File S1: Details on the Metropolis–Hastings within Gibbs MCMC algorithm implemented in the BayPass program

The purpose is to sample from the posterior distributions of the parameters defined in the models represented in Figure 1. To that end, each parameter is initialised to standard moment-based estimate and sequentially updated using a standard Metropolis–Hastings (M–H) within Gibbs MCMC algorithm. The same notations as in the main text are used in the following.

1 Update of the y_{ij}'s (Pool–Seq data):

The parameters y_{ij} are updated iteratively in each population, one locus at a time. The full conditional distribution has the form:

$$\begin{aligned} f\left(y_{ij} \mid .\right) &\propto f\left(y_{ij} \mid \alpha_{ij}\right) f\left(r_{ij} \mid y_{ij}, c_{ij}\right) \\ &\propto \binom{n_j}{y_{ij}} \alpha_{ij}^{y_{ij}} \left(1-\alpha_{ij}\right)^{n_j-y_{ij}} \left(\frac{y_{ij}}{n_j}\right)^{r_{ij}} \left(1-\frac{y_{ij}}{n_j}\right)^{c_{ij}-r_{ij}} \end{aligned}$$

Because this full conditional is not of usual form, a Metropolis update is implemented. A candidate y_{ij}^c is sampled uniformly over the integer interval $\left\{k_{ij}^{(y)} \dots l_{ij}^{(y)}\right\}$ where $k_{ij}^{(y)} = y_{ij} - \delta_{ij}^{(y)}$ and $l_{ij}^{(y)} = y_{ij} + \delta_{ij}^{(y)}$. The (integer) $\delta_{ij}^{(y)}$ are adjusted for each y_{ij} during pilot runs to obtain acceptance rates ranging between $\tau_{\min}$ and $\tau_{\max}$. As a default, $\tau_{\min} = 0.25$ and $\tau_{\max} = 0.4$ as usually recommended (Gilks *et al.*, 1996). In practice however, it is important to note that each y_{ij}^c should take values within the (integer) interval $\left\{y_{ij}^{\min} \dots y_{ij}^{\max}\right\}$, with:

- $y_{ij}^{\min} = 0$ (if $r_{ij} = 0$) or $y_{ij}^{\min} = 1$ (if $r_{ij} > 0$)
- $y_{ij}^{\max} = n_j$ (if $r_{ij} = c_{ij}$) or $y_{ij}^{\max} = n_j - 1$ (if $r_{ij} < c_{ij}$)

To account for these constraints, if y_{ij}^c is outside this interval, the excess is reflected back; that is:

- if $y_{ij}^c < y_{ij}^{\min}$ then y_{ij}^c is reset to $2 \times y_{ij}^{\min} - y_{ij}^c$
- if $y_{ij}^c > y_{ij}^{\max}$ then it is reset to $2 \times y_{ij}^{\max} - y_{ij}^c$

This leads to a symmetric proposal (Yang, 2005) and the candidate value y_{ij}^c is thus accepted with probability $min(1, \psi_{ij}^{(y)})$ according to the Metropolis rule where :

$$\psi_{ij}^{(y)} = \frac{f\left(y_{ij}^c \mid .\right)}{f\left(y_{ij} \mid .\right)}$$

2 Update of the $\alpha^\star_{ij}$'s:

Two different algorithms are implemented in BayPass to update $\alpha^\star_{ij}$ (recall that $\alpha_{ij} = 1 \wedge (0 \vee \alpha^\star_{ij})$). The first algorithm is expected to have better mixing properties, in particular for unbalanced designs (in terms of population representativeness), because each $\alpha^\star_{ij}$ are updated in turn (this statement has not been tested extensively). Unless otherwise stated, the second algorithm identical to the one described by Coop *et al.* (2010), in which vectors of allele frequencies are updated iteratively, is however generally used because computationally (slightly) faster.

