Web Appendix

The Generalized Roy Model and the Cost-Benefit Analysis of Social Programs*

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February 14, 2014

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A. Additional Properties of the Model

In Section A.1, we establish sufficient conditions for those who select into treatment to have the highest benefit and the lowest cost of treatment, and show testable restrictions that follow from those conditions. In Section A.2, we establish sufficient conditions on our model for measurable separability of X and $P(X, Z)$ as invoked in Assumption (A-5) and used in Theorem 2 in the text. The proof of all theorems are contained in Appendix B.

A.1. Properties of Marginal Benefits and Marginal Costs.

The marginal surplus parameter is biggest for those who most want to participate in the program. Using Equation (3.3), we thus have that the average surplus among the treated is higher than the unconditional average surplus of treatment. In other words, given our maintained assumptions, $S^{MTE}(x, u_S)$ is monotonically decreasing in u_S, and $S^{TT}(x) > S^{ATE}(x)$. While it might seem intuitive that that those who most want to participate in the program would have the highest benefit and the lowest cost, neither statement need be true without further restrictions. The following Theorem provides sufficient conditions under which these intuitive properties for benefits and costs will hold.

Theorem 3. *Assume that Equations (2.1)–(2.4) and our Assumptions (A-1)–(A-4) hold.*

1. *Suppose that $U_C \perp\!\!\!\perp U_1 - U_0$. Then $C^{TT}(z) \leq C^{ATE}(z)$, $B^{TT}(x) \geq B^{ATE}(x)$.*

2. *Suppose that $U_C \perp\!\!\!\perp U_1 - U_0$, and that U_C and $U_1 - U_0$ have log concave densities. Then $C^{MTE}(z, u_S)$ is monotonically increasing in u_S and $B^{MTE}(x, u_S)$ is monotonically decreasing in u_S.*

The proof of Theorem 3 is contained in Appendix B. Results of this theorem are intuitive. If the unobservables related to the cost and benefit are independent, then the average benefit among those who select into treatment is larger than the unconditional average benefit. At the same time, the average cost among those who select into treatment is lower than the unconditional average cost. In other words, under independence of the unobservables related to benefits and costs, it is the high benefit and low cost individuals who select into treatment in the generalized Roy model. Part (2) of the theorem state that, under a regularity condition, the expected gain is decreasing while the expected cost is increasing in U_S. Note that the normal density as well as many other standard densities are log concave.[1]

We now derive testable implications of $E(Y|X = x, P = p)$ as a function of p that result from additional restrictions including those developed in Theorem 3.

Theorem 4. *Assume that Equations (2.1)–(2.4) and our Assumptions (A-1)–(A-4) hold.*

1. *Suppose that $U_1 - U_0$ is degenerate. Then $E(Y|X = x, P = p)$ is linear in p.*

2. *Suppose $U_1 - U_0 \perp\!\!\!\perp U_C$. For a fixed x, consider a line $a(x) + b(x)p$, where $a(x) = E(Y|X = x, P(X,Z) = 0)$ and $b(x) = E(Y|X = x, P(X,Z) = 1) - E(Y|X = x, P(X,Z) = 0)$. Then $E(Y|X = x, P(X,Z) = p) \geq a(x) + b(x)p$ for all $p \in Supp(P|X = x)$.*

3. *Suppose $U_1 - U_0 \perp\!\!\!\perp U_C$, and suppose $U_1 - U_0$ and U_C have log concave densities. Then $E(Y|X = x, P(X,Z) = p)$ is a concave function of p.*

The proof of Theorem 4 is contained in Appendix B.

Remark A.1. The conditions of Theorem 3 provide sufficient conditions for the stated results to hold. The conditions are not necessary. For example, suppose that (U_1, U_0, U_C) is distributed joint normal. Then it can easily be shown that $C^{MTE}(z, u_S)$ is monotonically increasing in u_S if and only if $\text{Var}(U_C) > \text{Cov}(U_C, U_1 - U_0)$, and $B^{MTE}(x, u_S)$ is monotonically decreasing in u_S if and only if $\text{Var}(U_1 - U_0) > \text{Cov}(U_C, U_1 - U_0)$. Thus, if (U_1, U_0, U_C) is distributed joint normal, than $C^{MTE}(z, u_S)$ can be monotonically increasing in u_S and $B^{MTE}(x, u_S)$ monotonically decreasing in u_S even if unobserved benefits and costs are correlated, as long as they are not correlated too strongly. □

[1] Heckman and Honoré (1990) exploit the restriction of log-concave density functions for the disturbance terms in a Roy model with zero costs. See Bagnoli and Bergstrom (2005) for a review of log concave densities and economic applications.

