Supplementary Online Appendix

Dynamics of non-cohabiting sex partnering in sub-Saharan Africa

Ryosuke Omori[1,2,3], Hiam Chemaitelly[2] and Laith J. Abu-Raddad[2,3,4]

[1]*Division of Bioinformatics, Research Center for Zoonosis Control, Hokkaido University, Sapporo, Hokkaido, Japan*

[2]*Infectious Disease Epidemiology Group, Weill Cornell Medical College - Qatar, Cornell University, Qatar Foundation - Education City, Doha, Qatar*

[3]*Department of Healthcare Policy and Research, Weill Cornell Medical College, Cornell University, New York, New York, USA*

[4]*Vaccine and Infectious Disease Division, Fred Hutchinson Cancer Research Center, Seattle, Washington, USA*

Reprints or correspondence: Ryosuke Omori, PhD, Division of Bioinformatics, Research Center for Zoonosis Control, Hokkaido University. North 20, West 10, Kita-ku, Sapporo, Hokkaido, Japan. Telephone: +(81) 11-706-7307. Fax: +(81) 11-706-7310. E-mail: omori@czc.hokudai.ac.jp

Derivation S1

We assumed that the formation or dissolution of a non-cohabiting sex partnership follows a Poisson stochastic process. We assumed that there is a fixed hazard per unit time to form a partnership, p_x. Moreover, if a partnership is formed, there is a fixed hazard per unit time for this partnership to be dissolved, μ_x.

The equilibrium distribution for the number of partners for individual x, F_x, is obtained using the Kolmogorov forward equation for this process. The probability that the individual x has m partners at a specific time $t + \Delta t$, $\Pr(n_x = m, t + \Delta t)$, is described in terms of the probabilities that this individual has $m-1$ ($\Pr(n_x = m-1, t)$), m ($\Pr(n_x = m, t)$), and $m+1$ ($\Pr(n_x = m+1, t)$) partners at t. It follows then, to first order in Δt, that

$$
\begin{aligned}
\Pr(n_x = m, t + \Delta t) = {} & (1 - m\mu_x \Delta t - p_x \Delta t)\Pr(n_x = m, t) \\
& + p_x \Delta t \Pr(n_x = m - 1, t) \\
& + (m+1)\mu_x \Delta t \Pr(n_x = m + 1, t)
\end{aligned}
$$

$$
\begin{aligned}
\Pr(n_x = 0, t + \Delta t) = {} & (1 - p_x \Delta t)\Pr(n_x = 0, t) \\
& + \mu_x \Delta t \Pr(n_x = 1, t)
\end{aligned}
$$

As $Dt \rightarrow 0$, we obtain the Kolmogorov differential equation:

$$
\frac{d\Pr(n_x = m, t)}{dt} = -\left(p_x + m\mu_x\right)\Pr(n_x = m, t) + p_x \Pr(n_x = m - 1, t) + \left(m+1\right)\mu_x \Pr(n_x = m + 1, t)
$$

$$
\frac{d\Pr(n_x = 0, t)}{dt} = -p_x \Pr(n_x = 0, t) + \mu_x \Pr(n_x = 1, t)
$$

Accordingly, the probability for having m partners at equilibrium is given by

$$
\Pr(n_x = m)^* = \frac{\exp\left[-\dfrac{p_x}{\mu_x}\right]\left(\dfrac{p_x}{\mu_x}\right)^m}{m!}.
$$

This probability mass function is equivalent to the probability mass function of a Poisson

distribution with a mean of $\dfrac{p_x}{\mu_x}$. Therefore, the equilibrium distribution for the number of non-

cohabiting sex partners for individual x is described by

$$F_x \equiv Poisson\left(\frac{p_x}{\mu_x}\right).$$

Table S1: Descriptive statistics for the reported number of non-cohabiting sex partners in the last 12 months across 25 countries in sub-Saharan Africa.