2.1 Algorithm 1:

The parameters $\alpha^\star_{ij}$ are updated iteratively in each population, one locus at a time. The full conditional distribution has the general form:

$$\begin{aligned} f\left(\alpha^\star_{ij} \mid .\right) &\propto f\left(\alpha^\star_{ij} \mid \boldsymbol{\alpha}^\star_{\boldsymbol{i,-j}}, \boldsymbol{\Lambda}, \pi_i, \beta_i, \delta_i\right) f\left(y_{ij} \mid \alpha_{ij}, n_{ij}\right) \\ &\propto f\left(\widetilde{\alpha^\star_{ij}} \mid \widetilde{\boldsymbol{\alpha}^\star_{\boldsymbol{i,-j}}}, \boldsymbol{\Lambda}\right) f\left(y_{ij} \mid \alpha_{ij}, n_{ij}\right) \end{aligned}$$

where $\widetilde{\boldsymbol{\alpha}^\star_{\boldsymbol{i}}} = \left\{\frac{\alpha^\star_{ij} - \pi_i - \delta_i\beta_i \boldsymbol{Z_j}}{\sqrt{\pi(1-\pi_i)}}\right\}_{(1..J)}$ and thus $\widetilde{\boldsymbol{\alpha}^\star_{\boldsymbol{i}}} \mid \boldsymbol{\Lambda} \sim \mathrm{N}_J\left(\mathbf{I_J}; \boldsymbol{\Omega} = \boldsymbol{\Lambda}^{-1}\right)$. Note that, for the core model model $\beta_i = 0$ and for the STD model $\delta_i = 1$. Hence, from the properties of the multivariate Gaussian distribution: $\widetilde{\alpha^\star_{ij}} \mid \widetilde{\boldsymbol{\alpha}^\star_{\boldsymbol{i,-j}}}, \boldsymbol{\Lambda} \sim N\left(\mu_{(\alpha),ij}, \sigma^2_{(\alpha),ij}\right)$, where $\mu_{(\alpha),ij} = \mathbf{C}\widetilde{\boldsymbol{\alpha_{i,-j}}}$ and $\sigma^2_{(\alpha),ij} = \omega_{jj} - \mathbf{C}\boldsymbol{\Omega_{kj}}$. Here $\mathbf{C} = \boldsymbol{\Omega_{jk}}\boldsymbol{\Omega^{-1}_{kk}}$ represents the matrix of regression coefficients and, $\boldsymbol{\Omega_{kj}}$, $\boldsymbol{\Omega_{kk}}$ and $\boldsymbol{\Omega_{jj}}$ are blocks of the matrice $\boldsymbol{\Omega}$. For instance, for $j = 1$:

$$\boldsymbol{\Omega} = \begin{pmatrix} \omega_{11} & \boldsymbol{\Omega_{12}} \\ \boldsymbol{\Omega_{21}} & \boldsymbol{\Omega_{22}} \end{pmatrix}$$

As a consequence:

$$f\left(\alpha^\star_{ij} \mid .\right) \propto e^{-\frac{1}{2\sigma^2_{(\alpha),ij}}\left(\alpha^\star_{ij} - \mu_{(\alpha),ij}\right)^2} \times \alpha_{ij}^{y_{ij}} \left(1 - \alpha_{ij}\right)^{n_j - y_{ij}}$$

Because this full conditional is not of usual form, a (random walk) Metropolis update is implemented. A candidate $\widetilde{\alpha^{\star,(c)}_{ij}}$ is sampled from a uniform distribution: $\mathrm{Unif}\left(\widetilde{\alpha^\star_{ij}} - \delta^{(\alpha)}_{ij}; \widetilde{\alpha^\star_{ij}} + \delta^{(\alpha)}_{ij}\right)$ The $\delta^{(\alpha)}_{ij}$'s' are adjusted for each $\alpha^\star_{ij}$ during pilot runs to obtain acceptance rates ranging between $\tau_{\min}$ and $\tau_{\max}$. As a default, $\tau_{\min} = 0.25$ and $\tau_{\max} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal is symmetric, the candidate value $\widetilde{\alpha^{\star,(c)}_{ij}}$ is accepted with probability $min(1, \psi^{(\alpha)}_{ij})$ according to the Metropolis rule where :

$$\psi^{(\alpha)}_{ij} = \frac{f\left(\alpha^{\star,(c)}_{ij} \mid .\right)}{f\left(\alpha^\star_{ij} \mid .\right)}$$

