

## SUPPLEMENTARY MATERIALS

### A COMPARISON OF THE $\beta$ -SUBSTITUTION METHOD AND A BAYESIAN METHOD FOR ANALYZING LEFT-CENSORED DATA

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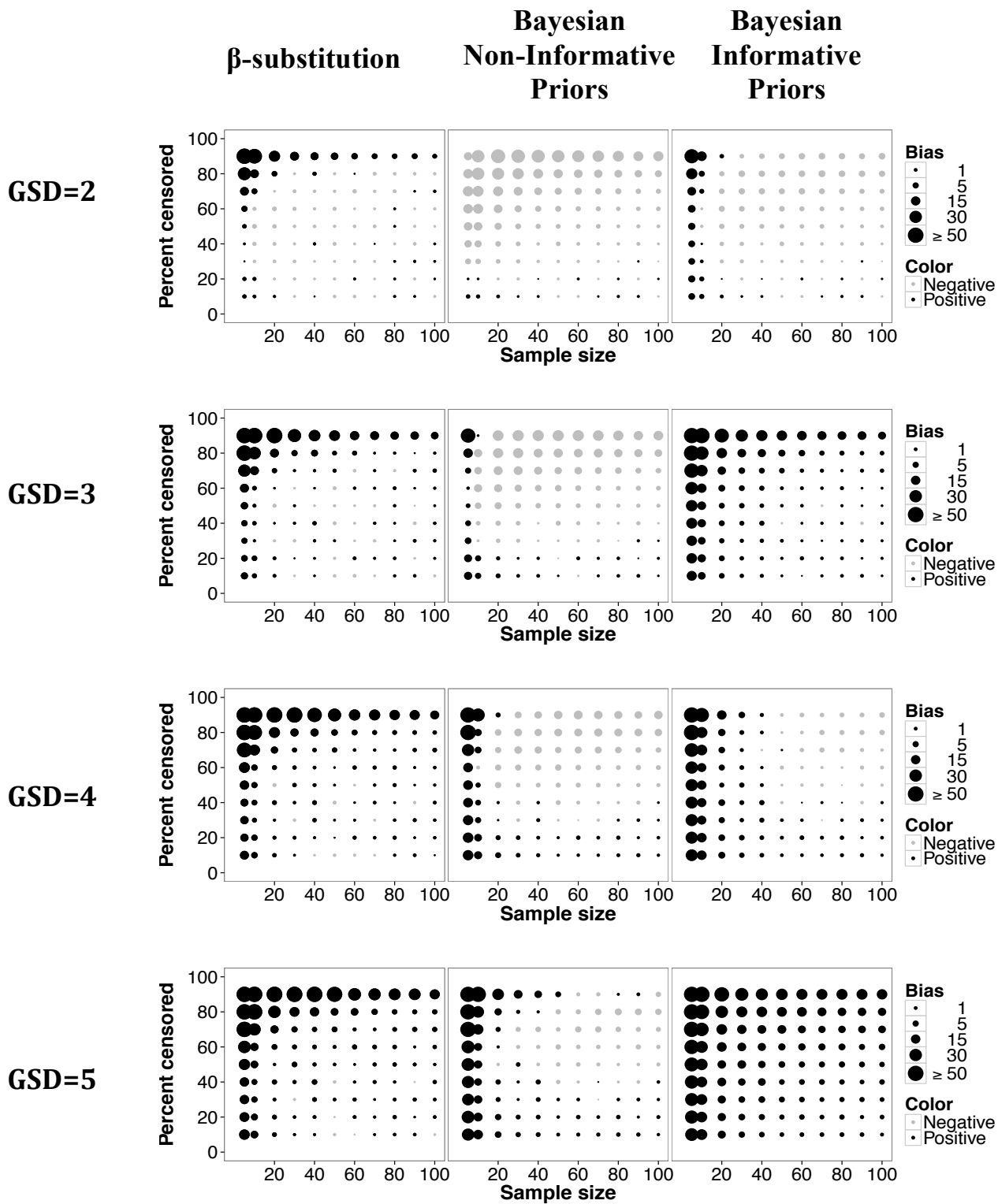


Figure 1: Relative bias in the estimate of the GM of a lognormal distribution and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.

