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Online Appendix A

For the first simulation study, we examined absolute bias, relative bias, and standardized bias within each design cell for seven parameters: the mean of *X*, the mean of *Y*, variances of *X* and *Y*, covariance between *X* and *Y*, correlation between *X* and *Y*, and regression coefficient. Standardized bias is reported in the manuscript; absolute bias and relative bias are reported here as supplemental material. Bias refers to the difference between the average parameter estimate across the 1000 replications within a given design cell and the corresponding population parameter. Because the categorization procedure makes it difficult to derive the population values, we generated 1000 complete data sets within each design cell, and we used the average parameter estimates from these data sets as the population parameters. Absolute bias refers to the absolute value of bias. To compute relative bias, we divided raw bias (i.e., the average parameter estimate minus the population parameter) by the population parameter. Consistent with previous research, we considered values less than 5% as negligible bias, values between 5% and 10% as moderate bias, and values greater than 10% as substantial bias (e.g., Flora & Curran, 2004; Kaplan, 1989; Curran, West, & Finch, 1996; Rhemtulla, Brosseau-Liard, & Savalei, 2012).

Values of absolute and relative bias for conditions corresponding to a 25% item-level missing data rate are reported in Tables A1 to A3; these conditions match those found in Tables 1 to 3 of the manuscript. As shown in Table A1, absolute bias ranged from 0 to 0.0363 (M = 0.0075) and relative bias ranged from -0.0189 to 0.0292 (i.e., the average parameter estimate differed from the population parameter by up to 2.92%) when the item means and inter-item correlations were uniform. As shown in Table A2, absolute bias ranged from 0 to 0.0410 (*M* = 0.0151) and relative bias ranged from -0.1032 to 0.0018 (i.e., the average parameter estimate differed from the population parameter by up to 10.32%) when the item means were uniform but the inter-item correlations varied. As shown in Table A3, absolute bias ranged from 0.0007 to 0.1686 (*M* = 0.0517) and relative bias ranged from -0.0331 to 0.1949 (i.e., the average parameter estimate differed from the population parameter by up to 19.49%) when the inter-item correlations were uniform but the item means varied. Consistent with the results reported in the manuscript, proration resulted in negligible bias when the item means and inter-item correlations were uniform but resulted in non-negligible bias when either the item means or inter-item correlations varied.

Table A1

Simulation Study 1, Absolute Bias and Relative Bias from Proration – Uniform Item Means, Uniform Inter-Item Correlations

Parameter	Items Per Scale	Sample Size	MCAR Mechanism		MAR Mechanism Due to External Variable		MAR Mechanism Due to Complete Items on the Scale	
			Absolute Bias	Relative Bias	Absolute Bias	Relative Bias	Absolute Bias	Relative Bias
Mean of *X*	8	200	0.0003	-0.0001	0.0006	-0.0001	0.0126	0.0031
		500	0.0000	0.0000	0.0001	0.0000	0.0129	0.0032
	16	200	0.0006	0.0002	0.0002	0.0001	0.0067	0.0017
		500	0.0001	0.0000	0.0000	0.0000	0.0062	0.0015
Mean of *Y*	8	200	0.0002	0.0000	0.0001	0.0000	0.0129	0.0032
		500	0.0001	0.0000	0.0000	0.0000	0.0130	0.0032
	16	200	0.0002	0.0000	0.0001	0.0000	0.0063	0.0016
		500	0.0000	0.0000	0.0001	0.0000	0.0059	0.0015
Variance of *X*	8	200	0.0197	0.0159	0.0216	0.0174	0.0350	0.0283
		500	0.0196	0.0158	0.0218	0.0175	0.0351	0.0282
	16	200	0.0096	0.0082	0.0108	0.0092	0.0174	0.0148
		500	0.0095	0.0080	0.0107	0.0090	0.0170	0.0144
Variance of *Y*	8	200	0.0199	0.0161	0.0217	0.0175	0.0356	0.0287
		500	0.0199	0.0160	0.0224	0.0180	0.0363	0.0292
	16	200	0.0088	0.0075	0.0098	0.0083	0.0163	0.0138
		500	0.0091	0.0077	0.0102	0.0086	0.0161	0.0136
Covariance	8	200	0.0008	-0.0024	0.0001	-0.0003	0.0030	0.0090
		500	0.0004	0.0012	0.0004	0.0012	0.0039	0.0117
	16	200	0.0002	0.0006	0.0000	0.0000	0.0017	0.0051
		500	0.0001	-0.0003	0.0002	-0.0006	0.0017	0.0051
Correlation	8	200	0.0050	-0.0186	0.0048	-0.0178	0.0051	-0.0189
		500	0.0039	-0.0146	0.0044	-0.0164	0.0044	-0.0164
	16	200	0.0020	-0.0071	0.0024	-0.0085	0.0026	-0.0092
		500	0.0023	-0.0081	0.0026	-0.0092	0.0025	-0.0088
Regression Coefficient	8	200	0.0050	-0.0185	0.0048	-0.0178	0.0051	-0.0189
		500	0.0039	-0.0145	0.0044	-0.0164	0.0043	-0.0160
	16	200	0.0020	-0.0070	0.0025	-0.0088	0.0026	-0.0091
		500	0.0022	-0.0078	0.0026	-0.0092	0.0025	-0.0088