Country	Year	Males						Females					
		Married			Unmarried			Married			Unmarried		
		n	Mean	Variance	n	Mean	Variance	n	Mean	Variance	n	Mean	Variance
Burkina Faso	2010	4536	0.09830	0.1310	2761	0.441	0.562	13390	0.01410	0.01680	3694	0.2480	0.2330
Burundi	2010	2423	0.02680	0.1680	1855	0.103	0.365	5259	0.00323	0.00360	4126	0.0589	0.0588
Cameroon	2011	3618	0.54900	3.0300	3549	1.090	11.40	9792	0.09760	0.12200	5614	0.5280	0.7120
Congo	2007	2757	0.23700	6.9400	1971	0.796	2.040	6560	0.05050	1.48000	3400	0.4340	0.8280
Congo-Brazzaville	2009	3062	0.52200	1.4300	2785	1.180	2.160	3943	0.05810	0.05980	2596	0.8100	0.9970
Cote d'Ivoire	2012	2679	0.43100	1.6900	2446	1.210	3.740	6442	0.02790	0.03030	3597	0.6730	0.5050
Ethiopia	2011	7909	0.00872	0.0132	6178	0.134	0.183	10188	0.01920	0.94600	6303	0.2300	14.500
Ghana	2003	2724	0.16600	0.3450	2286	0.453	0.712	3691	0.03500	0.03860	1997	0.3440	0.2730
Guinea	2005	1877	0.32400	0.9500	1287	0.860	1.040	6313	0.04180	0.04990	1618	0.4020	0.3260
Kenya	2008-09	1818	0.07970	0.1050	1639	0.662	11.90	5028	0.00875	0.00868	3395	0.3170	0.3840
Lesotho	2009	1378	0.32700	0.4380	1885	0.901	1.430	4099	0.08470	0.08920	3435	0.5210	2.9500
Liberia	2007	3274	0.36000	0.6950	2653	0.917	1.390	4367	0.07650	0.07840	2532	0.7630	0.5510
Malawi	2010	4196	0.07600	0.1020	2960	0.452	0.485	15423	0.00772	0.00766	7563	0.1960	0.2260
Mali	2006	2744	0.17100	0.8090	1430	0.382	1.350	12301	0.04610	0.05320	2257	0.1160	0.1350
Mozambique	2009	3084	0.19300	0.2310	1696	0.785	0.574	4251	0.02990	0.03180	2160	0.4760	0.3750
Niger	2006	2115	0.02510	0.0538	1409	0.179	0.365	7413	0.00418	0.00444	1786	0.1060	5.2600
Rwanda	2010	3260	0.04140	0.0465	3068	0.199	0.545	6832	0.00542	0.00539	6835	0.1230	1.4400
Sao Tome and Principe	2008-09	1177	0.19200	0.7860	1101	0.699	0.592	1740	0.00805	0.01260	859	0.3200	0.2460
Senegal	2010-11	2158	0.13900	2.2200	2770	0.293	0.547	10804	0.01030	0.01050	4884	0.0913	0.0912
Sierra Leone	2008	1895	0.26000	0.3400	1215	0.698	1.390	5209	0.04530	0.04440	1977	0.5580	0.3560
Swaziland	2006-07	1265	0.19100	0.2100	2870	0.574	0.839	2067	0.05030	0.21800	2917	0.5130	0.2970
Tanzania	2012	4329	0.23000	0.5410	4005	0.688	1.140	6816	0.04090	0.05830	4127	0.4340	0.3390
Uganda	2011	5712	0.31200	7.8600	3873	0.562	6.020	7626	0.05060	1.31000	4523	0.2920	0.2550
Zambia	2007	3618	0.15600	0.2230	2864	0.571	0.914	4314	0.00719	0.00760	2828	0.3370	0.2650
Zimbabwe	2011	3747	0.24400	7.8100	3635	0.462	1.340	5518	0.01090	0.01870	3537	0.2410	1.1300

[*]n denotes the sample size; mean and variance denote the mean and variance of the reported number of non-cohabiting sex partners in the last 12 months as reported in the cross-sectional Demographic and Health Survey.