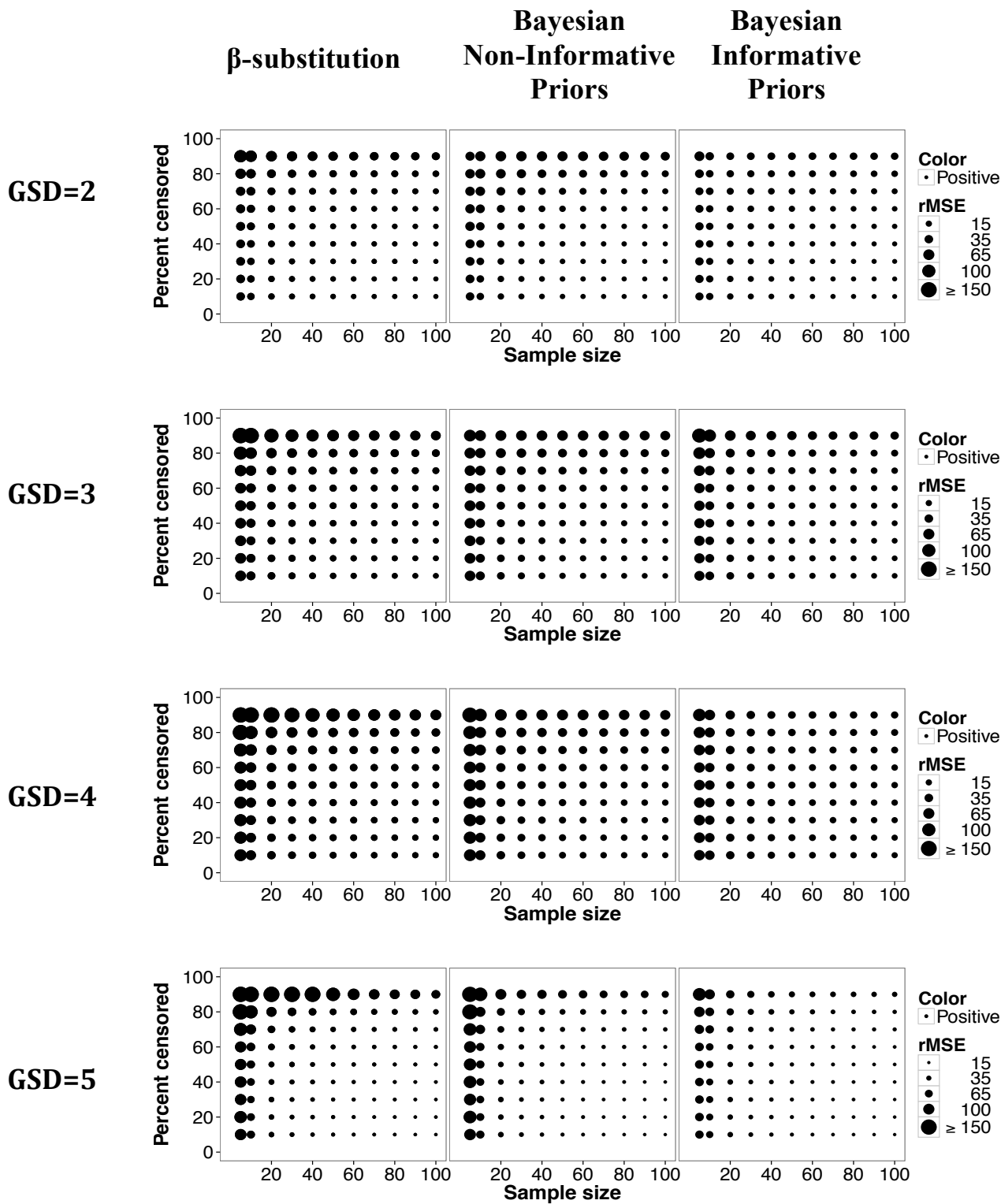


Figure 2: Relative rMSE in the estimate of the GM of a lognormal distribution and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.

