Online Appendix for the following November *JCMG* article

TITLE: Characterization and Quantification of Vortex Flow in the Human Left Ventricle by Contrast Echocardiography Using Vector Particle Image Velocimetry

AUTHORS: Geu-Ru Hong, MD, PHD, Gianni Pedrizzetti, PHD, Giovanni Tonti, MD, Peng Li, MD, PHD, Zhao Wei, MD, PHD, Jin Kyung Kim, MD, PHD, Abinav Baweja, Shizhen Liu, MD, PHD, Namsik Chung, MD, PHD, Helene Houle, RDCS, Jagat Narula, MD, PHD, FACC, Mani A. Vannan, MBBS, FACC

APPENDIX

.

Quantification of Vortex Flow From Particle Image Velocimetry

The flow structure, specifically the vorticity, was quantified. At every point (e.g., at a point with coordinates *x and y*), the vorticity $\omega\ (x, y, t)$ was a periodic function of time that can be expressed in Fourier series as:

$$\omega(x,y,t)=\omega_0(x,y)+\omega_1(x,y)\cos\left(\tfrac{2\pi}{T}t+\varphi_1(x,y)\right)+\sum_{k=2,N}\omega_k(x,y)\cos\left(\tfrac{2k\pi}{T}t+\varphi_k(x,y)\right)$$

where $\omega_0\ (x, y)$ is the average flow field in 1 heartbeat, so-called "steady-streaming" component of the flow; $\omega\square(x, y)$ is the fundamental harmonic (sinusoidal of period T) that represents the main pulsatile contribution to the flow, and $\omega\square(x, y)$ is the corresponding phase that is a measure of the synchronicity of such a pulsatile contribution. The further terms with subscript k are the higher harmonics that correspond to the additional unsteady contributions. The same Fourier time-decomposition is employed for the velocities. Quantification of vortex position and size were performed.

Thus, physiological coordinates *h* and *s* (the longitudinal *h*, in the direction of base-apex, and the transverse *s*, normal to *h*) are employed in place of the image coordinates *(x, y)*. The coordinates *h* and *s* are function of *(x, y)*; they are not a function of time because they are used to analyze time-averaged field, the zeroth and first vorticity harmonics. The origin is placed on the center of the mitral plane (between the first and last points in the border), with the *h* pointing to the apex. The integrals taken on the vortex were simply evaluated over the points where vorticity was of the same sign of the vortex (positive, counterclockwise, in A4C projection; negative in APLX). The 2-D integration was also performed by summation for each pixel content (times the area of a pixel); this is an apparently crude formula, but resolution is high (pixel size is less that 0.3 mm), and the accuracy is not comparable with the accuracy of available image data.

Then, the total vortex vorticity is computed as:

$$\Omega_0 = \int_{vortex} \omega_0(x,y)dxdy;$$

where the integral is extended over the region of the LV, in which vorticity is of the same sign of the vortex (otherwise, the total vorticity would be zero, based on the law of conservation), positive (counterclockwise) in A4C projection, and negative in A3C. Designating H_{LV} the LV long axis base-apex length, the following dimensionless parameters of average vortex geometry were computed:

$$VD = \frac{1}{\Omega_0} \int_{vortex} \omega_0(x,y)h(x,y)dxdy;$$

$$VT = \frac{1}{\Omega_0} \int_{Vvortex} \omega_0(x, y) s(x, y) dxdy;$$

$$VL = \frac{1}{H_{LV}} \sqrt{\frac{1}{\Omega_0} \int_{vortex} \omega_0(x, y)(h - VD)^2 dxdy};$$

$$VW = \frac{1}{H_{LV}} \sqrt{\frac{1}{\Omega_0} \int_{vortex} \omega_0(x, y)(s - VT)^2 dxdy};$$

where *VD* and *VT* are the vortex depth and transversal displacement, respectively; *VL* and *VW* are the vortex length and width, respectively, all of them made relative to LV length. These represent the simplest possible measures of a vorticity distribution. *VD* and *VT* are the first order moments (average position, the barycenter), and *VL* and *VW* are the second order moments (dispersion about the center, the standard deviation). From these, a vortex sphericity index *SI = VL/VW* is computed.

The relative strength (*RS*) of pulsatile contribution with respect to the time-average flow is a measure of flow “vitality”, a global measure generated by:

$$RS = \frac{1}{\Omega_0} \int_{vortex} \omega_1(x, y) dxdy;$$

where now the integral is taken along the entire LV because the pulsatility is not restricted to be on the vortex (and is a positive quantity; therefore, there are no cancellations).

$$VRS = \frac{1}{\Omega_0} \int_{vortex} \omega_1(x, y) dxdy;$$

VRS is the strength of the vortex pulsatile vorticity (*VRS*, the integral is taken on the vortex only instead of that on the entire LV) relative to the same steady streaming vorticity inside the vortex.

$$VPC = \frac{A_{vortex}}{{\Omega_0}^2} \int_{vortex} \omega_0(x,y)\omega_1(x,y)dxdy;$$

VPC is the correlation between steady and pulsatile vorticity (*VPC*) in the vortex, normalized with the vortex strength and area to make a dimensionless parameter. It is larger when pulsatility vorticity is located where the steady vorticity is also present. It represents a measure of pulsatility weighted with the vortex strength. The larger it is, the more the vortex is pulsatile. All parameters were computed dimensionally then made dimensionless afterward. Formulae have been contracted for sake of space, and this incongruence has been corrected.