Figure S1: Comparison of model-predicted distributions and empirical distributions for married males. Distribution of the number of non-cohabiting sex partners over the last 12 months among married males across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.
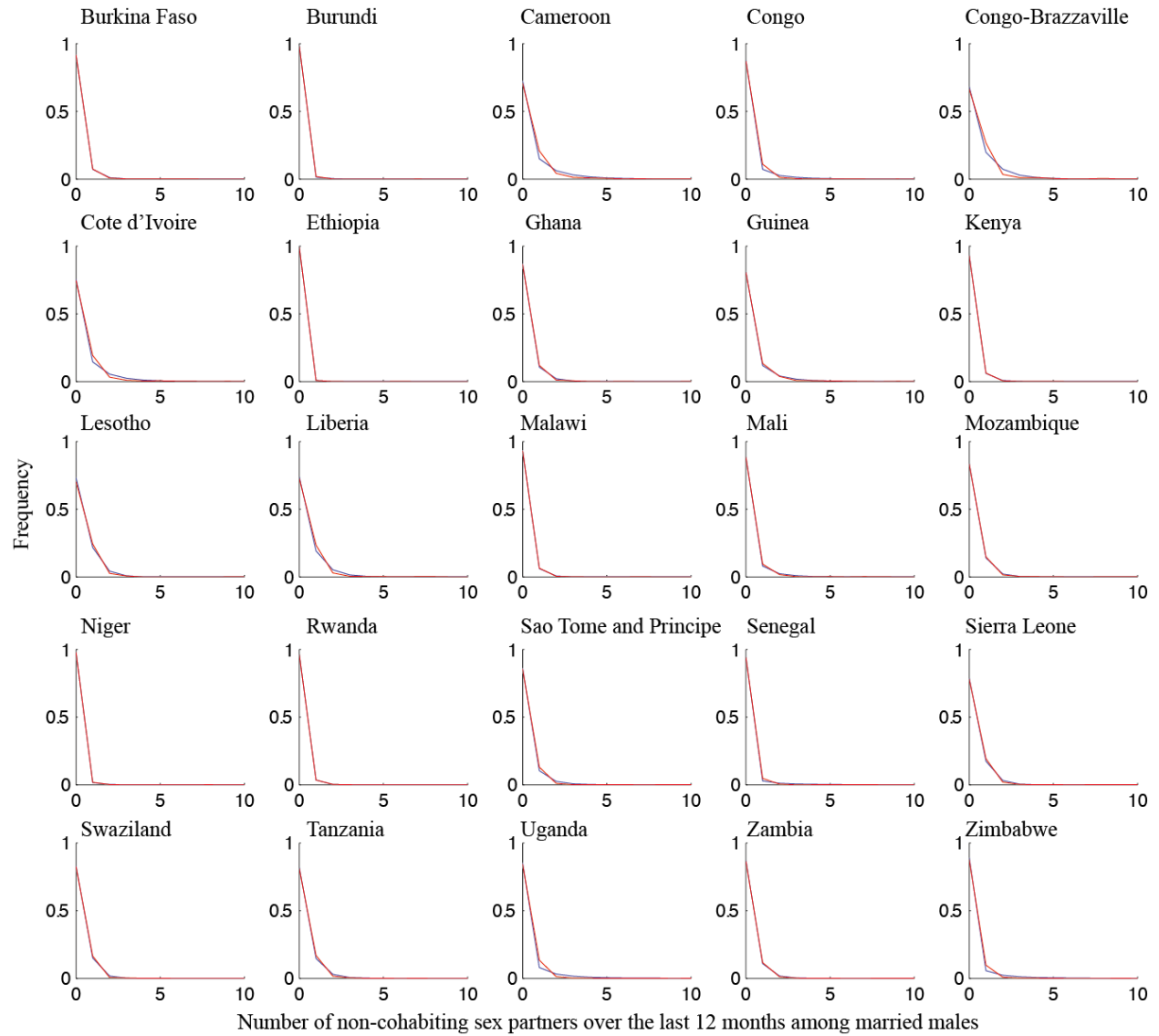
