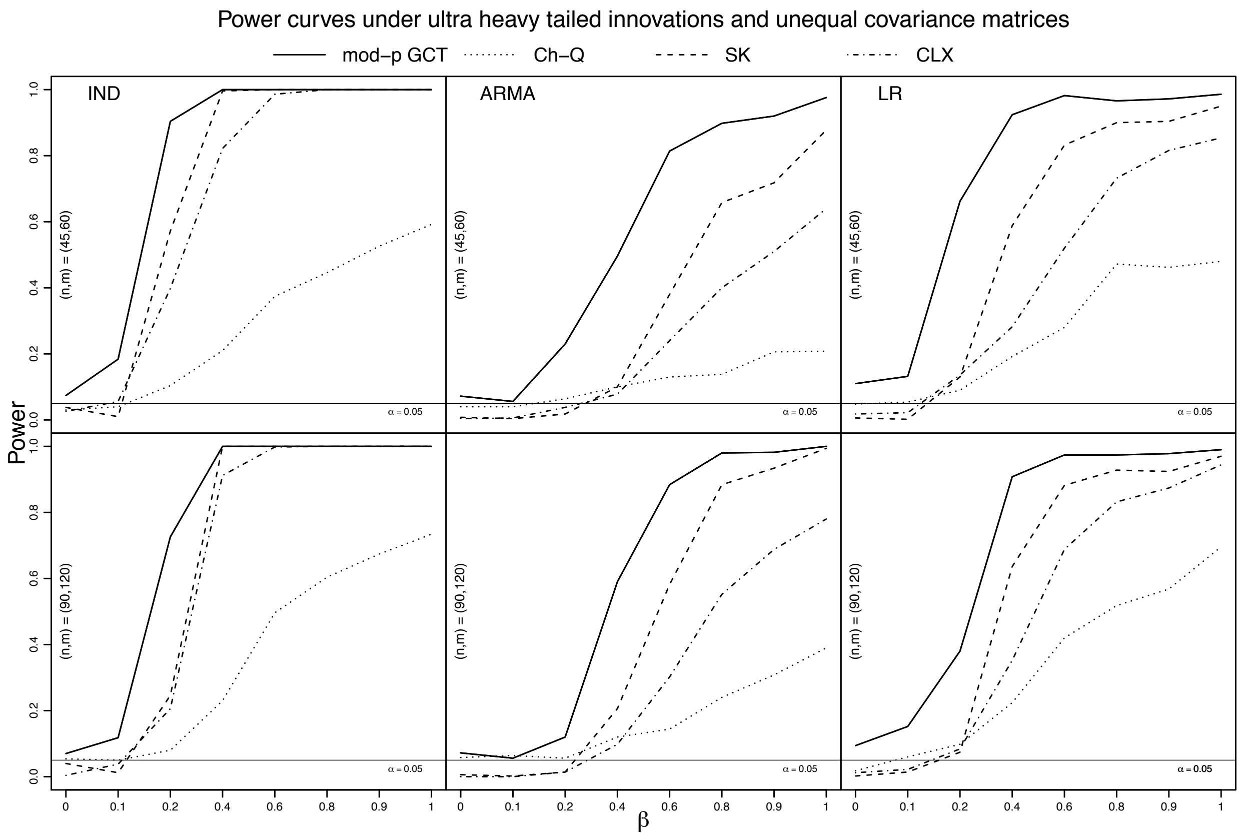

Figure S.9: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with double Pareto(1.5,1) innovations and $\Sigma_2 = 2\Sigma_1$. Based on $S = 500$ simulations.

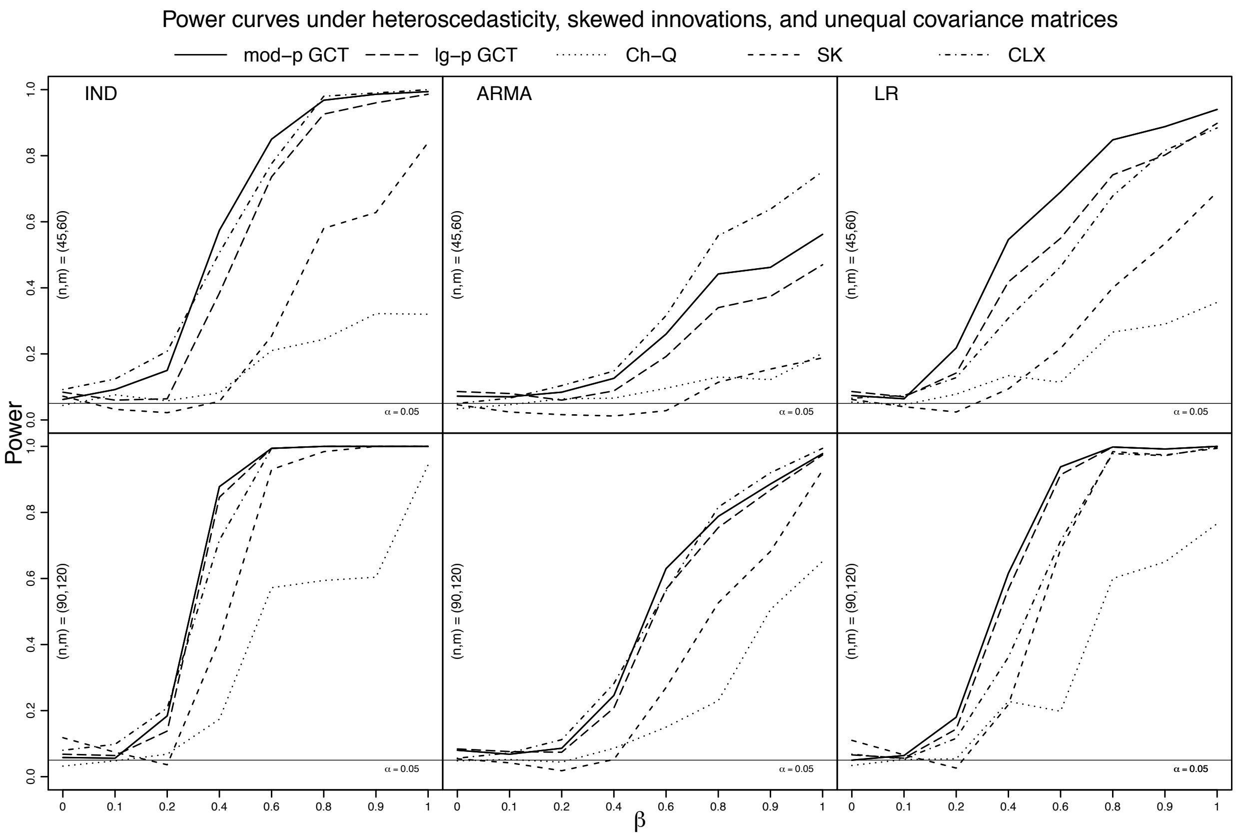

Figure S.10: Power curves at sample sizes $(n, m) = (45, 60), (90, 120)$ for the moderate- and large-p GCT, Ch-Q, SK, and CLX tests against the proportion of nonzero mean differences β under IND, ARMA, and LR dependence (left to right) with heteroscedastic centered gamma$(4, 2)$ innovations and $\Sigma_2 = 2\Sigma_1$. Based on $S = 500$ simulations.