## Supplementary Materials

## ADNI data analysis with APOE4

As one reviewer suggested, we included APOE4 as a covariates in addition to age, gender and education. Since 8 subjects had a missing value for APOE4 and were excluded, the new analyses were based on 22 AD patients and 38 CNs. Demographic information is listed in Table S1. The genetic variant APOE4 was significant between the two groups (p-value 2.75e-06). In Figure S1, the p-values were plotted at various connection densities, e.g.

```
c \in \{0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 1\}.
```

The p-value of taSPU was 0.0169 while that of taNBS was 0.0669, which were similar to those of the previous analysis without APOE4.

Table S1: Demographic information of the participants in the ADNI data.

| group                   | No. of subjects | APOE $4(0/1/2)$ | Gender(F/M) | Age              | Years of education |
|-------------------------|-----------------|-----------------|-------------|------------------|--------------------|
| AD                      | 22              | 2/13/7          | 14F/8M      | $73.20 \pm 7.87$ | $15.18 \pm 2.65$   |
| CN                      | 38              | 25/12/1         | 20F/18M     | $75.22 \pm 6.12$ | $16.23 \pm 2.29$   |
| Two sample test p-value | -               | 2.75e-06        | 0.576       | 0.307            | 0.126              |

Figure S1: P-values for testing brain network differences between the AD group and CN group in the ADNI data. The left column (a) shows the p-values using correlations. The middle column (b) shows the p-values with using partial correlations. The last column (c) taSPU and taNBS shows the p-values by combining two inferences evaluated with correlations or partial correlations.