Note. The table contains values of absolute bias and relative bias for conditions with a 25% item-level missing data rate. The item means and inter-item correlations were uniform.

Table A2

Simulation Study 1, Absolute Bias and Relative Bias from Proration – Uniform Item Means, Varied Inter-Item Correlations

Parameter	Items Per Scale	Sample Size	MCAR Mechanism		MAR Mechanism Due to External Variable		MAR Mechanism Due to Complete Items on the Scale	
			Absolute Bias	Relative Bias	Absolute Bias	Relative Bias	Absolute Bias	Relative Bias
Mean of *X*	8	200	0.0003	-0.0001	0.0158	-0.0039	0.0071	0.0018
		500	0.0001	0.0000	0.0153	-0.0038	0.0072	0.0018
	16	200	0.0006	0.0002	0.0146	-0.0037	0.0026	-0.0007
		500	0.0000	0.0000	0.0150	-0.0037	0.0032	-0.0008
Mean of *Y*	8	200	0.0002	0.0000	0.0151	-0.0038	0.0072	0.0018
		500	0.0000	0.0000	0.0150	-0.0037	0.0072	0.0018
	16	200	0.0002	0.0000	0.0149	-0.0037	0.0033	-0.0008
		500	0.0000	0.0000	0.0149	-0.0037	0.0036	-0.0009
Variance of *X*	8	200	0.0143	-0.0153	0.0208	-0.0222	0.0089	-0.0095
		500	0.0147	-0.0157	0.0211	-0.0225	0.0092	-0.0098
	16	200	0.0300	-0.0351	0.0398	-0.0466	0.0321	-0.0376
		500	0.0303	-0.0353	0.0404	-0.0470	0.0329	-0.0383
Variance of *Y*	8	200	0.0143	-0.0153	0.0210	-0.0224	0.0090	-0.0096
		500	0.0143	-0.0152	0.0205	-0.0218	0.0084	-0.0089
	16	200	0.0308	-0.0360	0.0410	-0.0479	0.0333	-0.0389
		500	0.0306	-0.0357	0.0408	-0.0476	0.0336	-0.0392
Covariance	8	200	0.0155	-0.0667†	0.0240	-0.1032*	0.0150	-0.0645†
		500	0.0145	-0.0627†	0.0235	-0.1016*	0.0142	-0.0614†
	16	200	0.0136	-0.0586†	0.0235	-0.1013*	0.0153	-0.0659†
		500	0.0140	-0.0602†	0.0237	-0.1019*	0.0153	-0.0658†
Correlation	8	200	0.0129	-0.0521†	0.0204	-0.0824†	0.0137	-0.0553†
		500	0.0118	-0.0480	0.0200	-0.0813†	0.0129	-0.0524†
	16	200	0.0066	-0.0244	0.0153	-0.0565†	0.0078	-0.0288
		500	0.0069	-0.0255	0.0154	-0.0569†	0.0075	-0.0277
Regression Coefficient	8	200	0.0131	-0.0528†	0.0205	-0.0826†	0.0138	-0.0556†
		500	0.0119	-0.0483	0.0200	-0.0812†	0.0129	-0.0520†
	16	200	0.0067	-0.0247	0.0154	-0.0567†	0.0079	-0.0291
		500	0.0069	-0.0255	0.0155	-0.0573†	0.0076	-0.0281

Note. The table contains values of absolute bias and relative bias for conditions with a 25% item-level missing data rate. The item means were uniform but the inter-item correlations varied. Absolute values of relative bias between .05 and .10 are denoted by a dagger (†); those greater than .10 are denoted by an asterisk (*).

