Supporting Text

For the model without patient isolation (Eq. **1-3**) the basic reproduction number, R_0, is given by:

$$R_0 = \frac{\beta n(\nu + \lambda + \gamma)(\xi + \lambda)}{(\mu + \lambda)(\nu + \lambda + \gamma)(\xi + \lambda) - \mu\nu(\xi + \lambda) - \gamma\mu\xi}$$

This is the product of the number of secondary cases in a single stay ($R_A = \beta n/(\mu + \lambda)$, which is itself the product of the mean duration of colonization in a single stay and the rate at which new cases occur) and the mean number of stays per patient while still colonized (given by $1/(1 - P)$, where P is the probability that a hospitalized colonized patient is discharged and readmitted at least once while still colonized). Thus, even though $R_A < 1$, and there is insufficient transmission to sustain the epidemic over a short time scale, patient readmission may be sufficient to allow R_0 to be greater than one, leading to an epidemic over a longer time scale.

Note that were transmission in the community assumed to be important the more general (but less intuitive) next-generation matrix approach to calculating R_0 would be required (27); in the present case the two approaches give the same answer.

Setting Eq. 1-3 to zero and solving gives the stable endemic prevalence, y^*:

$$y^* = n - \frac{\beta(\lambda + \gamma + \nu)(\lambda + \xi)(\lambda + \mu) - \mu\nu(\lambda + \xi) - \mu\xi\gamma}{\beta(\lambda + \gamma + \nu)(\lambda + \xi)},$$

provided that $R_0 > 1$. If $R_0 < 1$ then $y^* = 0$.

The instantaneous ratio of the rate of new hospital acquisitions to imported cases, η, is gven by:

$$\eta = \frac{\beta y(n-y)}{\nu y_c + \xi y'_c}.$$

When $R_0 > 1$, evaluating this at the stable equilibrium values, y^*, $y_c{}^*$ and $y'_c{}^*$, gives

$$\eta^* = \frac{(\lambda+\mu)(\lambda+\gamma+\nu)(\lambda+\xi)}{\mu\nu(\lambda+\xi)+\gamma\mu\xi} - 1,$$

which is independent of the transmission parameter, β.

The model does not explicitly consider birth and death. However, births would enter into the susceptible class and mortality can be absorbed into the parameter λ, the rate at which colonization is cleared. In practice, the contribution of mortality to λ will be very small compared to that caused by loss of colonization and can be neglected.

For this model, the requirement that the three populations (hospital, and community with high and low admission rates) remain fixed in size determines the sizes of the community populations (chosen to ensure that flows into and out of each population remain constant). The total community population size is given by

$$N_c = \frac{(\xi+\gamma)\mu n}{(\nu+\gamma)\xi}.$$

The fixed population sizes also enable this model (without isolation) to be specified by the reduced set of differential equations (Eq. **1-3**). For the model with isolation populations sizes do not remain constant and a full set of equations is required (Eq. **4-11**).