

Tumor Growth

$$\mathbf{V}_{boundary} = \mathbf{v}_c \cdot \mathbf{n}$$

Evolution of tumor (\mathbf{n} denotes boundary normal direction)

$$\mathbf{v}_c = -\mu \nabla P + \chi_E \nabla E$$

Non-dimensional tumor velocity \mathbf{v}_c (**Eq. 1**; Ref. 35, Eq. 7)

$$\nabla \cdot \mathbf{v}_c = \lambda_p$$

Growth of tumor (**Eq. 2**; Ref. 35, Eq. 8)

$$\lambda_p = \lambda_M \sigma - \lambda_A \text{ in } \Omega_P \text{ and } \lambda_p = \lambda_N \text{ in } \Omega_N$$

Proliferation rate λ_p (**Eq. 12**; Ref. 16, Eq. 13)

$$0 = \nabla \cdot (D_\sigma \nabla \sigma) + \lambda_{ev}^\sigma(\mathbf{x}, t, \mathbf{1}_{vessel}, p, \sigma, h) - \lambda^\sigma(\sigma) \sigma$$

Transport of oxygen σ (**Eq. 4**; Ref. 34, Eq. 1)

$$\lambda_{ev}^\sigma = \bar{\lambda}_{ev}^\sigma \mathbf{1}_{vessel}(\mathbf{x}, t) \left(\frac{h}{H_D} - \bar{h}_{min} \right)^+ \left(1 - k_{p_i} \frac{p_i}{p_e} \right) (1 - \sigma)$$

Oxygen extravasation rate λ_{ev}^σ (**Eq. 5**; Ref. 34, Eq. 3)

Angiogenesis

$$\frac{\partial n}{\partial t} = \nabla \cdot (D \nabla n) - \nabla \cdot (\chi_{sprout}^T(T) n \nabla T) - \nabla \cdot (\chi_{sprout}^E n \nabla E)$$

Endothelial cell density n (**Eq. 3**; Ref. 35, Eq. 21)
(discrete and stochastic implementation)

$$0 = \nabla \cdot (D_T \nabla T) + \bar{\lambda}_{prod}^T (1 - T) \mathbf{1}_{\Omega_H} - \bar{\lambda}_{decay}^T T - \bar{\lambda}_{binding}^T \mathbf{1}_{sprout\ tips}$$

TAF concentration T (Ref. 35, Eq. 20)

$$\frac{\partial E}{\partial t} = -\bar{\lambda}_{degradation}^E E M + \bar{\lambda}_{prod}^E (1 - E) \mathbf{1}_{\Omega_V} + \bar{\lambda}_{sprout\ prod}^E \mathbf{1}_{sprout\ tips}$$

ECM density E (Ref. 35, Eq. 18)

$$\frac{\partial M}{\partial t} = \nabla \cdot (D_M \nabla M) + \bar{\lambda}_{prod}^M (1 - M) \mathbf{1}_{\Omega_V} - \bar{\lambda}_{decay}^M M + \bar{\lambda}_{sprout\ prod}^M \mathbf{1}_{sprout\ tips}$$

MDE concentration M (Ref. 35, Eq. 17)

Blood Flow

$$\sum_q Q_{qp} = O_V$$

Flow conservation for each vascular node (Ref. 34, Eq. 22)

$$O_V = K_{V_f} S_V (P_c - P_f)$$

Fluid extravasation at a node (Ref. 34, Eq. 23)

$$\Delta R = (S_{wss} + S_p + S_m - S_s) R \Delta t$$

Vessel radius R adaptation (Ref. 34, Eq. 25)

Nanoparticles

$$N(d, R_u, S_u) = S_u \alpha d^{\delta_1} \exp(-\beta(1 + \gamma d^{\delta_2}) S r t_u)$$

Particle number N attached in a vessel segment (**Eq. 7**; Ref. 13, Eq. 14)

$$S r t_u = 4 Q_u / \pi R_u^3$$

Shear rate $S r t$ in vessel segment u (Ref. 13)

$$(1 + \frac{\Delta t}{V_p} \sum_u Q_u) C_p^{t+\Delta t} = C_p^t + \frac{\Delta t}{V_p} (\sum_u C_u^t Q_u (1 - N(d, R_u, S_u)))$$

Particle concentration C_p in blood (**Eq. 8**; Ref. 13, Eq. 15)

$$C_{pS}^{t+\Delta t} = C_{pS}^t + \frac{\Delta t}{S_p} \sum_u C_u^t Q_u N(d, R_u, S_u)$$

Particle concentration C_{pS} on vessel surface (**Eq. 9**; Ref. 13, Eq. 16)

Drug

$$\frac{\partial G}{\partial t} = \nabla \cdot (D_G \nabla G) + \lambda_{release}^G(t, C_{pS}, d) - \bar{\lambda}_{decay}^G G$$

Drug concentration G (**Eq. 10**)

$$\lambda_{release}^G = k C_{pS} \sqrt{d}$$

Drug release rate $\lambda_{release}^G$ (**Eq. 11**)

$$\lambda_p = \lambda_M \sigma (1 - \bar{\lambda}_{effect}^G G) - \lambda_A$$

Effect of drug on tumor net proliferation λ_p (**Eq. 12**; Ref. 16, Eq. 13)