Additional file 1. Further details of calculations of expected values of relevant functions of genotype scores.

$$
\begin{aligned}
E(x_i) &= 2P(\mathrm{AA}_i^*) + P(\mathrm{AB}_i^*) \\
&= 2p^2 + 2p(1-p)F_i + 4p(1-p)(1-F_i)K_i + 2p(1-p)(1-F_i)(1-2K_i) \\
&= 2p^2 + 2p(1-p)[F_i + 2(1-F_i)K_i + (1-F_i)(1-2K_i)] \\
&= 2p^2 + 2p(1-p)[F_i + 2K_i - 2F_iK_i + 1 - F_i - 2K_i + 2F_iK_i] \\
&= 2p^2 + 2p(1-p) \\
&= 2p(p + 1 - p) \\
&= 2p \\
E(x_i^2) &= 4P(\mathrm{AA}_i^*) + P(\mathrm{AB}_i^*) \\
&= 4p^2 + 4p(1-p)F_i + 8p(1-p)(1-F_i)K_i + 2p(1-p)(1-F_i)(1-2K_i) \\
&= 4p^2 + 2p(1-p)[2F_i + 4(1-F_i)K_i + (1-F_i)(1-2K_i)] \\
&= 4p^2 + 2p(1-p)[2F_i + 4K_i - 4F_iK_i + 1 - F_i - 2K_i + 2F_iK_i] \\
&= 4p^2 + 2p(1-p)(1 + F_i + 2K_i - 2F_iK_i) \\
E(x_ix_i') &= 4P(\mathrm{AA}_i^*, \mathrm{AA}_{i'}^*) + 2P(\mathrm{AA}_i^*, \mathrm{AB}_{i'}^*) + 2P(\mathrm{AB}_i^*, \mathrm{AA}_{i'}^*) + P(\mathrm{AB}_i^*, \mathrm{AB}_{i'}^*) \\
&= 4P(\mathrm{AA}_i, \mathrm{AA}_{i'}) + 2P(\mathrm{AA}_i, \mathrm{AB}_{i'})(2K_{i'} + 1 - 2K_{i'}) + 2P(\mathrm{AB}_i, \mathrm{AB}_{i'}) \\
&\quad + P(\mathrm{AB}_i, \mathrm{AB}_{i'})(4K_iK_{i'} + 2K_i(1-2K_{i'}) + 2K_{i'}(1-2K_i) + (1-2K_i)(1 \\
&\quad - 2K_{i'})) \\
&= 4P(\mathrm{AA}_i, \mathrm{AA}_{i'}) + 4P(\mathrm{AA}_i, \mathrm{AB}_{i'}) + P(\mathrm{AB}_i, \mathrm{AB}_{i'}) \\
&= 4p(1-p)(1-6p+6p^2)\delta + 12p^2(1-p)^2\Delta + 16p^2(1-p)(1-2p)\gamma \\
&\quad + 24p^3(1-p)\theta + 4p^4 + 8p(1-p)[-(1-6p+6p^2)\delta - 3p(1-p)\Delta + (1 \\
&\quad - 2p)(1-4p)\gamma + 3p(1-2p)\theta + p^2] + 4p(1-p)[(1-6p+6p^2)\delta \\
&\quad + 3p(1-p)\Delta - 2(1-2p)^2\gamma + (1-6p+6p^2)\theta + p(1-p)] \\
&= 4p(1-p)(1-6p+6p^2)\delta + 12p^2(1-p)^2\Delta + 16p^2(1-p)(1-2p)\gamma \\
&\quad + 24p^3(1-p)\theta + 4p^4 + [-8p(1-p)(1-6p+6p^2)\delta - 24p^2(1-p)^2\Delta \\
&\quad + 8p(1-p)(1-2p)(1-4p)\gamma + 24p^2(1-p)(1-2p)\theta + 8(1-p)p^3] \\
&\quad + [4p(1-p)(1-6p+6p^2)\delta + 12p^2(1-p)^2\Delta - 8p(1-p)(1-2p)^2\gamma \\
&\quad + 4p(1-p)(1-6p+6p^2)\theta + 4p^2(1-p)^2] \\
&= 4[1-2+1]p(1-p)(1-6p+6p^2)\delta + 12[1-2+1]p^2(1-p)^2\Delta + 8[2p \\
&\quad + (1-4p) - (1-2p)]p(1-p)(1-2p)\gamma + 4[6p^2 + 6p(1-2p) + (1 \\
&\quad - 6p + 6p^2)]p(1-p)\theta + 4(p^2 + 2(1-p)p + (1-p)^2)p^2 \\
&= 4[6p^2 - 12p^2 + 1 + 6p^2]p(1-p)\theta + 4(p^2 + 2p - 2p^2 + 1 - 2p + p^2)p^2 \\
&= 4p(1-p)\theta + 4p^2 \\
E(S_{ii}) &= E(x_i^2) - 4pE(x_i) + 4p^2 \\
&= 4p^2 + 2p(1-p)(1 + F_i + 2K_i - 2F_iK_i) - 8p^2 + 4p^2 \\
&= 2p(1-p)(1 + F_i + 2K_i - 2F_iK_i) \\
E(S_{ii'}) &= E(x_ix_{i'}) - 4pE(x_i) + 4p^2 \\
&= E(x_ix_{i'}) - 4p^2 \\
&= 4p(1-p)\theta_{ii'}
\end{aligned}
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