Applendix A: Joint likelihood function

For the ith individual in a sample of size n, let Y_i be the quantitative or binary trait and X_i the vector of covariates. For the kth SNP in a genetic region, let G_{ik} denote the unobserved true genotype, p_k the MAF and D_{ik} the observed sequenced data. Let $f(D_{ik}|G_{ik})$ be the given genotype likelihood function (GLF). We assume

$$f(G_{ik}|p_k) = \binom{2}{G_{ik}} p_k^{G_{ik}}(1-p_k)^{2-G_{ik}}, G_{ik} \in \{0,1,2\} \tag{1}$$

For generalized linear model,

$$f(Y_i|X_i, G_{ik}; \alpha_0, \alpha_1, \beta, \phi) = \exp\Big(\frac{Y_i\eta_i - b(\eta_i)}{a(\phi)} + c(Y_i, \phi)\Big) \tag{2}$$

with $\eta_i = \eta_{\alpha_0,\alpha_1,\beta}(X_i, G_{ik}) = \alpha_0 + \alpha_1^T X_i + \beta G_{ik}$. The exact forms of $a(\cdot), b(\cdot)$ and $c(\cdot)$ depend on whether Y_i is quantitative or binary. When Y_i is quantitative,

$$\phi = \sigma^2, a(\phi) = \phi, b(\eta) = \frac{\eta^2}{2}, c(Y_i, \phi) = -\frac{Y_i^2}{2\phi} - \frac{\log(2\pi\phi)}{2} \tag{3}$$

When Y_i is binary,

$$a(\phi) = 1, b(\eta) = \log(1 + \exp(\eta)), c(Y_i, \phi) = 0 \tag{4}$$

The joint log-likelihood function of the parameters $\{\alpha = (\alpha_0, \alpha_1^T)^T, \beta, \phi, p\}$ given the observed data $\{Y_i, X_i, D_{ik}, i = 1, \ldots, n\}$ can be written as:

$$l(\alpha, \beta, \phi, p_k|O) = \sum_{i=1}^{n} \log\Big[\sum_{G_{ik}\in(0,1,2)} \big\{f(Y_i|X_i, G_{ik}; \alpha, \beta, \phi) f(D_{ik}|G_{ik}) f(G_{ik}|p_k)\big\}\Big]. \tag{5}$$

Applendix B: UNC score test

UNC score test is developed upon the pioneer work of Skotte et al. 2012.

B.1 score function

We want to test the null hypothesis

$$H_0 : \beta = 0$$

by score test, which is based on the score function

$$
\begin{aligned}
S(\alpha,\beta,\phi,p_k) &= \begin{pmatrix} \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial \alpha} \\ \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial \beta} \\ \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial \phi} \\ \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k} \end{pmatrix} \\
&= \sum_{i=1}^{n} H^{-1}(Y_i, D_{ik}|X_i) \sum_{G_{ik}\in(0,1,2)} \begin{pmatrix} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})f(G_{ik}|p_k) \\ f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}G_{ik} f(D_{ik}|G_{ik})f(G_{ik}|p_k) \\ f(Y_i|X_i,G_{ik})\big[-\frac{Y_i\eta_i-b(\eta_i)}{a^2(\phi)}a'(\phi)+c'(\phi)\big]f(D_{ik}|G_{ik})f(G_{ik}|p_k) \\ f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q(G_{ik},p_k) \end{pmatrix}
\end{aligned}
\tag{6}
$$

where $H(Y_i, D_{ik}|X_i) = \sum_{G_{ik}\in(0,1,2)} \big\{f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})f(G_{ik}|p_k)\big\}$ and

$$
Q(G_{ik},p_k) = \begin{cases} 2(p_k-1), & G_{ik}=0 \\ 2-4p_k, & G_{ik}=1 \\ 2p_k, & G_{ik}=2 \end{cases} .
$$

Denote the maximum likelihood estimator under $H_0 : \beta = 0$ by $(\tilde{\alpha}, 0, \tilde{\phi}, \tilde{p}_k)$. If we evaluate the score function by $(\tilde{\alpha}, 0, \tilde{\phi}, \tilde{p}_k)$, the score function above simplifies to

$$
S(\tilde{\alpha},0,\tilde{\phi},\tilde{p}_k) = \begin{pmatrix} 0 \\ \sum_{i=1}^{n} \frac{Y_i - b'(\tilde{\alpha}^T X_i)}{a(\tilde{\phi})} E(G_{ik}|D_{ik}) \\ 0 \\ 0 \end{pmatrix} \tag{7}
$$

where $E(G_{ik}|D_{ik}) = \frac{\sum_{G_{ik}\in(0,1,2)} G_{ik} f(D_{ik}|G_{ik})f(G_{ik}|\tilde{p}_k)}{\sum_{G_{ik}\in(0,1,2)} f(D_{ik}|G_{ik})f(G_{ik}|\tilde{p}_k)}$ is the posterior expectation of the genotype of the kth SNP for individual i.

