Supplementary Information Appendix for
The metabolic cost of changing walking speeds is significant, implies lower optimal speeds for shorter distances, and increases daily energy estimates

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S1 Deriving step-to-step transition work for changing speeds

The mechanical work done to redirect the velocity of the center of mass from downward to upward when transitioning from one step to the next is thought to be a major determinant of the metabolic cost of walking at a constant speed [1, 2, 3].

Mathematical expressions for this step-to-step transition cost have previously been derived for such steady state walking, with the simplifying assumption that humans walk with an inverted pendulum gait [4, 1, 2, 3]. When a person walks at continuously varying speeds, as in our experiments, the step-to-step transition will include a change in both the magnitude and direction of the COM velocity. Here, we derive an expression for the work done in the step-to-step transition when walking at changing (non-constant) speeds, thereby generalizing previous work [4, 1, 2, 3]. We allow that the length of the leg to change during the step-to-step transition (unequal θ_{before} and θ_{after}) while having constant leg length during the stance phase.

Push-off before heel-strike. Figure S1 describes the transition from one inverted pendulum to the next using push-off and heel-strike impulses; panels a-c describe situations in which the push-off happens entirely before heel-strike, which we consider first. In particular, Figure S1a-b shows a finite reduction in speed being accomplished during the step-to-step transition, with push-off before heel-strike. We focus on Figure S1b in the following derivation. Here, vector $\overrightarrow{\mathrm{OA}}$ with magnitude $\mathrm{OA} = V_{\text{before}}$ and making angle θ_{before} with horizontal, is the body velocity just before push-off at the end of one inverted pendulum phase. Vector $\overrightarrow{\mathrm{OC}}$, with magnitude $\mathrm{OC} = V_{\text{after}}$ and making angle θ_{after}, is the body velocity just after heel-strike at the beginning of the next inverted pendulum phase.

A push-off impulse is applied along the trailing leg to change velocity $\overrightarrow{\mathrm{OA}}$ to $\overrightarrow{\mathrm{OB}}$, along $\overrightarrow{\mathrm{AB}}$. Then, a heel-strike impulse is applied along the leading leg to change velocity $\overrightarrow{\mathrm{OB}}$ to $\overrightarrow{\mathrm{OC}}$, along $\overrightarrow{\mathrm{BC}}$. The push-off positive work W_{pos} is the kinetic energy change from $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$ given by $\frac{1}{2}m\mathrm{OB}^2 - \frac{1}{2}m\mathrm{OA}^2$ and the heel-strike negative work is the kinetic energy change from $\overrightarrow{\mathrm{OB}}$ to $\overrightarrow{\mathrm{OC}}$ given by W_{neg} given by $\frac{1}{2}m\mathrm{OC}^2 - \frac{1}{2}m\mathrm{OB}^2$, which simplify to:

$$W_{\text{pos}} = \frac{1}{2}m\,(\mathrm{AB})^2 \quad \text{and} \quad W_{\text{neg}} = \frac{1}{2}m\,(\mathrm{BC})^2 \quad \text{respectively.} \tag{1}$$

As opposed to the steady walking situation [4, 1, 2, 3], when changing speeds, the step-to-step push-off positive work W_{pos} and the step-to-step heel-strike negative work W_{neg} will be unequal.

First, we note that angle that in triangle EBD, $\angle\mathrm{DEB} = \pi/2 - \theta_{\text{after}}$, $\angle\mathrm{BDE} = \pi/2 - \theta_{\text{before}}$, and $\angle\mathrm{EBD} = \theta_{\text{after}} + \theta_{\text{before}}$. We use the geometric relations that AB = AD-BD, BC = EB+CE, $\mathrm{AD} = \mathrm{OA}\tan\theta_{\text{before}}$, CE =