Table A3

Simulation Study 1, Absolute Bias and Relative Bias from Proration – Varied Item Means, Uniform Inter-Item Correlations

Parameter	Items Per Scale	Sample Size	MCAR Mechanism		MAR Mechanism Due to External Variable		MAR Mechanism Due to Complete Items on the Scale	
			Absolute Bias	Relative Bias	Absolute Bias	Relative Bias	Absolute Bias	Relative Bias
Mean of *X*	8	200	0.0568	0.0130	0.0626	0.0144	0.1099	0.0252
		500	0.0571	0.0131	0.0628	0.0144	0.1100	0.0253
	16	200	0.0538	0.0124	0.0597	0.0137	0.0957	0.0220
		500	0.0532	0.0122	0.0594	0.0136	0.0951	0.0219
Mean of *Y*	8	200	0.0571	0.0131	0.0630	0.0145	0.1100	0.0253
		500	0.0569	0.0131	0.0629	0.0145	0.1101	0.0253
	16	200	0.0536	0.0123	0.0592	0.0136	0.0954	0.0219
		500	0.0534	0.0123	0.0594	0.0136	0.0951	0.0219
Variance of *X*	8	200	0.0190	0.0157	0.0788	0.0650†	0.1681	0.1386*
		500	0.0185	0.0152	0.0787	0.0647†	0.1678	0.1379*
	16	200	0.0074	0.0064	0.0647	0.0561†	0.1296	0.1125*
		500	0.0073	0.0063	0.0649	0.0560†	0.1292	0.1116*
Variance of *Y*	8	200	0.0187	0.0154	0.0786	0.0647†	0.1681	0.1383*
		500	0.0186	0.0153	0.0789	0.0647†	0.1686	0.1383*
	16	200	0.0062	0.0054	0.0633	0.0549†	0.1282	0.1112*
		500	0.0066	0.0057	0.0640	0.0554†	0.1287	0.1114*
Covariance	8	200	0.0017	-0.0052	0.0632	0.1931*	0.0329	0.1005*
		500	0.0007	-0.0021	0.0635	0.1949*	0.0338	0.1037*
	16	200	0.0008	-0.0024	0.0604	0.1846*	0.0307	0.0938†
		500	0.0012	-0.0037	0.0603	0.1842*	0.0305	0.0932†
Correlation	8	200	0.0055	-0.0205	0.0324	0.1206*	0.0089	-0.0331
		500	0.0046	-0.0172	0.0326	0.1220*	0.0081	-0.0303
	16	200	0.0024	-0.0085	0.0346	0.1221*	0.0045	-0.0159
		500	0.0027	-0.0096	0.0345	0.1220*	0.0045	-0.0159
Regression Coefficient	8	200	0.0055	-0.0204	0.0325	0.1206*	0.0089	-0.0330
		500	0.0046	-0.0172	0.0327	0.1222*	0.0079	-0.0295
	16	200	0.0025	-0.0088	0.0345	0.1215*	0.0047	-0.0165
		500	0.0027	-0.0096	0.0344	0.1217*	0.0046	-0.0163

Note. The table contains values of absolute bias and relative bias for conditions with a 25% item-level missing data rate. The inter-item correlations were uniform but the item means varied. Absolute values of relative bias between .05 and .10 are denoted by a dagger (†); those greater than .10 are denoted by an asterisk (*).