2.2 Algorithm 2 (Coop *et al.*, 2010):

This Metropolis update was adapted from Coop *et al.* (2010) (Appendix A). The vectors $\boldsymbol{\alpha}_i^\star$ are updated iteratively one locus at a time. he full conditional distribution has the general form:

$$\begin{aligned} f\left(\boldsymbol{\alpha}_i^\star \mid .\right) &\propto f\left(\boldsymbol{\alpha}_i^\star \mid \boldsymbol{\Lambda}, \pi_i, \beta_i, \delta_i\right) \prod_{j=1}^{j=J} \left(f\left(y_{ij} \mid \alpha_{ij}, n_{ij}\right)\right) \\ &\propto f\left(\widetilde{\boldsymbol{\alpha}_i^\star} \mid \boldsymbol{\Lambda}\right) \prod_{j=1}^{j=J} \left(f\left(y_{ij} \mid \alpha_{ij}, n_{ij}\right)\right) \\ &\propto e^{-\frac{1}{2}\ {}^t\widetilde{\boldsymbol{\alpha}_i^\star} \boldsymbol{\Lambda} \widetilde{\boldsymbol{\alpha}_i^\star}} \prod_{j=1}^{j=J} \left(\alpha_{ij}^{y_{ij}} \left(1-\alpha_{ij}\right)^{n_j - y_{ij}}\right) \end{aligned}$$

(see 2.1 for a definition of $\widetilde{\boldsymbol{\alpha}_i^\star}$)

Because this full conditional is not of usual form, a joint Metropolis update is implemented. A vector of candidate values $\widetilde{\boldsymbol{\alpha}_i^{\star,(c)}}$ is sampled from the following Multivariate Gaussian proposal:

$$\widetilde{\boldsymbol{\alpha}_i^{\star,(c)}} \sim MNV\left(\widetilde{\boldsymbol{\alpha}_i^\star}, \Gamma \delta_i^{(\alpha)}\right)$$

where Γ is obtain from the Cholesky decomposition of $\boldsymbol{\Omega} = \boldsymbol{\Lambda}^{-1}$ (i.e., $\Omega = {}^t\Gamma\Gamma$) and the $\delta_i^{(\alpha)}$ are adjusted during pilot runs to obtain acceptance rates ranging between $\tau_{\min}$ and $\tau_{\max}$. As a default, $\tau_{\min} = 0.25$ and $\tau_{\max} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal is symmetric, the candidate vector $\widetilde{\alpha_i^{\star,(c)}}$ is accepted with probability $min(1, \psi_i^{(\alpha)})$ according to the Metropolis rule where :

$$\psi_i^{(\alpha)} = \frac{f\left(\alpha_i^{\star,(c)} \mid .\right)}{f\left(\alpha_i^\star \mid .\right)}$$

3 Update of the π_i's:

The parameters π_i are updated iteratively one locus at a time. The full conditional distribution has the general form:

$$\begin{aligned} f\left(\pi_i \mid .\right) &\propto f\left(\alpha_{ij}^\star \mid \boldsymbol{\Lambda}, \pi_i, \beta_i, \delta_i\right) f\left(\pi_i \mid a_\pi, b_\pi\right) \\ &\propto \pi_i^{-\frac{J}{2}} (1-\pi_i)^{-\frac{J}{2}} e^{-\frac{1}{2}\ {}^t\widetilde{\boldsymbol{\alpha}_i^\star} \boldsymbol{\Lambda} \widetilde{\boldsymbol{\alpha}_i^\star}} \times \pi_i^{a_\pi - 1} (1-\pi_i)^{(b_\pi - 1)} \\ &\propto \pi_i^{a_\pi - 1 - \frac{J}{2}} (1-\pi_i)^{b_\pi - 1 - \frac{J}{2}} e^{-\frac{1}{2}\ {}^t\widetilde{\boldsymbol{\alpha}_i^\star} \boldsymbol{\Lambda} \widetilde{\boldsymbol{\alpha}_i^\star}} \end{aligned}$$

(see 2.1 for a definition of $\widetilde{\boldsymbol{\alpha}_i^\star}$)

Because this full conditional is not of usual form, a (random walk) Metropolis–Hastings update is implemented. A candidate $\pi_i^{(c)}$ is sampled from a uniform distribution whose support is centred on the current value of π_i:

$$\pi_i^{(c)} \sim \mathrm{Unif}\left(\max(\epsilon, \pi_i - \delta_i^{(\pi)}), \min(1-\epsilon, \pi_i - \delta_i^{(\pi)})\right)$$

($\epsilon = 10^{-8}$ in BayPass)