A.2. Measurable Separability of X and $P(X, Z)$

We now show sufficient conditions for X and $P(X,Z) \equiv F_V(\mu_S(X,Z))$ to be measurably separated as invoked in Assumption (A-5) in the text. Consider the following assumptions:

(B-1) *X can be partitioned as $(X^{(d)}, X^{(c)})'$, where $X^{(d)}$ is a discrete random vector and $X^{(c)}$ is a $K-$dimensional continuous random vector. Define $\mathcal{X}^{(d)}$ to be the support of the distribution of $X^{(d)}$. For all $x^{(d)} \in \mathcal{X}^{(d)}$, the distribution of $(X^{(c)}, \mu_S(X,Z))$ conditional on $X^{(d)} = x^{(d)}$ is absolutely continuous with respect to Lebesgue measure on $\Re^{K+1}$, with a density whose support is connected and any point in the interior of its support has a neighborhood such that the density is strictly positive within it.*

(B-2) *Let $S_{x^{(d)}}$ denote the support of the distribution of $\mu_S(X,Z)$ conditional on $X^{(d)} = x^{(d)}$. Then, for any $\tilde{x}^{(d)}, \bar{x}^{(d)} \in \mathcal{X}^{(d)}$, there exists a sequence $x_1^{(d)}, ..., x_J^{(d)}$ in $\mathcal{X}^{(d)}$ with $x_1^{(d)} \equiv \tilde{x}^{(d)}$, $x_J^{(d)} \equiv \bar{x}^{(d)}$, such that $S_{x_j^{(d)}} \cap S_{x_{j+1}^{(d)}}$ contains an open interval for $j = 1, ..., J-1$.*

(B-3) *V has a continuous, everywhere positive density with respect to Lebesgue measure.*

Assumption (B-1) strengthens Assumption (A-2), while Assumption (B-3) strengthens Assumption (A-3). The requirement that X and Z both contain at least one continuous element are necessary conditions for Assumption (A-1) to hold. Assumption (B-2) is a support condition, which allows the support of the distribution of $\mu_S(X,Z)$ conditional on the discrete covariates $X^{(d)}$ to depend on $X^{(d)}$, while requiring overlap in the supports of the conditional distributions. We now show that these three assumptions are sufficient for X and $P(X,Z) \equiv F_V(\mu_S(X,Z))$ to be measurably separated. Our proof relies upon Theorem 2 of Florens et al. (2008). In particular, we show that, under Assumptions (B-1) and (B-3), the conditions of Florens et al. (2008) are satisfied for X and $P(X,Z)$ conditional on the discrete covariates, so that X and $P(X,Z)$ are measurably separated conditional on the discrete covariates. Assumption (B-2) allows us to piece together each conditional statement to conclude that X and $P(X,Z)$ are measurably separated.

Theorem 5. *Suppose that Assumptions (B-1), (B-2), and (B-3) hold. Then X and $P(X, Z)$ are measurably separated.*

The proof of the theorem is contained in Appendix B.

B. Proofs

Theorem 2

Proof. Consider part (i) of the theorem. Let $\mu_{10}(\cdot) = \mu_1(\cdot) - \mu_0(\cdot)$, and let $\Upsilon(p) = E(U_1 - U_0 \mid U_S = p)$. From our previous analysis, we have

$$\frac{\partial}{\partial p} E(Y|X = x, P = p) = \mu_{10}(x) + \Upsilon(p) \quad \text{a.e. } (x, p).$$

Let (μ_{10}^*, Υ^*) denote candidate functions that also satisfy Equation (B). We then have $\mu_{10}^*(x) - \mu_{10}(x) = \Upsilon(p) - \Upsilon^*(p)$ for a.e. (x, p). By the rank condition (A-5), we have that, for some constant C: $\mu_{10}^*(x) - \mu_{10}(x) = C$ for a.e. x, and $\Upsilon^*(p) - \Upsilon(p) = -C$ for a.e. p. We thus have that $\mu_{10}^*(x) + \Upsilon^*(p) = \mu_{10}(x) + \Upsilon(p) = B^{MTE}(x, p)$ for a.e. x and a.e. p. We have thus established identification of $B^{MTE}(x, p)$ for $(x, p) \in \text{Supp}(X) \times \text{Supp}(P)$. The same argument *mutatis mutandis* shows identification of $C^{MTE}(z, u_S)$ for $(z, u_S) \in \text{Supp}(Z) \times \text{Supp}(P)$, and we thus have identification of $S^{MTE}(x, z, u_S)$ for $(x, z, u_S) \in \text{Supp}(X) \times \text{Supp}(Z) \times \text{Supp}(P)$. Parts (ii) and (iii) of the theorem now follow using part (i) of the theorem and the representation of the ATE and TT parameters as integrals of the MTE parameters. □

