Proof for Eq. (15)

Here, we prove that a mutant on the cycle with dB updating has lower fixation probability than in the well-mixed population for general N. We have to show that $\phi_{dB}^{M} - \phi_{dB}^{\circ} > 0$ for $r > 0, r \neq 1$. Under dB updating, the fixation probability in the well-mixed population is given by

$$\phi_{dB}^{M} = \frac{N-1}{N} \frac{1-\frac{1}{r}}{1-\frac{1}{r^{N-1}}}. \tag{1}$$

On the cycle, the respective fixation probability is given by (Eq. (5.3) in [1])

$$\phi_{dB}^{\circ} = \frac{2(r-1)}{3r-1+(r-3)r^{2-N}}. \tag{2}$$

Then the difference is given by

$$\phi_{dB}^{M} - \phi_{dB}^{\circ} = \frac{(r-1)r^{N-2}}{N} \cdot \frac{(N-3)r^{N+1} - (N-1)r^{N} + (N-1)r^{3} - (N-3)r^{2}}{(r^{N-1}-1)(3r^{N+1} - r^{N} + r^{3} - 3r^{2})} \tag{3}$$

This expression can be written in the form of

$$\frac{(r-1)^2 r^{N-2}}{N} \cdot \frac{\sum_{k=0}^{N-4} r^k((N-2)(k+1)-(k+1)^2)}{\left(\sum_{k=0}^{N-2} r^k\right)\left(3 + \sum_{k=1}^{N-3} 2r^k + 3r^{N-2}\right)}. \tag{4}$$

As Eq. (4) contains only positive coefficients in r, the difference is always positive for $r > 0$ and $r \neq 1$. For $r = 1$, it is zero. Thus Eq. (15) is fulfilled for all $r > 0$, $r \neq 1$.

References

[1] Kaveh K, Komarova NL, Kohandel M. The duality of spatial death-birth and birth-death processes and limitations of the isothermal theorem. Journal of the Royal Society Open Science. 2015;2(140465).