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Appendix E1

The typical multicomponent spin-echo T2-mapping technique requires acquisition of images at varying echo time, and the image data are fit using a multiexponential decay model. In comparison, the mcDESPOT method fits observed spoiled gradient-recalled echo and balanced steady-state free precession signals at varying flip angle. The observed spoiled gradient-recalled echo and balanced steady-state free precession signals can be modeled by using a two-pool water exchanging system at steady-state condition as described below.

Consider a composite spin system including two exchanging water pools (fast (F) and slow (S) relaxing water pools) described in the mcDESPOT method. On the basis of the Bloch-McConnell equation, the spin evolution of magnetization can be expressed as follows:

$$\frac{dM}{dt} = AM + C\ ,$$

where vector $M = \begin{bmatrix} M_{\mathrm{x,F}} & M_{\mathrm{x,S}} & M_{\mathrm{y,F}} & M_{\mathrm{y,S}} & M_{\mathrm{z,F}} & M_{\mathrm{z,S}} \end{bmatrix}^{\mathrm{T}}$ contains components of the transverse (x, y) and longitudinal (z) magnetizations of each pool, and vector $C = \rho\begin{bmatrix} 0 & 0 & 0 & 0 & R_{1,\mathrm{F}} f_{\mathrm{F}} & R_{1,\mathrm{S}} f_{\mathrm{S}} \end{bmatrix}^{\mathrm{T}}$. Here, ‘T’ denotes the transpose operation and ρ is the proton density term, f_{F} and f_{S} are fractional magnetizations for fast and slow water proton pools normalized by total water magnetization, respectively. $R_1 = 1/T_1$ is longitudinal relaxation rate. d is the notation symbol for differentiation , t is time. All relaxation and exchange terms are included into a matrix, A, defined as follows:

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -R_{2,\mathrm{F}} - k_{\mathrm{FS}} & k_{\mathrm{SF}} & \Delta\omega & 0 \\ k_{\mathrm{FS}} & -R_{2,S} - k_{\mathrm{SF}} & 0 & \Delta\omega \\ -\Delta\omega & 0 & -R_{2,\mathrm{F}} - k_{\mathrm{FS}} & k_{\mathrm{SF}} \\ 0 & -\Delta\omega & k_{\mathrm{FS}} & -R_{2,\mathrm{S}} - k_{\mathrm{SF}} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -R_{1,\mathrm{F}} - k_{\mathrm{FS}} & k_{\mathrm{SF}} \\ k_{\mathrm{FS}} & -R_{1,\mathrm{S}} - k_{\mathrm{SF}} \end{pmatrix},$$

where k_{SF}, k_{FS} are rates of (diffusion-driven) magnetization exchange between S and F pools and vice versa, respectively, and $\Delta\omega$ is the off-resonance frequency caused by B_0 field inhomogeneity. $R_2 = 1/T_2$ is the transverse relaxation rate. At chemical equilibrium, the fractional magnetization and exchange rates are related by:

$$k_{\mathrm{FB}} f_{\mathrm{F}} = k_{\mathrm{SF}} f_{\mathrm{S}}$$

For the balanced steady-state free precession signals (bSSFP) signal, an analytical solution of the above equation can be derived for the steady-state magnetization, $M_{\mathrm{bSSFP}}^{\mathrm{SS}}$:

$$M_{\mathrm{bSSFP}}^{\mathrm{SS}} = \left[I - e^{\mathrm{A} \cdot \mathrm{TR}} R_{\alpha} \right]^{-1} \left(e^{\mathrm{A} \cdot \mathrm{TR}} - I \right) A^{-1} C$$

where $M_{\mathrm{bSSFP}}^{\mathrm{SS}} = \left[M_{\mathrm{x,F}}^{\mathrm{SS}} \quad M_{\mathrm{x,S}}^{\mathrm{SS}} \quad M_{\mathrm{y,F}}^{\mathrm{SS}} \quad M_{\mathrm{y,S}}^{\mathrm{SS}} \quad M_{\mathrm{z,F}}^{\mathrm{SS}} \quad M_{\mathrm{z,S}}^{\mathrm{SS}} \right]^{\mathrm{T}}$, I is the 6 × 6 identity matrix, TR is the sequence repetition time and matrix R_{α} is a rotation matrix with the prescribed flip angle α, expanded as follows:

$$R_{\alpha} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 0 & 0 & \cos\alpha & 0 & \sin\alpha \\ 0 & 0 & -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

The observed bSSFP signal, $M_{\mathrm{bSSFP}}^{\mathrm{obs}}$, is calculated as the magnitude of the complex summation of the transverse magnetizations of water components F and S:

$$M_{\mathrm{bSSFP}}^{\mathrm{obs}} = \left| \left(M_{\mathrm{x,F}}^{\mathrm{SS}} + i M_{\mathrm{y,F}}^{\mathrm{SS}} \right) + \left(M_{\mathrm{x,S}}^{\mathrm{SS}} + i M_{\mathrm{y,S}}^{\mathrm{SS}} \right) \right|$$

Spoiled gradient-recalled echo (SPGR) signal can be derived in a similar fashion. An analytical solution to the equation for the steady-state signal at SPGR sequence, $M_{\mathrm{SPGR}}^{\mathrm{SS}}$, can be expressed in the matrix form yielding the steady-state magnetization:

$$M_{\mathrm{SPGR}}^{\mathrm{SS}} = \rho \left[I - e^{\mathrm{A}_2 \cdot \mathrm{TR}} \cos\alpha \right]^{-1} \left(I - e^{\mathrm{A}_2 \cdot \mathrm{TR}} \right) \begin{bmatrix} f_{\mathrm{F}} \\ f_{\mathrm{S}} \end{bmatrix}$$

where $M_{\mathrm{SPGR}}^{\mathrm{SS}} = \left[M_{\mathrm{z,F}}^{\mathrm{SS}} \quad M_{\mathrm{z,S}}^{\mathrm{SS}} \right]^{\mathrm{T}}$, and I is now the 2 × 2 identity matrix. The observed SPGR signal, $M_{\mathrm{SPGR}}^{\mathrm{obs}}$, can be calculated as follows:

$$M_{\mathrm{SPGR}}^{\mathrm{obs}} = \sin\alpha \left(M_{\mathrm{z,F}}^{\mathrm{SS}} + M_{\mathrm{z,S}}^{\mathrm{SS}} \right)$$