Online Appendix B

MSE ratios comparing FIML with items as auxiliary variables to scale-level FIML are reported in the manuscript. *MSE* ratios comparing FIML with items as auxiliary variables to proration are reported here as supplementary material. Recall that *MSE* equals the squared bias plus the sampling variance of the parameter estimate. The results of the first simulation study indicated that the mean and covariance structures dictate the performance of proration; proration resulted in negligible bias when the item means and inter-item correlations were uniform but resulted in non-negligible bias when either the item means or inter-item correlations varied. By contrast, FIML with items as auxiliary variables provided unbiased parameter estimates across all conditions (see the results of the second simulation study). However, we expected proration to be more efficient (i.e., provide lower sampling variance) than FIML with items as auxiliary variables because proration yields a complete data set after singly imputing each participant's missing scores with the mean of his or her observed scores. When considering the tradeoff between bias and efficiency, we expected the *MSE* ratios to favor proration when the item means and inter-item correlations were uniform but to favor FIML with items as auxiliary variables when either the item means or inter-item correlations varied.

We computed *MSE* ratios by dividing the *MSE* from proration by the *MSE* from FIML with items as auxiliary variables, such that values less than 1 favor proration and values greater than 1 favor FIML with items as auxiliary variables. *MSE* ratios for conditions corresponding to a sample size of 500 are reported in Table B1. As shown in Table B1, the *MSE* ratios ranged from 0.9000 to 1.0172 (M = 0.9818) when the item means and inter-item correlations were uniform, from 0.9429 to 1.5000 (M = 0.9818) when the item means were uniform but the inter-item correlations varied, and from 1.0000 to 2.6000 (*M* = 1.5071) when the inter-item correlations were uniform but the item means varied.

Table B1

Simulation Study 2, MSE Ratios Comparing FIML with All But One Item from Each Scale as Auxiliary Variables to Proration

			MSE Ratio		
Parameter	Items Per Scale	Item-Level Missing Data Rate	Uniform Item Means, Uniform Inter-Item Correlations	Uniform Item Means, Varied Inter-Item Correlations	Varied Item Means, Uniform Inter-Item Correlations
Mean of *X*	8	5%	1.0000	1.0000	1.0385
		15%	1.0000	0.9524	1.5000
		25%	0.9310	1.0000	2.3929
	16	5%	1.0000	1.0000	1.0455
		15%	1.0000	1.0000	1.5000
		25%	0.9565	1.0000	2.5217
Mean of *Y*	8	5%	1.0000	0.9474	1.0417
		15%	1.0000	1.0000	1.4800
		25%	1.0000	1.0000	2.6000
	16	5%	1.0000	1.0000	1.0435
		15%	1.0000	1.0000	1.4583
		25%	0.9600	1.0000	2.4000
Variance of *X*	8	5%	0.9818	0.9688	1.0588
		15%	0.9825	0.9429	1.4717
		25%	1.0000	0.9459	2.1071
	16	5%	1.0000	1.0000	1.0732
		15%	1.0000	1.2000	1.4286
		25%	1.0000	1.5000	2.0233
Variance of *Y*	8	5%	1.0000	0.9677	1.0612
		15%	1.0000	0.9697	1.4706
		25%	1.0172	0.9429	2.1698
	16	5%	1.0000	1.0000	1.0435
		15%	0.9800	1.1429	1.3913
		25%	1.0000	1.4643	1.9565
Covariance	8	5%	0.9714	0.9500	1.0606
		15%	0.9459	1.0000	1.5294
		25%	0.9474	1.0435	2.2286
	16	5%	1.0000	1.0000	1.1111
		15%	1.0000	1.0625	1.6667
		25%	0.9667	1.1875	2.3929
Correlation	8	5%	1.0000	1.0000	1.0000

		15%	0.9474	1.0000	1.1579
		25%	0.9000	1.0952	1.4737
	16	5%	1.0000	1.0000	1.0000
		15%	1.0000	1.0000	1.2941
		25%	0.9444	1.1111	1.6471
Regression Coefficient	8	5%	1.0000	1.0000	1.0000
		15%	0.9500	1.0000	1.1500
		25%	0.9048	1.0455	1.4500
	16	5%	1.0000	1.0000	1.0556
		15%	1.0000	1.0526	1.2778
		25%	0.9474	1.0500	1.5263

Note. The table contains *MSE* ratios for conditions with a sample size of 500. We computed *MSE* ratios by dividing the *MSE* from proration by the *MSE* from FIML with items as auxiliary variables, such that values less than 1 favor proration and values greater than 1 favor FIML with items as auxiliary variables.

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