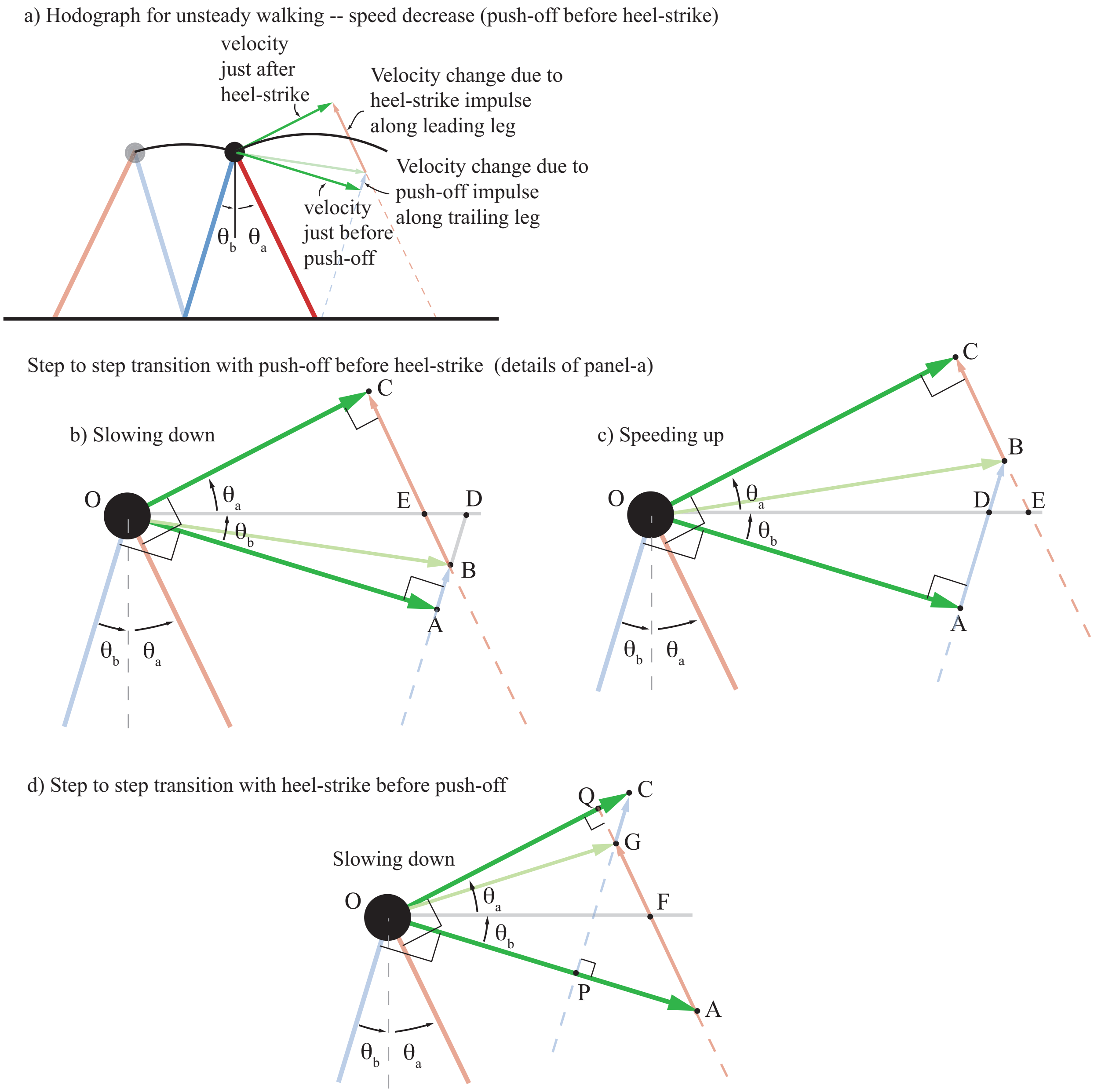

Figure S1: Step-to-step transition to change speed. a) The walking motion is assumed to be inverted pendulum-like with the transitions from one inverted pendulum step to the next accomplished using push-off and heel-strike impulses. Overlaid is the 'hodograph' (a depiction of velocity changes) during the step-to-step transition, when push-off happens entirely before heel-strike. b) Details of the velocity changes during step-to-step transition, with push-off before heel-strike and slowing down. c) Analogous to panel-c, except the walking speeds up during the transition. d) Velocity changes and impulses when the heel-strike precedes push-off entirely.

OC $\tan\theta_{\text{after}}$,

$$\text{ED} = \frac{\text{OA}}{\cos\theta_{\text{before}}} - \frac{\text{OC}}{\cos\theta_{\text{after}}}, \quad \frac{\text{BD}}{\text{ED}} = \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}}+\theta_{\text{before}})}, \quad \text{and} \quad \frac{\text{EB}}{\text{ED}} = \frac{\cos\theta_{\text{before}}}{\sin(\theta_{\text{after}}+\theta_{\text{before}})} \tag{2}$$

in Eq. 1 to obtain:

$$W_{\text{pos}} = \frac{1}{2}m\left[V_{\text{before}}\tan\theta_{\text{before}} - \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}}+\theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)\right]^2 \text{ and} \tag{3}$$