The $\delta_i^{(\pi)}$'s' are adjusted for each π_i during pilot runs to obtain acceptance rates ranging between $\tau_{\min}$ and $\tau_{\max}$. As a default, $\tau_{\min} = 0.25$ and $\tau_{\max} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal may not be symmetric, the candidate value ${\pi_i}^{(c)}$ is accepted with probability $min(1, \psi_i^{(\pi)})$ according to the Metropolis–Hastings rule where :

$$\psi_i^{(\pi)} = \frac{f\left(\pi_i^{(c)} \mid .\right) q(\pi_i \mid \pi_i^{(c)})}{f\,(\pi_i \mid .)\, q(\pi_i^{(c)} \mid \pi_i)}$$

with:

- $q(\pi_i^{(c)} \mid \pi_i) = \frac{1}{\min\left(1-\epsilon;\pi_i-\delta_i^{(\pi)}\right)-\max\left(\epsilon;\pi_i-\delta_i^{(\pi)}\right)}$
- $q(\pi_i \mid \pi_i^{(c)}) = \frac{1}{\min\left(1-\epsilon;\pi_i^{cdt}-\delta_i^{(\pi)}\right)-\max\left(\epsilon;\pi_i^{cdt}-\delta_i^{(\pi)}\right)}$

4 Update of Λ:

From the conjugacy properties a simple Gibbs update is possible consisting in directly sampling $\Lambda = \Omega^{-1}$ in its full conditional distribution. Indeed the full conditional of Λ is a Wishart distribution:

$$\begin{aligned} f(\mathbf{\Lambda} \mid .) &\propto f(\mathbf{\Lambda}) \times \prod_{i=1}^{i=I} f(\boldsymbol{\alpha}_i^{\star} \mid \mathbf{\Lambda}) \\ &\propto |\mathbf{\Lambda}|^{\frac{\rho-J+I-1}{2}} \exp\left(-\frac{1}{2}\left(\rho \mathrm{tr}\,(\mathbf{\Lambda}) + \sum_{i=1}^{i=I}\left({}^t\widetilde{\boldsymbol{\alpha}_i^{\star}}\mathbf{\Lambda}\widetilde{\boldsymbol{\alpha}_i^{\star}}\right)\right)\right) \\ &\propto |\mathbf{\Lambda}|^{\frac{\rho-J+I-1}{2}} \exp\left(-\frac{1}{2}\left(\rho \mathrm{tr}\,(\mathbf{\Lambda}) + \mathrm{tr}\left(\sum_{i=1}^{i=I}\left(\widetilde{\boldsymbol{\alpha}_i^{\star}}\,{}^t\widetilde{\boldsymbol{\alpha}_i^{\star}}\mathbf{\Lambda}\right)\right)\right)\right) \\ &\propto |\mathbf{\Lambda}|^{\frac{\rho-J+I-1}{2}} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\rho \mathbf{I}_{\mathrm{J}} + \sum_{i=1}^{i=I}\widetilde{\boldsymbol{\alpha}_i^{\star}}\,{}^t\widetilde{\boldsymbol{\alpha}_i^{\star}}\right)\mathbf{\Lambda}\right)\right) \end{aligned}$$

(see 2.1 for a definition of $\widetilde{\boldsymbol{\alpha}_i^{\star}}$)

Hence:

$$\mathbf{\Lambda} \mid . \sim W\left(\left(\rho \mathbf{I}_{\mathrm{J}} + \sum_{i=1}^{i=J}\widetilde{\boldsymbol{\alpha}_i}\,{}^t\widetilde{\boldsymbol{\alpha}_i}\right)^{-1}; \rho + I\right)$$

Working on $\mathbf{\Omega}$, Coop *et al.* (2010) found an equivalent result (Appendix A)[1]

5 Update of a_π and b_π:

To update a_π and b_π, we follow the reparametrization in terms of mean $\mu_\pi = \frac{a_\pi}{a_\pi + b_\pi}$ and "sample size" $\nu_\pi = a_\pi + b_\pi$ (e.g. Kruschke, 2014). Equivalently, $a_\pi = \mu_\pi \nu_\pi$ and $b_\pi = (1-\mu_\pi)\nu_\pi$. The parameters μ_π and ν_π are updated one at a time.