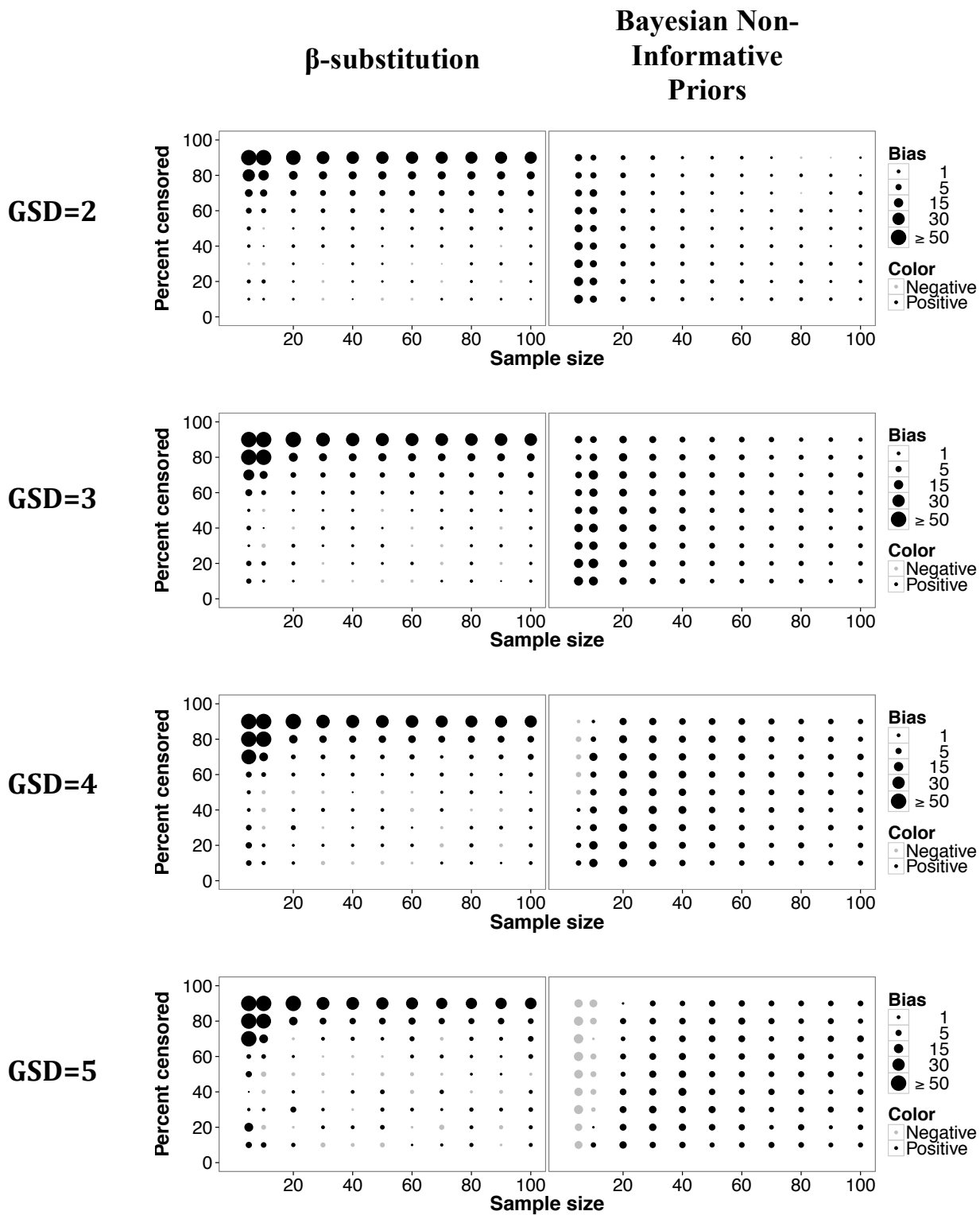


Figure 3: Relative bias in the estimate of the AM of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60% (the average for the three LODs).