$$W_{\text{neg}} = \frac{1}{2}m\left[V_{\text{after}}\tan\theta_{\text{after}} + \frac{\cos\theta_{\text{before}}}{\sin(\theta_{\text{after}}+\theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)\right]^2, \tag{4}$$

with OA $= V_{\text{before}}$ and OC $= V_{\text{after}}$. We obtain exactly the same expressions for speeding up during step-to-step transition as shown in figure S1c (though the figure looks superficially different, a couple of negative signs cancel, thereby giving the same answers). Note that the main qualitative difference between panels b and c of figure S1 is that the velocity $\overrightarrow{\text{OB}}$ is above or below the horizontal.

An implicit assumption in the above derivation is that the push-off and heel-strike impulses do not require tensional leg forces (the leg cannot pull on the ground) and are in the directions shown. This requirement is satisfied when the ratio of the two speeds obey the following condition:

$$\cos(\theta_{\text{after}}+\theta_{\text{before}}) \le \frac{V_{\text{after}}}{V_{\text{before}}} \le \frac{1}{\cos(\theta_{\text{after}}+\theta_{\text{before}})}.$$

When $V_{\text{after}}/V_{\text{before}} = \cos(\theta_{\text{after}}+\theta_{\text{before}})$, the necessary push-off impulse becomes zero and when $V_{\text{before}}/V_{\text{after}} = \cos(\theta_{\text{after}}+\theta_{\text{before}})$, the necessary heel-strike impulse becomes zero.

Heel-strike before push-off. When the heel-strike impulse precedes the push-off impulse, the negative work W_{neg} by the heel-strike and the positive work by the push-off W_{pos} are given by the respective kinetic energy changes:

$$W_{\text{neg}} = \frac{1}{2}m(\text{OA}^2 - \text{OG}^2) \quad \text{and} \quad W_{\text{pos}} = \frac{1}{2}m(\text{OC}^2 - \text{OG}^2), \tag{5}$$

where OA $= V_{\text{before}}$, OC $= V_{\text{after}}$, OG^2 = OQ^2 + QG^2, OQ = OA $\cos(\theta_{\text{after}}+\theta_{\text{before}})$, angle $\angle$QGC $= \theta_{\text{after}} + \theta_{\text{before}}$, QG = CG $\cos(\theta_{\text{after}}+\theta_{\text{before}})$, and CG = AB in figure S1b. i.e.,

$$\text{CG} = \text{AB} = V_{\text{before}}\tan\theta_{\text{before}} - \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}}+\theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)$$

using the derivation for push-off before heel-strike; the final expression is easily obtained by substituting these relations into equation 5. As has been shown before for steady state walking [4, 2], we find that a transition with heel-strike before push-off requires more metabolic cost than push-off before heel-strike even when changing speeds with the simplifying assumption that $\theta_{\text{before}} = \theta_{\text{after}}$.

S2 Optimal multi-step inverted pendulum gaits satisfying experimental protocol

We use a metabolic cost that is a sum of two terms: (1) the step-to-step transition cost and (2) a swing cost.

Step-to-step transition cost. The step-to-step transition cost E_{s2s} is a weighted sum of the push-off work W_{pos} and heel-strike work W_{neg}, summed over all steps, and scaled by the approximate efficiencies of positive and negative work respectively:

$$E_{\text{s2s}} = \sum_{\text{steps}} \eta_{\text{pos}}^{-1} W_{\text{pos}} + \eta_{\text{neg}}^{-1} W_{\text{neg}}$$

using the equations 3 and 4 for the work expressions.

Swing cost. The step-to-step transition cost does not account for the work required to swing the legs. We use a simple model of the metabolic cost required to swing the legs forward [5] equal to

$$E_{\text{swing}} = \mu D_{\text{stride}}/T_{\text{step}}^3$$

where T_{step} is the step duration, D_{stride} is the distance travelled by the swing foot during the step (distance between previous and successive foot contact points), and the proportionality constant $\mu = 0.06$ when all other quantities are non-dimensional, chosen so as to best fit steady walking metabolic costs [5].

Representing a multi-step inverted pendulum walking motion. Each step of the inverted pendulum walking motion was represented using five variables: the initial leg angle θ_0, the initial (post-heel-strike) angular velocity $\dot{\theta}_0$, step duration T_{step}, the constant leg-length over the step ℓ_{leg}, and the foot-ground contact position in the forward direction x_{contact}. Nonlinear equality constraints make sure that the body position at the end of one step is equal to that at the beginning of the next step.

