

Supplement to “Log-gamma linear mixed-effects models for multiple outcomes with application to a longitudinal glaucoma study”

1 Two figures

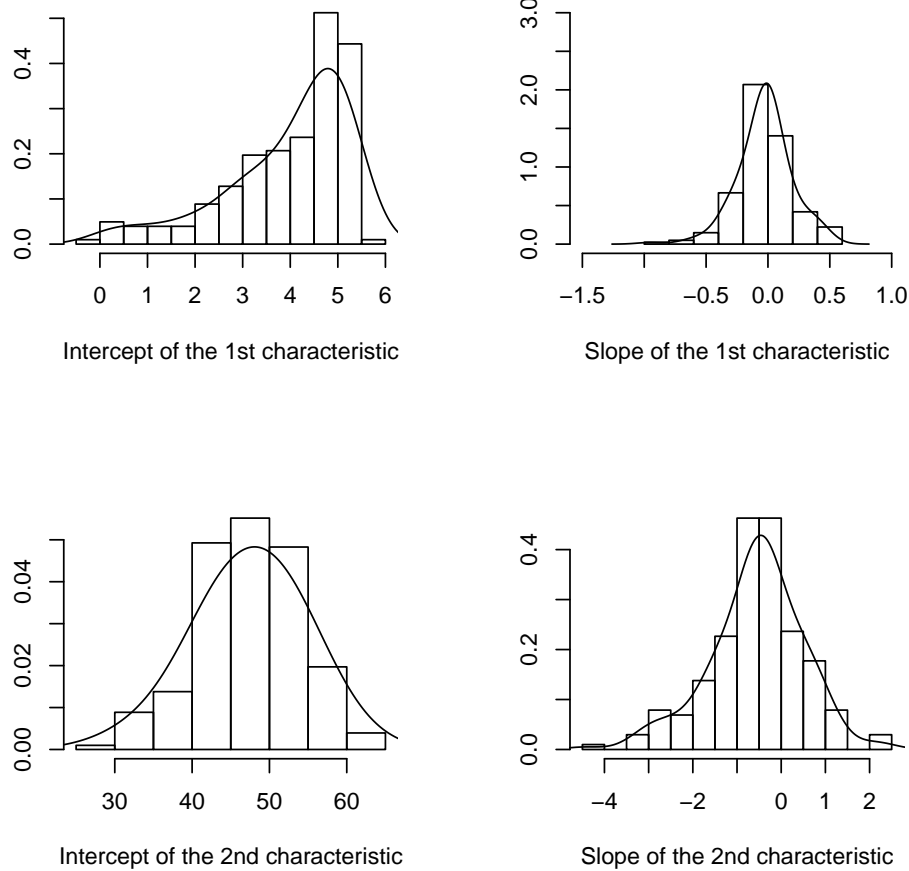


Figure 1 Coefficients for the within-subject regressions of two outcomes on time

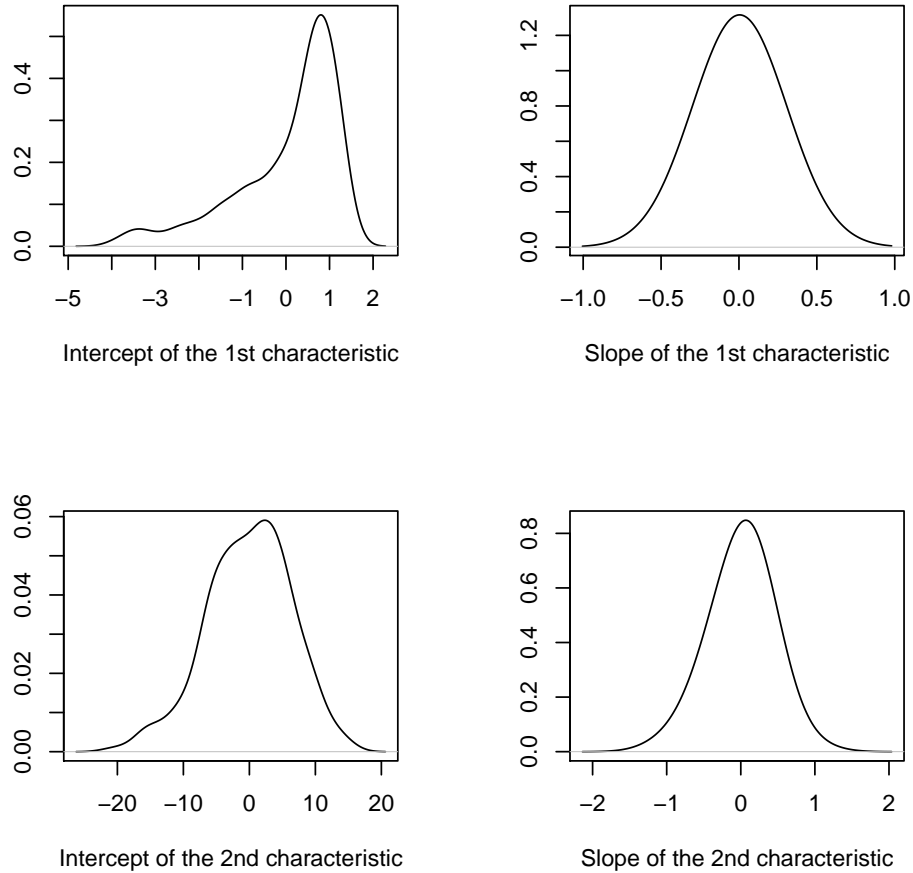


Figure 2 Distributions of the empirical estimates of the random effects under the log-gamma model

2 Log-gamma distribution

In this section, we give a brief introduction to the log-gamma distribution. The log-gamma distribution can be symmetric, negatively skewed, positively skewed, or very skewed. The probability density function of the standard log-gamma random variable W is