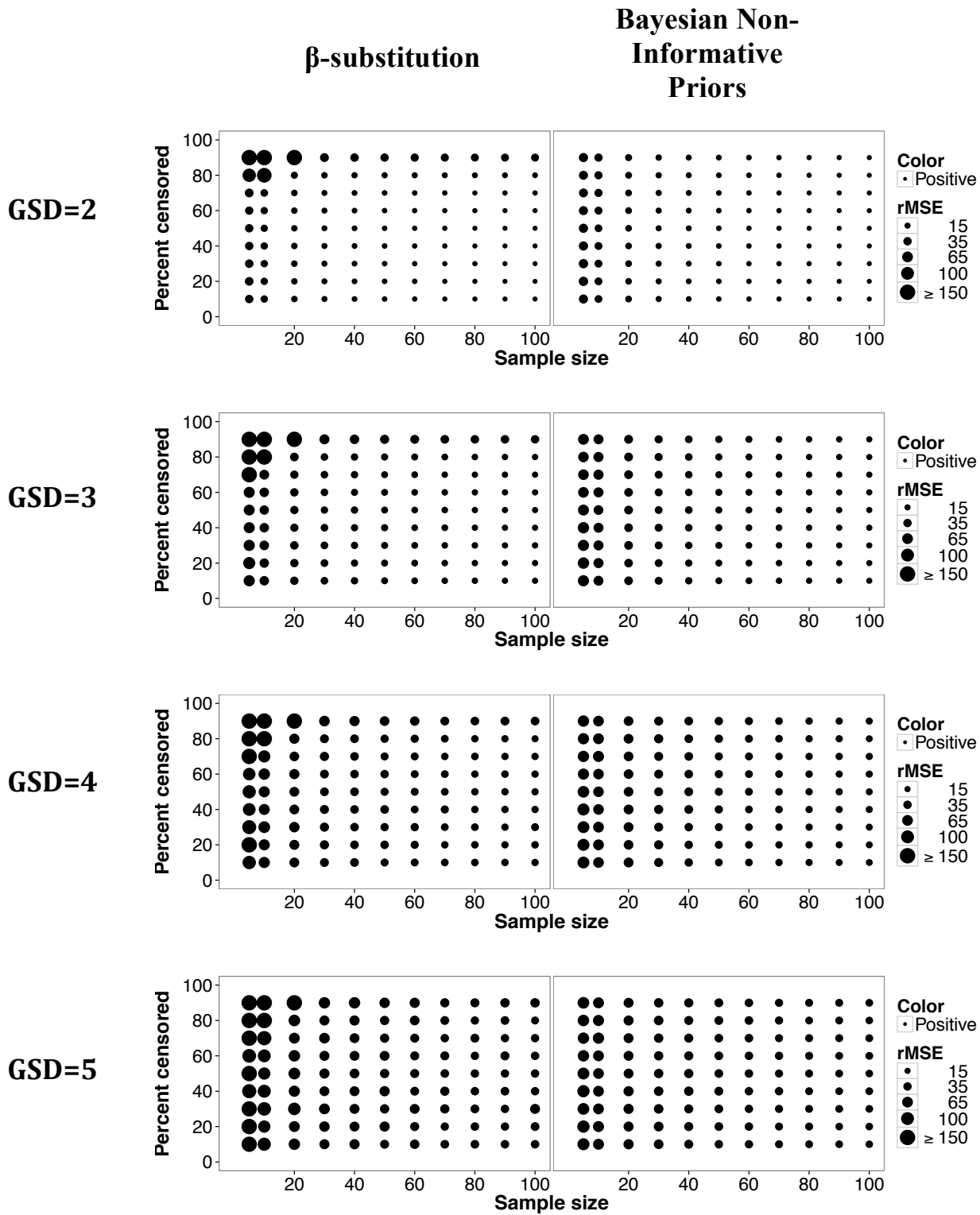


Figure 4: Relative rMSE in the estimate of the AM of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