Numerical optimization. We used numerical optimization to determine the multi-step walking motion that satisfies the oscillating-speed experimental protocol and minimizes the model metabolic cost as described above. The biped model alternates between a higher speed $v_{\text{avg}} + L/T_{\text{fwd}}$ and a lower speed $v_{\text{avg}} - L/T_{\text{bck}}$, each lasting a few steps, so that the net average speed is v_{avg}, and the forward and backward movement in lab frame have periods equal to T_{fwd} and T_{bck}. The number of steps for the forward and backward movements are chosen based on the number of steady walking steps in the durations T_{fwd} and T_{bck}. Other constraints included an upper bound on the leg length ($< \ell_{\text{max}}$) and a periodicity constraint on the body height over one period of back and forth walking. The optimization problem was solved in `MATLAB` using the optimization software `SNOPT`, which employs the sequential quadratic programming technique [6]. At each average speed, we also computed the optimal constant-speed inverted pendulum walking gait (repeating calculations in [3, 7]), so as to subtract from the optimal oscillating-speed walking cost.

Leg force cost. We repeated the calculations above with a cost for leg force, proportional to the integral of the leg force, with a proportionality constant as in [**?**]. We found that this leg force cost did not change our overall predictions for the difference between oscillating-speed and constant-speed metabolic costs, as both these costs increase by the almost same amount due to the leg force cost. This result can be explained intuitively as follows: because the legs make relatively small angle with the vertical, as explained in [7], the average leg force is approximately equal to the average vertical force, which has to be equal to the total body weight for periodic motion – be it constant-speed walking or oscillating-speed walking.

S3 Daily energy budget for starting and stopping

Subjects in [8] performed a majority of the walking over a day in short bouts; they walked in 43914 bouts and took a total of 1717730 steps. Assuming a typical step length of 0.6 m [9], the subjects walked a total distance of 1030638 m. Assuming the subjects walked the whole distance at a constant speed of 1.4 m/s, we can predict a total constant-speed energy expenditure to be 2262600 J, based on a parabolic relationship given by $\dot{E}_{\text{steady}} = a + bv^2$ with $a = 2.22$ W/kg and $b = 1.15$ W/kg/(ms^{-1})^2 [10]. But, such a cost would ignore the cost of accelerating from and to rest at the start and the end of the bout. We can approximate this daily unsteady cost for the 43914 bouts to be 137380 J by extrapolating our results from the kinetic energy-based model with the unsteady cost for one bout given by, $\lambda_{ke}(\eta_{\text{pos}}^{-1} + \eta_{\text{neg}}^{-1})v^2/2$, where $\lambda_{ke} = 0.67$. The ratio of the cost of changing speed to cost of walking is thus found to be 0.06, in other words, the unsteady cost of walking per day is 6% of the steady cost on average. Using the 95% C.I. for λ_{ke} gives us 4-8% as reported in the main manuscript. This approximate calculation shows that the cost of changing speeds is a significant fraction of the energy humans consume in daily walking.

S4 Comparison with a previous study

Our oscillating speed protocols had speed fluctuations between ±0.13 and ±0.27 m/s. As noted in the main manuscript, one previous article [11] attempted to measure the cost of changing speeds, with greater speed fluctuations (±0.15 to ±0.56 m/s) and higher kinetic energy fluctuations per unit time than our study. The rate of kinetic energy fluctuations for both experiments can be compared by comparing $v\Delta v/T$, for a fluctuation between speeds $v - \Delta v$ and $v + \Delta v$ in T seconds. In [11] $v\Delta v/T$ ranges between to 0.0226 and 0.127 $\mathrm{m^2s^{-3}}$ and in our protocol, $v\Delta v/T$ ranges between 0.026 and 0.1108 $\mathrm{m^2s^{-3}}$. Thus, the kinetic energy fluctuation rates were similar in the two studies. Nevertheless, the study [11] found significant increase only for their highest speed fluctuation but not lower. As noted in the main manuscript, this study [11] required walking on oscillating-speed treadmill belts or controlling step durations in overground walking (derived from oscillating-speed treadmill). An oscillating-speed treadmill, being a non-inertial reference frame, can perform mechanical work on the subject, and is not mechanically equivalent to oscillating-speed walking overground. Further, controlling step durations [11] will produce incorrect speed fluctuations that do not obey the speed-step-duration relation for directly controlling walking speed, as established by Bertram and Ruina [12] in the case of steady walking.

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