$$f_W(w) = \frac{1}{\Gamma(\kappa)} e^{-e^w + \kappa w}, \quad w \in \mathbf{R}, \quad (1)$$

where $\kappa > 0$ is the shape parameter. For the location parameter $\mu \in \mathbf{R}$ and the scale parameter $\eta > 0$, the log-gamma location-scale family of probability density functions $(1/\eta)f_W((w-\mu)/\eta)$ has mean $\mu + \eta\psi(\kappa)$ and variance $\eta^2\psi'(\kappa)$, where ψ and ψ' are the digamma and trigamma functions. Figure 3 shows the log-gamma density functions with mean zero, scale parameter $\eta = 1$, and shape parameter κ changing from 0.1 to 5 by 0.1.

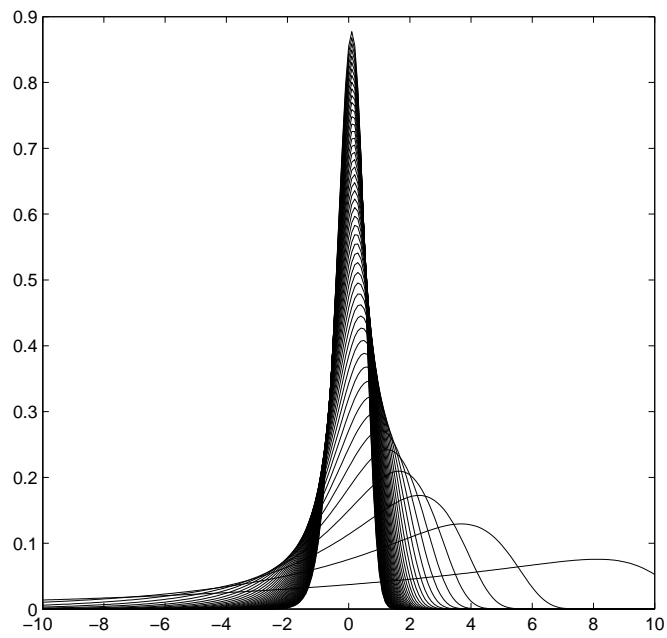


Figure 3 Log-gamma density functions with different shape parameters