When Y_i is quantitative,

$$
a'(\phi) = 1, b'(\eta) = \eta, c'(Y_i,\phi) = \frac{Y_i^2}{2\phi^2} - \frac{1}{2\phi} \tag{8}
$$

When Y_i is binary,

$$
a'(\phi) = 0, b'(\eta) = \frac{\exp(\eta)}{1+\exp(\eta)}, c'(Y_i,\phi) = 0 \tag{9}
$$

B.2 Information matrix

The observed information matrix is

$$
\begin{aligned}
I(\alpha,\beta,\phi,p_k|O) &= -\begin{bmatrix}
\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\alpha} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\beta} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\phi} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial p_k} \\
\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial\alpha} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial\beta} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial\phi} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial p_k} \\
\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial\alpha} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial\beta} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial\phi} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial p_k} \\
\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k\partial\alpha} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k\partial\beta} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k\partial\phi} & \frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k\partial p_k}
\end{bmatrix} \\
&= \begin{bmatrix}
I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha\phi} & I_{\alpha p_k} \\
I_{\beta\alpha} & I_{\beta\beta} & I_{\beta\phi} & I_{\beta p_k} \\
I_{\phi\alpha} & I_{\phi\beta} & I_{\phi\phi} & I_{\phi p_k} \\
I_{p_k\alpha} & I_{p_k\beta} & I_{p_k\phi} & I_{p_k p_k}
\end{bmatrix} \qquad (10)
\end{aligned}
$$

where

$$
\begin{aligned}
& I_{\alpha\alpha} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\alpha} \\
& = -\sum_{i=1}^{n}\Bigg[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
& \cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i^T f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
& +H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big(\frac{(Y_i-b'(\eta_i))^2}{a^2(\phi)}-\frac{b''(\eta_i)}{a(\phi)}\Big)X_iX_i^T f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Bigg]
\end{aligned} \qquad (11)
$$

$$
\begin{aligned}
& I_{\alpha\beta} = I_{\beta\alpha} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\beta} \\
& = -\sum_{i=1}^{n}\Bigg[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
& \cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}G_{ik} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
& +H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big(\frac{(Y_i-b'(\eta_i))^2}{a^2(\phi)}-\frac{b''(\eta_i)}{a(\phi)}\Big)X_iG_{ik} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Bigg]
\end{aligned} \qquad (12)
$$