Theorem 3

Proof. Assertion (1) was proved in the discussion of the theorem in Appendix A. For Assertion (2), first consider the cost parameters. $C^{ATE}(z) - C^{TT}(z) = E(U_C) - E(U_C|Z = z, D = 1)$, and $E(U_C|Z = z, D = 1) = \int E(U_C|Z = z, X = x, U_S \leq P(x, z)) dF_{X|Z,D}(x|z, 1) = \int E(U_C|U_S \leq P(x, z)) dF_{X|Z,D}(x|z, 1)$ using $(X, Z) \perp\!\!\!\perp (U_C, U_S)$. Thus, using that $U_S = F_V(V)$, it will be sufficient to show that $E(U_C|V \leq t) \leq E(U_C)$ for all t, and thus sufficient to show that $\Pr[U_C \leq s | U_C - (U_1 - U_0) \leq t] \geq \Pr[U_C \leq s]$ for all s. Using Bayes' rule, this is equivalent to $\Pr[U_C - (U_1 - U_0) \leq t | U_C \leq s] \geq \Pr[U_C - (U_1 - U_0) \leq t]$, and this last assertion can now easily be shown using $U_C \perp\!\!\!\perp (U_1 - U_0)$. We can thus conclude that $C^{ATE}(z) - C^{TT}(z) \geq 0$. The same argument *mutatis mutandis* shows that $B^{ATE}(x) - B^{TT}(x) \leq 0$. Now consider Assertion (3). The densities of U_C and $U_1 - U_0$ being log concave is equivalent to their densities being Polya frequency functions of order 2 (PF2) (Karlin, 1968). Using that $U_1 - U_0 \perp\!\!\!\perp U_C$, one can now easily verify that $(U_C, U_C - (U_1 - U_0))$ and $(-(U_1 - U_0), U_C - (U_1 - U_0))$ have joint densities that are totally positive of order 2 (TP2). By Joe (1997) (Theorems 2.2, 2.3), $(U_C, U_C - (U_1 - U_0))$ and $(-(U_1 - U_0), U_C - (U_1 - U_0))$ having TP2 densities implies that U_C and $-(U_1 - U_0)$ are stochastically increasing in $U_C - (U_1 - U_0)$ and thus stochastically increasing in U_S using that U_S is a strictly monotonic function of $U_C - (U_1 - U_0)$. Thus $E(U_C|U_S = u_S)$ is increasing in u_S while $E(U_1 - U_0|U_S = u_S)$ is decreasing in u_S, establishing the assertion. □

Theorem 4

Proof. Assertion (1) follows from Equation (4.1) and $B^{MTE}(x, u_S) = \mu_1(x) - \mu_0(x)$ if $U_1 - U_0$ is degenerate. Assertion (2) follows from $E(Y|X = x, P(X,Z) = 1) - E(Y|X = x, P(X,Z) = 0) = B^{ATE}(x)$, $[E(Y|X = x, P(X,Z) = p) - E(Y|X = x, P(X,Z) = 0)]/p = E(B|X = x, P(X,Z) = p, D = 1)$, and that $B^{ATE}(x) \leq E(B|X = x, P(X,Z) = p, D = 1)$ by the arguments used to prove Assertion (2) of Theorem 3. Assertion (3) follows from Equation (4.1) and Assertion (3) of Theorem 3. □

Theorem 5

Proof. Fix an arbitrary $x^{(d)} \in \mathcal{X}^{(d)}$. Using that $P(X,Z) = F_V(\mu_S(X,Z))$, Assumptions (B-1) and (B-3) imply that the distribution of $(X, P(X,Z))$ conditional on $X^{(d)} = x^{(d)}$ is absolutely continuous with respect to Lebesgue measure and that any point in the the interior of its support has a neighborhood such that the density is strictly positive within it. The mapping from $(X, \mu_S(X,Z))$ to $(X, P(X,Z))$ is continuous, and thus, using Assumption (B-1), the interior of the support of the conditional distribution of $(X, P(X,Z))$ is connected. Since the interior of the conditional distribution of $(X, P(X,Z))$ is a connected open subset of a Euclidean space, it is path connected. It now follows from Theorem 2 of Florens et al. (2008) that X and $P(X,Z)$ are measurably separated conditional on $X^{(d)}$.