[1] Note that there is a typo in eq. A1 since the equation $\hat{S}$ should read $\hat{S} = \frac{1}{L}\sum_{l=1}^{l=L}\left(\frac{1}{\epsilon_l(1-\epsilon_l)}(\theta_l-\epsilon_l)(\theta_l-\epsilon_l)^T\right)$. According to the notations employed here: $\sum_{l=1}^{l=L}\left(\frac{1}{\epsilon_l(1-\epsilon_l)}(\theta_l-\epsilon_l)(\theta_l-\epsilon_l)^T\right) \equiv \sum_{i=1}^{i=J}\widetilde{\boldsymbol{\alpha}_i}\,{}^t\widetilde{\boldsymbol{\alpha}_i}$

5.1 Update of μ_π:

The full conditional distribution of μ_π has the form:

$$\begin{aligned} f(\mu_\pi \mid .) &\propto f(\mu_\pi)\prod_{i=1}^{j=I} f(\pi_i \mid \mu_\pi, \nu_\pi) \\ &\propto \left(\frac{1}{\Gamma(\nu_\pi\mu_\pi)\,\Gamma(\nu_\pi(1-\mu_\pi))}\right)^I \prod_{i=1}^{i=I}\left(\pi_i^{\nu_\pi\mu_\pi-1}(1-\pi_i)^{\nu_\pi(1-\mu_\pi)-1}\right) \end{aligned}$$

Because this full conditional is not of usual form, a (random walk) Metropolis–Hastings update is implemented. A candidate $\mu_\pi^{(c)}$ is sampled from a uniform distribution whose support is centred on the current value of μ_π:

$$\mu_\pi^{(c)} \sim \text{Unif}\left(\max(\epsilon, \pi_i - \delta^{(\mu)}), \min(1-\epsilon, \pi_i - \delta^{(\mu)})\right)$$

($\epsilon = 10^{-4}$ in BayPass)

$\delta^{(\mu)}$ is adjusted during pilot runs to obtain an acceptance rate ranging between τ_{min} and τ_{max}. As a default, $\tau_{\text{min}} = 0.25$ and $\tau_{\text{max}} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal may not be symmetric, the candidate value $\mu_\pi^{(c)}$ is accepted with probability $min(1, \psi^{(\mu)})$ according to the Metropolis–Hastings rule where :

$$\psi^{(\mu)} = \frac{f\left(\mu_\pi^{(c)} \mid .\right) q(\mu_\pi \mid \mu_\pi^{(c)})}{f(\mu_\pi \mid .)\, q(\mu_\pi^{(c)} \mid \mu_\pi)}$$

with:

- $q(\mu_\pi^{(c)} \mid \mu_\pi) = \frac{1}{\min\left(1-\epsilon;\mu_\pi-\delta^{(\mu)}\right)-\max\left(\epsilon;\mu_\pi-\delta^{(\mu)}\right)}$
- $q(\mu_\pi \mid \mu_\pi^{(c)}) = \frac{1}{\min\left(1-\epsilon;\mu_\pi^{(c)}-\delta^{(\mu)}\right)-\max\left(\epsilon;\mu_\pi^{(c)}-\delta^{(\mu)}\right)}$

5.2 Update of ν_π:

The full conditional distribution of nu_π has the form:

$$\begin{aligned} f(\nu_\pi \mid .) &\propto f(\nu_\pi)\prod_{i=1}^{j=I} f(\pi_i \mid \mu_\pi, \nu_\pi) \\ &\propto e^{-\nu_\pi}\left(\frac{\Gamma(\nu_\pi)}{\Gamma(\nu_\pi\mu_\pi)\,\Gamma(\nu_\pi(1-\mu_\pi))}\right)^I \prod_{i=1}^{i=I}\left(\pi_i^{\nu_\pi\mu_\pi-1}(1-\pi_i)^{\nu_\pi(1-\mu_\pi)-1}\right) \end{aligned}$$

Because this full conditional is not of usual form, a (random walk) Metropolis–Hastings update is implemented. A candidate $\nu_\pi^{(c)}$ is sampled from a log–normal distribution is centred on the current value of ν_π:

$$\log\left(\nu_\pi^{(c)}\right) \sim \text{N}\left(\log\left(\nu_\pi^{(c)}\right); \delta^{(\mu)}\right)$$