$$
\begin{aligned}
&I_{\alpha\phi} = I_{\phi\alpha} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial\phi} \\
&= -\sum_{i=1}^{n}\Big[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&\cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\big\{-\frac{Y_i\eta_i-b(\eta_i)}{a^2(\phi)}a'(\phi)+c'(\phi)\big\}f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&+H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big\{\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i\big(-\frac{Y_i\eta_i-b(\eta_i)}{a^2(\phi)}a'(\phi)+c'(\phi)\big) \\
&-\frac{Y_i-b'(\eta_i)}{a^2(\phi)}X_i a'(\phi)\Big\}f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big]
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&I_{\alpha p_k} = I_{p_k\alpha} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\alpha\partial p_k} \\
&= -\sum_{i=1}^{n}\Big[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&\cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big) \\
&+H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}X_i f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big]
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&I_{\beta\beta} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial\beta} \\
&= -\sum_{i=1}^{n}\Big[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i-b'(\eta_i)}{a(\phi)}G_{ik} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big)^2 \\
&+H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big(\frac{(Y_i-b'(\eta_i))^2}{a^2(\phi)}-\frac{b''(\eta_i)}{a(\phi)}\Big)G_{ik}^2 f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big]
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&I_{\beta\phi} = I_{\phi\beta} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial\phi} \\
&= -\sum_{i=1}^{n}\Big[- H^{-2}(Y_i, D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i - b'(\eta_i)}{a(\phi)}G_{ik}f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&\cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\big\{-\frac{Y_i\eta_i - b(\eta_i)}{a^2(\phi)}a'(\phi) + c'(\phi)\big\} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&+H^{-1}(Y_i, D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big\{\frac{Y_i - b'(\eta_i)}{a(\phi)}G_{ik}\big(-\frac{Y_i\eta_i - b(\eta_i)}{a^2(\phi)}a'(\phi) + c'(\phi)\big) \\
&-\frac{Y_i - b'(\eta_i)}{a^2(\phi)}G_{ik}a'(\phi)\Big\} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big]
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&I_{\beta p_k} = I_{p_k\beta} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\beta\partial p_k} \\
&= -\sum_{i=1}^{n}\Big[- H^{-2}(Y_i, D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i - b'(\eta_i)}{a(\phi)}G_{ik}f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big) \\
&\cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big) \\
&+H^{-1}(Y_i, D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\frac{Y_i - b'(\eta_i)}{a(\phi)}G_{ik}f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big]
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&I_{\phi\phi} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial\phi} \\
&= -\sum_{i=1}^{n}\Big[- H^{-2}(Y_i, D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\big\{-\frac{Y_i\eta_i - b(\eta_i)}{a^2(\phi)}a'(\phi) + c'(\phi)\big\} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big)^2 \\
&+H^{-1}(Y_i, D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\Big\{\big(-\frac{Y_i\eta_i - b(\eta_i)}{a^2(\phi)}a'(\phi) + c'(\phi)\big)^2 \\
&+\big(Y_i\eta_i - b(\eta_i)\big)\Big(\frac{2a'^2(\phi)}{a^3(\phi)} - \frac{a''(\phi)}{a^2(\phi)}\Big) + c''(\phi)\Big\} f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big]
\end{aligned}
\tag{18}
$$

$$I_{\phi p_k} = I_{p_k\phi} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial\phi\partial p_k}$$
$$= -\sum_{i=1}^{n}\Big[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\big\{-\frac{Y_i\eta_i-b(\eta_i)}{a^2(\phi)}a'(\phi)+c'(\phi)\big\}f(D_{ik}|G_{ik})f(G_{ik}|p_k)\Big)$$
$$\cdot\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big)$$
$$+H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})\big\{-\frac{Y_i\eta_i-b(\eta_i)}{a^2(\phi)}a'(\phi)+c'(\phi)\big\}f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big] \quad (19)$$

$$I_{\phi p_k} = -\frac{\partial l(\alpha,\beta,\phi,p_k|O)}{\partial p_k\partial p_k}$$
$$= -\sum_{i=1}^{n}\Big[-H^{-2}(Y_i,D_{ik}|X_i)\Big(\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q(G_{ik},p_k)\Big)^2$$
$$+H^{-1}(Y_i,D_{ik}|X_i)\sum_{G_{ik}\in(0,1,2)} f(Y_i|X_i,G_{ik})f(D_{ik}|G_{ik})Q'(G_{ik},p_k)\Big] \quad (20)$$

where $Q'(G_{ik},p_k) = \begin{cases} 2, & G_{ik}=0 \\ -4, & G_{ik}=1 \\ 2, & G_{ik}=2 \end{cases}$

When Y_i is quantitative,

$$a''(\phi)=0, b''(\eta)=1, c''(Y_i,\phi) = -\frac{Y_i^2}{\phi^3}+\frac{1}{2\phi^2} \quad (21)$$

When Y_i is binary,

$$a''(\phi)=0, b''(\eta)=\frac{\exp(\eta)(1+\exp(\eta))-\exp^2(\eta)}{(1+\exp(\eta))^2}, c''(Y_i,\phi)=0 \quad (22)$$

The observed information matrix evaluated at $(\tilde{\alpha},0,\tilde{\phi},\tilde{p}_k)$ simplifies to

$$I(\tilde{\alpha},0,\tilde{\phi},\tilde{p}_k)|O) = \begin{bmatrix} I_{\tilde{\alpha}\tilde{\alpha}} & I_{\tilde{\alpha}0} & 0 & 0 \\ I_{0\tilde{\alpha}} & I_{00} & I_{0\tilde{\phi}} & I_{0\tilde{p}_k} \\ 0 & I_{\tilde{\phi}0} & I_{\tilde{\phi}\tilde{\phi}} & 0 \\ 0 & I_{\tilde{p}_k0} & 0 & I_{\tilde{p}_k\tilde{p}_k} \end{bmatrix} \quad (23)$$