Figure S2: Comparison of model-predicted distributions and empirical distributions for unmarried males. Distribution of the number of non-cohabiting sex partners over the last 12 months among unmarried males across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.
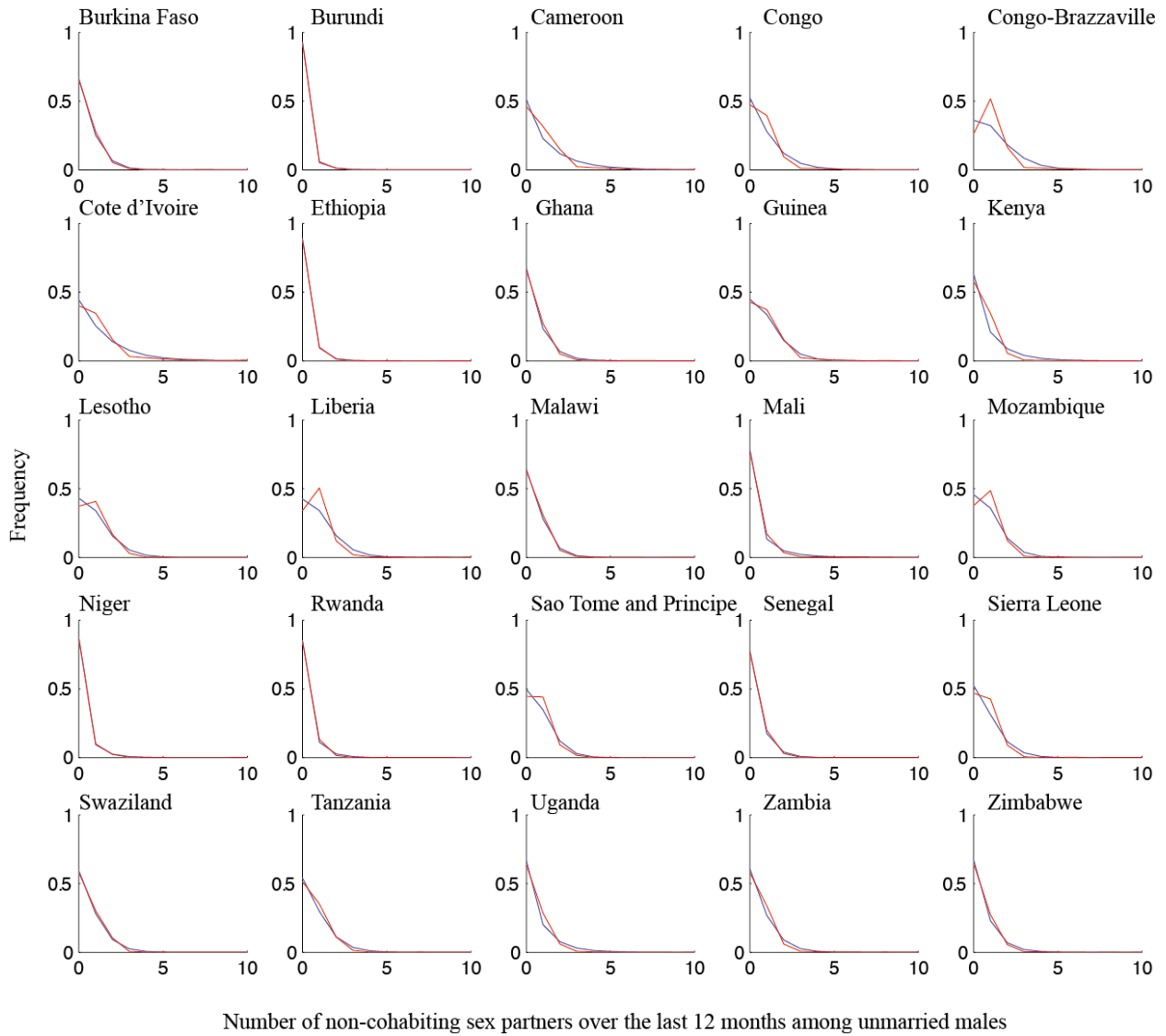