$\delta^{(\nu)}$ is adjusted during pilot runs to obtain an acceptance rate ranging between τ_{min} and τ_{max}. As a default, $\tau_{\text{min}} = 0.25$ and $\tau_{\text{max}} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal is not symmetric, the candidate value $\nu_\pi^{(c)}$ is accepted with probability $min(1, \psi^{(\nu)})$ according to the Metropolis–Hastings rule where :

$$\psi^{(\mu)} = \frac{f\left(\nu_\pi^{(c)} \mid .\right) q(\nu_\pi \mid \nu_\pi^{(c)})}{f(\nu_\pi \mid .)\, q(\nu_\pi^{(c)} \mid \nu_\pi)}$$

where $\frac{q(\nu_\pi|\nu_\pi^{(c)})}{q(\nu_\pi^{(c)}|\nu_\pi)} = \frac{\nu_\pi^{(c)}}{\nu_\pi}$ from the definition of the log–normal distribution.

6 Update of the β_i's (AUX and STD models):

The parameters β_i are updated iteratively one locus at a time.

6.1 In the STD model or if $\delta_i = 1$ in the AUX model:

To simplify further notations, let $\ddot{\boldsymbol{\alpha}}_i = \Gamma^{-1}\left\{\frac{\alpha^\star_{ij}-\pi_i}{\sqrt{\pi_i(1-\pi_i)}}\right\}_{(1..J)}$ where Γ results from the Choleski decomposition of $\boldsymbol{\Omega} = \boldsymbol{\Lambda}^{-1}$ (i.e., $\boldsymbol{\Omega} = {}^t\boldsymbol{\Gamma}\boldsymbol{\Gamma}$). Note that with this transformation, $\ddot{\boldsymbol{\alpha}}_i \sim N_J\left(\beta_i\widetilde{\boldsymbol{\Phi}}, \mathbf{I}_J\right)$ where $\widetilde{\boldsymbol{\Phi}} = \left\{\phi_j\right\}_{(1..J)} = \Gamma^{-1}\left\{\frac{Z_j}{\sqrt{\pi_i(1-\pi_i)}}\right\}$.

The full conditional distribution of β_i has the form:

$$
\begin{aligned}
f(\nu_\pi \mid .) &\propto f(\beta_i)\, f(\ddot{\boldsymbol{\alpha}}_i \mid \beta_i) \\
&\propto \prod_{j=1}^{J} e^{-\frac{1}{2}(\ddot{\alpha}_{ij}-\beta_i\widetilde{\phi}_j)^2} \\
&\propto e^{\beta_i\left(\sum_{j=1}^{j=J}\widetilde{\phi}_j\ddot{\alpha}_{ij}-\frac{\beta_i}{2}\sum_{j=1}^{j=J}\widetilde{\phi}_j^{\,2}\right)}
\end{aligned}
$$

Because this full conditional is not of usual form, a (random walk) Metropolis–Hastings update is implemented. A candidate $\beta_i^{(c)}$ is sampled from a uniform distribution whose support is centred on the current value of β_i:

$$\beta_i^{(c)} \sim \text{Unif}\left(\max(\min_\beta, \beta_i - \delta_i^{(\beta)}), \min(\max_\beta, \beta_i - \delta_i^{(\beta)})\right)$$

The $\delta_i^{(\beta)}$'s' are adjusted for each β_i during pilot runs to obtain acceptance rates ranging between $\tau_{\min}$ and $\tau_{\max}$. As a default, $\tau_{\min} = 0.25$ and $\tau_{\max} = 0.4$ as usually recommended (Gilks *et al.*, 1996).

As the proposal may not be symmetric, the candidate value $\beta_i^{(c)}$ is accepted with probability $min(1, \psi_i^{(\beta)})$ according to the Metropolis–Hastings rule where :

$$\psi_i^{(\beta)} = \frac{f\left(\beta_i^{(c)} \mid .\right) q(\beta_i \mid \beta_i^{(c)})}{f(\beta_i \mid .)\, q(\beta_i^{(c)} \mid \beta_i)}$$

with:

- $q(\beta_i^{(c)} \mid \beta_i) = \frac{1}{\min\left(\max_\beta; \beta_i - \delta_i^{(\beta)}\right) - \max\left(\min_\beta; \beta_i - \delta_i^{(\beta)}\right)}$
- $q(\beta_i \mid \beta_i^{(c)}) = \frac{1}{\min\left(\max_\beta; \beta_i^{cdt} - \delta_i^{(\beta)}\right) - \max\left(\min_\beta; \beta_i^{cdt} - \delta_i^{(\beta)}\right)}$