Suppose $h(X) = g(P(X,Z))$ a.s.. Then $h(X) = g(P(X,Z))$ a.s. conditional on $X^{(d)}$. Since we have shown that X and $P(X,Z)$ are measurably separated conditional on $X^{(d)}$, we have that $g(P(X,Z))$ a.s. equals a constant conditional on $X^{(d)}$. Let $\mathcal{P}_{x^{(d)}}$ denote the support of the distribution of $P(X,Z)$ conditional on $X^{(d)} = x^{(d)}$. We have that g is (a.s.) constant on $\mathcal{P}_{x^{(d)}}$ for all $x^{(d)} \in \mathcal{X}^{(d)}$. Consider any $\tilde{x}^{(d)}, \bar{x}^{(d)} \in \mathcal{X}^{(d)}$. Using Assumption (B-2), and that $P(X,Z) = F_V(\mu_S(X,Z))$ is a strictly increasing function of $\mu_S(X,Z)$ by Assumption (B-3), there exists a sequence $x_1^{(d)}, ..., x_J^{(d)}$ in $\mathcal{X}$ with $x_1^{(d)} \equiv \tilde{x}^{(d)}$, $x_J^{(d)} \equiv \bar{x}^{(d)}$, such that $\mathcal{P}_{x_j^{(d)}} \cap \mathcal{P}_{x_{j+1}^{(d)}}$ contains an open interval for $j = 1, ..., J-1$. We have that g is (a.s.) constant on $\mathcal{P}_{x_1^{(d)}}$, is (a.s.) constant on $\mathcal{P}_{x_2^{(d)}}$, and, since $\mathcal{P}_{x_j^{(d)}} \cap \mathcal{P}_{x_{j+1}^{(d)}}$ is non-negligible, g is (a.s.) constant on $\mathcal{P}_{x_1^{(d)}} \cup \mathcal{P}_{x_2^{(d)}}$. Iterating in this way along the sequence, it follows that g is (a.s.) constant on $\mathcal{P}_{\tilde{x}^{(d)}} \cup \mathcal{P}_{\bar{x}^{(d)}}$. Since this holds for arbitrary $\tilde{x}^{(d)}, \bar{x}^{(d)} \in \mathcal{X}^{(d)}$, it follows that g is (a.s.) constant on $\mathcal{P}$, and therefore g and thus h are a.s. equal to a constant. $\square$

C. Data Description

Our sample consists of white males from the National Longitudinal Survey of Youth of 1979 (NLSY79).[2] We define participation in college as having attended some college or having completed more than 12 grades in school. The wage variable that is used is an average of deflated (to 1983) non-missing hourly wages reported in 1989, 1990, 1991, 1992 and 1993. We delete all wage observations that are below 1 or above 100. Experience is actual work experience in weeks (we divide it by 52 to express it as a fraction of a year) accumulated from 1979 to 1991 (annual weeks worked are imputed to be zero if they are missing in any given year). The remaining variables that we include in the X and Z vectors are mother's years of schooling, number of siblings, urban residence at 14, schooling corrected AFQT, dummies indicating the year of birth, the presence of a four-year college in the county of residence at age 14 (Kling, 2001)[3], average tuition in public four year colleges in the county of residence at age 17 (deflated to 1993), and local average wages in the county of residence at 17. Permanent local wages are computed by location of residence at 17 (county level), by averaging values of (deflated) local labor market variables between 1973 and 2000. County wages correspond to the average wage per job in the county constructed using data from the Bureau of Economic Analysis, deflated to 2000. Annual records on tuition, enrollment, and location of all public four year colleges in the United States were constructed from the Department of Education's annual Higher Education General Information Survey and Integrated Postsecondary Education Data System "Institutional Characteristics" surveys. By matching location with county of residence, we determined the presence of four-year colleges. Tuition measures are taken as enrollment weighted averages of all public four-year colleges in a person's county of residence (if available) or at the state level if no college is available. County and state of residence at 17 are not available for everyone in the NLSY, but only for the cohorts born in 1962, 1963, and 1964 (age 17 in 1979, 1980 and 1981). However, county and state of residence at age 14 is available for most respondents. Therefore, we impute location at 17

[2]For a description of the NLSY79, see Bureau of Labor Statistics (2001).

[3]The distance variable we use is the one used in Kling (2001), available at the Journal of Business and Economics Statistics website.

to be equal to location at 14 for cohorts born between 1957 and 1962 unless location at 14 is missing, in which case we use location in 1979 for the imputation. Many individuals report having obtained a bachelors degree or more and, at the same time, having attended only 15 years of schooling (or less). We recode years of schooling for these individuals to be 16. This variable is only used to annualize the returns to schooling. We divide the returns to college by 4, which is the average difference in years of schooling between individuals in each schooling group. The NLSY79 has an oversample of poor whites which we exclude from this analysis. We also exclude the military sample. To remove the effect of schooling on AFQT we implement the procedure of Hansen et al. (2004).

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