Applendix C: Prove of inequality (7) in main text

UNC score test statistics is

$$T_{UNC\ score}(Y_i, D_{ik}|X_i) = S^T(\tilde{\alpha}, 0, \tilde{\phi}, \tilde{p}_k) I^{-1}(\tilde{\alpha}, 0, \tilde{\phi}, \tilde{p}_k)|O) S(\tilde{\alpha}, 0, \tilde{\phi}, \tilde{p}_k), \quad (24)$$

In Skotte (2012) paper, their test statistics is

$$T_{SKA\ score}(Y_i, D_{ik}|X_i) = S^T_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi}) I^{-1}_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi})|O) S_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi}), \quad (25)$$

where

$$S_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi}) = \begin{pmatrix} 0 \\ \sum_{i=1}^{n} \frac{Y_i - b'(\tilde{\alpha}^T X_i)}{a(\phi)} E(G_{ik}|D_{ik}) \\ 0 \end{pmatrix} \quad (26)$$

and

$$I_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi})|O) = \begin{bmatrix} I_{\tilde{\alpha}\tilde{\alpha}} & I_{\tilde{\alpha}0} & 0 \\ I_{0\tilde{\alpha}} & I_{00} & I_{0\tilde{\phi}} \\ 0 & I_{\tilde{\phi}0} & I_{\tilde{\phi}\tilde{\phi}} \end{bmatrix} \quad (27)$$

$I_{Skotte}(\tilde{\alpha}, 0, \tilde{\phi}|O)$ contains only the first three rows and columns of matrix in (23). By some algebra,

$$T_{UNC\ score}(Y_i, D_{ik}|X_i) = \Big(\sum_{i=1}^{n} \frac{Y_i - b'(\tilde{\alpha}^T X_i)}{a(\phi)} E(G_{ik}|D_{ik}) \Big)^2 \frac{I_{\tilde{\alpha}\tilde{\alpha}} I_{\tilde{\phi}\tilde{\phi}}}{I_{\tilde{\alpha}\tilde{\alpha}} I_{00} I_{\tilde{\phi}\tilde{\phi}} - I_{\tilde{\alpha}\tilde{\alpha}} I^2_{0\tilde{\phi}} - I^2_{\tilde{\alpha}0} I_{\tilde{\phi}\tilde{\phi}} - I_{\tilde{\alpha}\tilde{\alpha}} I^2_{0\tilde{p}_k} I_{\tilde{\phi}\tilde{\phi}} / I_{\tilde{p}_k\tilde{p}_k}} \quad (28)$$

and

$$T_{SKA\ score}(Y_i, D_{ik}|X_i) = \Big(\sum_{i=1}^{n} \frac{Y_i - b'(\tilde{\alpha}^T X_i)}{a(\phi)} E(G_{ik}|D_{ik}) \Big)^2 \frac{I_{\tilde{\alpha}\tilde{\alpha}} I_{\tilde{\phi}\tilde{\phi}}}{I_{\tilde{\alpha}\tilde{\alpha}} I_{00} I_{\tilde{\phi}\tilde{\phi}} - I_{\tilde{\alpha}\tilde{\alpha}} I^2_{0\tilde{\phi}} - I^2_{\tilde{\alpha}0} I_{\tilde{\phi}\tilde{\phi}}} \quad (29)$$

Obviously,

$$T_{UNC\ score}(Y_i, D_{ik}|X_i) \geq T_{SKA\ score}(Y_i, D_{ik}|X_i) \quad (30)$$

If $\tilde{\beta}$ and $\tilde{p}_k$ is independent to each other, then $I_{\tilde{\beta}\tilde{p}_k} = 0$ and thus $I^2_{0\tilde{p}_k} = 0$. Consequently,

$$T_{UNC\ score}(Y_i, D_{ik}|X_i) = T_{SKA\ score}(Y_i, D_{ik}|X_i) \quad (31)$$

References

Skotte, L., T. S. Korneliussen, et al., 2012. Association Testing for Next-Generation Sequencing Data Using Score Statistics. *Genetic Epidemiology*, 36(5), 430–437.