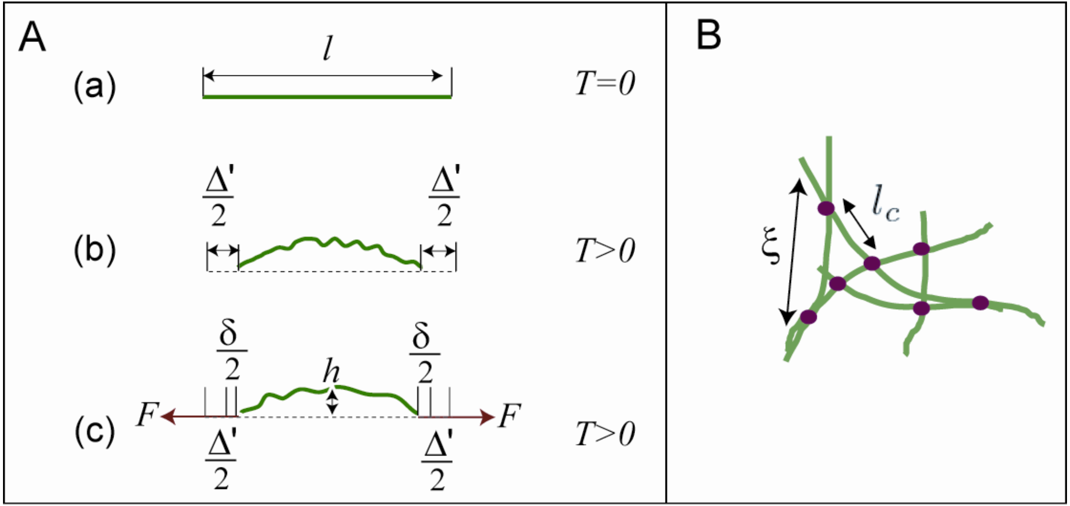
Fig. 5. Derivation of elasticity of a semi-flexible polymer: (*A*) A single inextensible semiflexible filament with a contour length of l_c. (*a*) At absolute zero temperature (T = 0), where thermal effects vanish, the end-to-end distance, l, of a semiflexible polymer equals l_c. (*b*) For $T > 0$, the filament undergoes thermal fluctuation of a magnitude h, and l contracts by Δ' due to the thermal bending such that $l = l_c - \Delta'$. The corresponding bending energy is $U_B \sim \kappa_0 l(h/l^2)^2$, where κ_0 is the filament bending rigidity. (*c*) In the presence of an applied extensional force, F, the contracted filament extends by δ with the stretching energy of $U_F \sim Fl\varepsilon \sim Fh^2/l$, where ε is the net stain of the filament due to both the bending and stretching, $\varepsilon = (\Delta' - \delta)/l_c$. The elastic energy due to bending and stretching must balance the thermal energy by equipartition such that $U_B + U_F \sim (\kappa/l^2 + F)\varepsilon l \sim k_B T$. This energy balance equation can be rewritten for $\varepsilon \sim (l^2/l_p - Fl^4/k_B T l_p^2)/l$ with the persistence length, $l_p = \kappa/k_B T$, yielding the lateral displacements $\Delta' \sim l^2/l_p$ and $\delta \sim Fl^4/k_B T l_p^2$; thus, the linear force-extension is $F \sim (\kappa^2/k_B T l^4)\delta$. (*B*) For a cross-linked network characterized by a pore size ξ, the stress σ is defined by $\sigma \sim F/\xi^2$ and the imposed strain is $\gamma \sim \delta/l_c$, where l_c is now the distance between cross-links. This calculation leads to the network elastic modulus $G' \sim \sigma/\gamma \sim \kappa^2/(k_B T \xi^2 l_c^3)$.


A
l
(a)
T=0
Δ'/2
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