Number of non-cohabiting sex partners over the last 12 months among unmarried males

Figure S3: Comparison of model-predicted distributions and empirical distributions for married females. Distribution of the number of non-cohabiting sex partners over the last 12 months among married females across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.
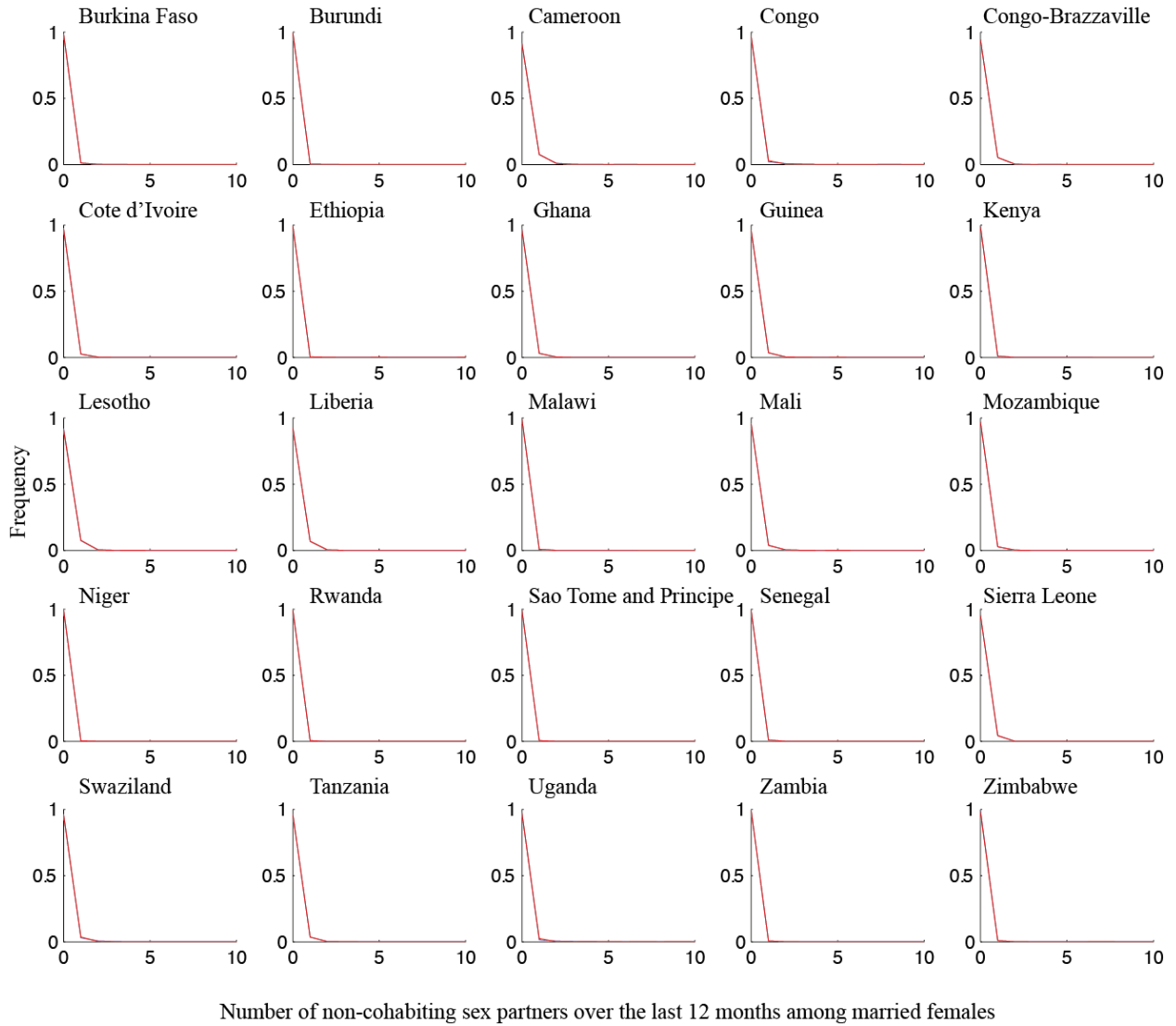

Number of non-cohabiting sex partners over the last 12 months among married females

Figure S4: Comparison of model-predicted distributions and empirical distributions for unmarried females. Distribution of the number of non-cohabiting sex partners over the last 12 months among unmarried females across the 25 studied countries in sub-Saharan Africa. The blue line shows the model-predicted distribution while the red line shows the empirical distribution per the Demographic and Health Survey data.

Number of non-cohabiting sex partners over the last 12 months among unmarried females