

## Appendix 1

Given the energetic cost per unit of structure  $[E_G]$  and the energy flux to growth  $\dot{p}_G$ , the change in structural volume is

$$\frac{dV}{dt} = \frac{\dot{p}_G}{[E_G]} \quad (2.1)$$

The change in reserve energy  $E$  can be expressed as the energy assimilation rate, minus the maintenance rate, minus the energy flux to growth  $\dot{p}_G$ :

$$\frac{dE}{dt} = f\{\dot{p}_{Am}\}V^{\frac{2}{3}} - [\dot{p}_M]V - \dot{p}_G \quad (2.2)$$

Using the chain rule for differentiation, the change in the reserve density  $[E]$  is

$$\frac{d[E]}{dt} = \frac{dE}{dt}V^{-1} - \frac{dV}{dt}EV^{-2} \quad (2.3)$$

Substituting equation (2.2) and (2.3) into (2.1) gives

$$\frac{dV}{dt} = V \frac{f\{\dot{p}_{Am}\}/V^{\frac{1}{3}} - [\dot{p}_M] - \frac{d[E]}{dt}}{[E_G] + [E]} \quad (2.4)$$

For constant food levels the ratio of reserve to structure is constant so (2.4) simplifies to

$$\frac{dV}{dt} = \frac{f\{\dot{p}_{Am}\}V^{\frac{2}{3}} - [\dot{p}_M]V}{[E_G] + [E]^*} \quad (2.5)$$

where the ratio of reserve to structure  $[E]^*$  is now constant. This equation is identical to the von Bertalanffy growth curve but parameters can now be interpreted in terms of their underlying processes.

To extend the formulation to non-steady states where food level may vary, we use a result from Kooijman (2010) that restricts how the reserve density may vary:

$$\frac{d[E]}{dt} = \frac{f\{\dot{p}_{Am}\} - [E]\dot{v}}{V^{\frac{1}{3}}} \quad (2.6)$$

The simple result of equation (2.6) relies on no further assumptions but requires an involved derivation that is not covered here (see [1,2]). The interpretation is that in the absence of assimilation the reserve density decreases according to first-order dynamics. The introduced parameter  $\dot{v}$  has dimensions length per time and is called energy conductance.

We may now solve reserve and structure at non-steady states at which the food level  $f$  and reserve density is not constant. Rearranging the previous equations give:

$$\frac{dE}{dt} = f\{\dot{p}_{Am}\}V^{\frac{2}{3}} - [E] \frac{[\dot{p}_M]V + [E_G]\dot{v}V^{\frac{2}{3}}}{[E_G] + [E]} \quad (2.7)$$

$$\frac{dV}{dt} = \frac{[E]\dot{v}V^{2/3} - [\dot{p}_M]V}{[E_G] + [E]} \quad (2.8)$$

To accommodate insects, which do not follow von Bertalanffy growth the assumption of constant specific assimilation is relaxed. The parameter for specific assimilation is scaled by the dimensionless term  $\left(\frac{V}{V_b}\right)^\alpha$  where  $\alpha$  is a free parameter and  $V_b$  is structure at birth. This results in:

$$\frac{dE}{dt} = f\{\dot{p}_{Am}\}V^{(2/3+\alpha)}/V_b^\alpha - [E]\frac{[\dot{p}_M]V + [E_G]\dot{v}V^{2/3}}{[E_G] + [E]} \quad (2.9)$$

This does not change the dependence of structure on reserve, only the rate of reserve accumulation. Structure (equation 2.8) and reserve (equation 2.9), which are in units of volume and energy respectively, can be converted to mass ( $m$ ) using the appropriate conversion coefficients.

$$\frac{dm}{dt} = d_V \frac{dV}{dt} + e_E^{-1} \frac{dE}{dt} \quad (2.10)$$

where  $d_V$  is the dry mass density of structure (dry mass/volume),  $e_E$  is the energy density of reserve (energy/dry mass).

## References

1. Maino, J. L., Kearney, M. R., Nisbet, R. M. & Kooijman, S. A. L. M. 2014 Reconciling theories for metabolic scaling. *J. Anim. Ecol.* **83**, 20–9. (doi:10.1111/1365-2656.12085)
2. Kooijman, S. A. L. M. 2010 *Dynamic energy budget theory for metabolic organisation*. Cambridge: Cambridge University Press.