6.2 If $\delta_i = 0$ in the AUX model

In this case, β_i is simply sampled from its prior distribution since:

$$\beta_i \mid \delta_i = 0, . \sim \text{Unif}(\min_\beta, \max_\beta)$$

7 Update of the δ_i's (AUX model)

The parameters δ_i are updated iteratively one locus at a time. Since these variables are binary auxiliary variables, the full conditional distribution is a Bernoulli distribution allowing a simple Gibbs update. Indeed:

$$
\begin{aligned}
\mathbb{P}(\delta_i \mid .) &\propto \mathbb{P}(\delta_i \mid P, \mathrm{b_{is}}, \boldsymbol{\delta_{-i}})\, f(\ddot{\boldsymbol{\alpha}}_i \mid \beta_i, \delta_i) \\
&\propto P^{\delta_i}(1-P)^{1-\delta_i} e^{\mathrm{b_{is}}\left(\mathbb{I}_{\delta_i=\delta_{i-1}}+\mathbb{I}_{\delta_i=\delta_{i+1}}\right)} \prod_{j=1}^{J} e^{-\frac{1}{2}(\ddot{\alpha}_{ij}-\delta_i\beta_i\widetilde{\phi}_j)^2}
\end{aligned}
$$

(see 6.1 for a definition of the definitions of $\ddot{\alpha}_{ij}$ and $\widetilde{\phi_j}$)

Hence:

$$\delta_i \mid . \sim \mathrm{Ber}\left(\frac{Pe^{\mathrm{b_{is}}\left(\mathbb{I}_{\delta_{i-1}=1}+\mathbb{I}_{\delta_{i+1}=1}\right)}\prod_{j=1}^{J}e^{-\frac{1}{2}(\ddot{\alpha}_{ij}-\beta_i\widetilde{\phi_j})^2}}{Pe^{\mathrm{b_{is}}\left(\mathbb{I}_{\delta_{i-1}=1}+\mathbb{I}_{\delta_{i+1}=1}\right)}\prod_{j=1}^{J}e^{-\frac{1}{2}(\ddot{\alpha}_{ij}-\beta_i\widetilde{\phi_j})^2}+(1-P)e^{\mathrm{b_{is}}\left(\mathbb{I}_{\delta_{i-1}=0}+\mathbb{I}_{\delta_{i+1}=0}\right)}\prod_{j=1}^{J}e^{-\frac{\ddot{\alpha}_{ij}^2}{2}}}\right)$$

8 Update of P (AUX model)

The full conditional distribution of P is a Beta distribution allowing a simple Gibbs update. Indeed:

$$\begin{aligned}
f(P \mid .) &\propto f(P)\,f(\boldsymbol{\delta} \mid P)\\
&\propto P^{a_P-1}(1-P)^{b_P-1}P^{\left[\sum_{i=1}^{I}\delta_i\right]}(1-P)^{\left[I-\sum_{i=1}^{I}\delta_i\right]}\\
&\propto P^{\left[a_P+\sum_{i=1}^{I}\delta_i-1\right]}(1-P)^{\left[b_P+I-\sum_{i=1}^{I}\delta_i-1\right]}
\end{aligned}$$

Hence:

$$P \mid . \sim Beta\left(a_P+\sum_{i=1}^{I}\delta_i; b_P+I-\sum_{i=1}^{I}\delta_i\right)$$

References

Coop, G., D. Witonsky, A. D. Rienzo, and J. K. Pritchard, 2010 Using environmental correlations to identify loci underlying local adaptation. Genetics 185: 1411–1423.

Gilks, W. R., S. Richardson, and D. J. Spiegelhalter, 1996 *Markov Chain Monte Carlo in Practice*. Chapman & Hall.

Kruschke, J., 2014 *Doing Bayesian Data Analysis, Second Edition: A Tutorial with R, JAGS, and Stan*. Academic Press, Amsterdam, 2 edition edition.

Yang, Z., 2005 Bayesian inference in molecular phylogenetics. In O. Gascuel, editor, *Mathematics of Evolution and Phylogeny*. Oxford University Press, 63–90.