

## Supplementary materials

# Multifrequency multi-qubit entanglement based on plasmonic hot spots

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According to the Ref. S1, the matrix elements  $(Q^{pq})_{\nu\mu}^{11}$ ,  $(Q^{pq})_{\nu\mu}^{21}$ ,  $(Q^{pq})_{\nu\mu}^{12}$  and  $(Q^{pq})_{\nu\mu}^{22}$  are expressed in the following:

$$(Q^{pq})_{\nu\mu}^{11} = \frac{jk_1^2}{\pi} \int_S \{ [\mathbf{n}(\mathbf{r}') \times \mathbf{M}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{N}_\nu^p(k_1\mathbf{r}') + \sqrt{\frac{\epsilon_2}{\epsilon_1}} [\mathbf{n}(\mathbf{r}') \times \mathbf{N}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{M}_\nu^p(k_1\mathbf{r}') \} dS(\mathbf{r}') , \quad (\text{S1})$$

$$(Q^{pq})_{\nu\mu}^{12} = \frac{jk_1^2}{\pi} \int_S \{ [\mathbf{n}(\mathbf{r}') \times \mathbf{N}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{N}_\nu^p(k_1\mathbf{r}') + \sqrt{\frac{\epsilon_2}{\epsilon_1}} [\mathbf{n}(\mathbf{r}') \times \mathbf{M}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{M}_\nu^p(k_1\mathbf{r}') \} dS(\mathbf{r}') , \quad (\text{S2})$$

$$(Q^{pq})_{\nu\mu}^{21} = \frac{jk_1^2}{\pi} \int_S \{ [\mathbf{n}(\mathbf{r}') \times \mathbf{M}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{M}_\nu^p(k_1\mathbf{r}') + \sqrt{\frac{\epsilon_2}{\epsilon_1}} [\mathbf{n}(\mathbf{r}') \times \mathbf{N}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{N}_\nu^p(k_1\mathbf{r}') \} dS(\mathbf{r}') , \quad (\text{S3})$$

$$(Q^{pq})_{\nu\mu}^{22} = \frac{jk_1^2}{\pi} \int_S \{ [\mathbf{n}(\mathbf{r}') \times \mathbf{N}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{M}_\nu^p(k_1\mathbf{r}') + \sqrt{\frac{\epsilon_2}{\epsilon_1}} [\mathbf{n}(\mathbf{r}') \times \mathbf{M}_\mu^q(k_2\mathbf{r}')] \cdot \mathbf{N}_\nu^p(k_1\mathbf{r}') \} dS(\mathbf{r}') . \quad (\text{S4})$$

The line-width  $\delta$  and Rabi-frequency  $\Omega$  for the two-, three- and four-qubit cases of the systems at  $d = 1nm$  are given in the following tables:

Table 1 Two-qubit cases in Fig. 2 at  $d = 1nm$

Peak in Fig. 2(a)	$\omega$ (THz)	$\Gamma$ (THz)	$\delta$ (THz)	$\Omega$ (THz)
1	4530	2540	111	751
2	4800	3290	68.4	671
3	5470	13500	57.7	1250

Table 2 Three-qubit cases in Fig. 4 at  $d = 1nm$

Peak in Fig. 4(b)	$\omega$ (THz)	$\Gamma$ (THz)	$\delta$ (THz)	$\Omega$ (THz)
1	4460	2240	48.6	467
2	4710	3300	134	940
3	5440	8080	59.3	979

Table 3 Four-qubit cases in Fig. 5 at  $d = 1nm$

Peak in Fig. 5(b)	$\omega$ (THz)	$\Gamma$ (THz)	$\delta$ (THz)	$\Omega$ (THz)
1	4540	3820	100	874
2	4950	2850	56.2	566
3	5440	10800	59.3	1130

## References

- [S1]. Doicu, A., Wriedt, T. & Eremin, Y. A. *Light Scattering by Systems of Particles*. (Springer-Verlag, Berlin, Heidelberg, 2006).