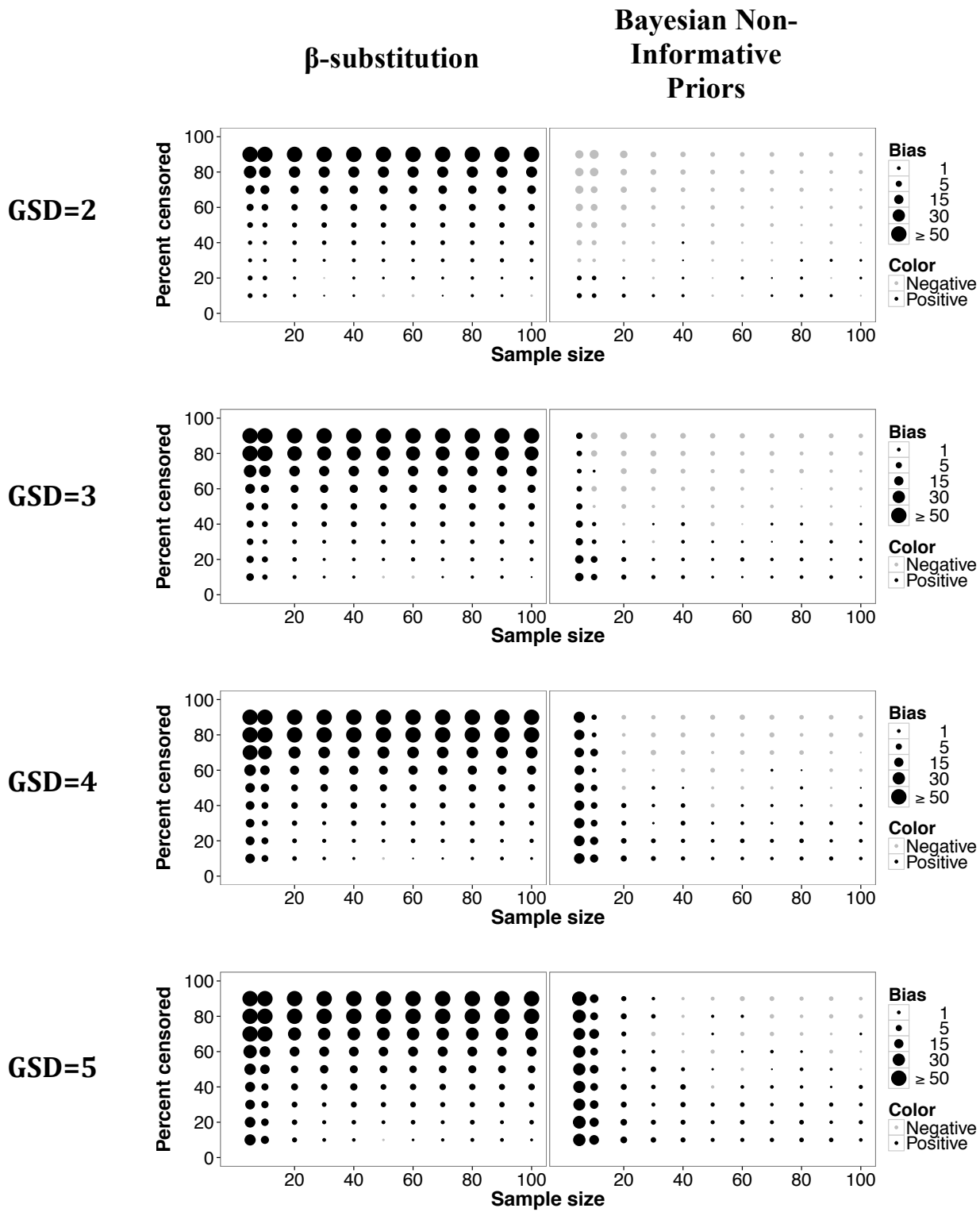


Figure 5: Relative bias in the estimate of the GM of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

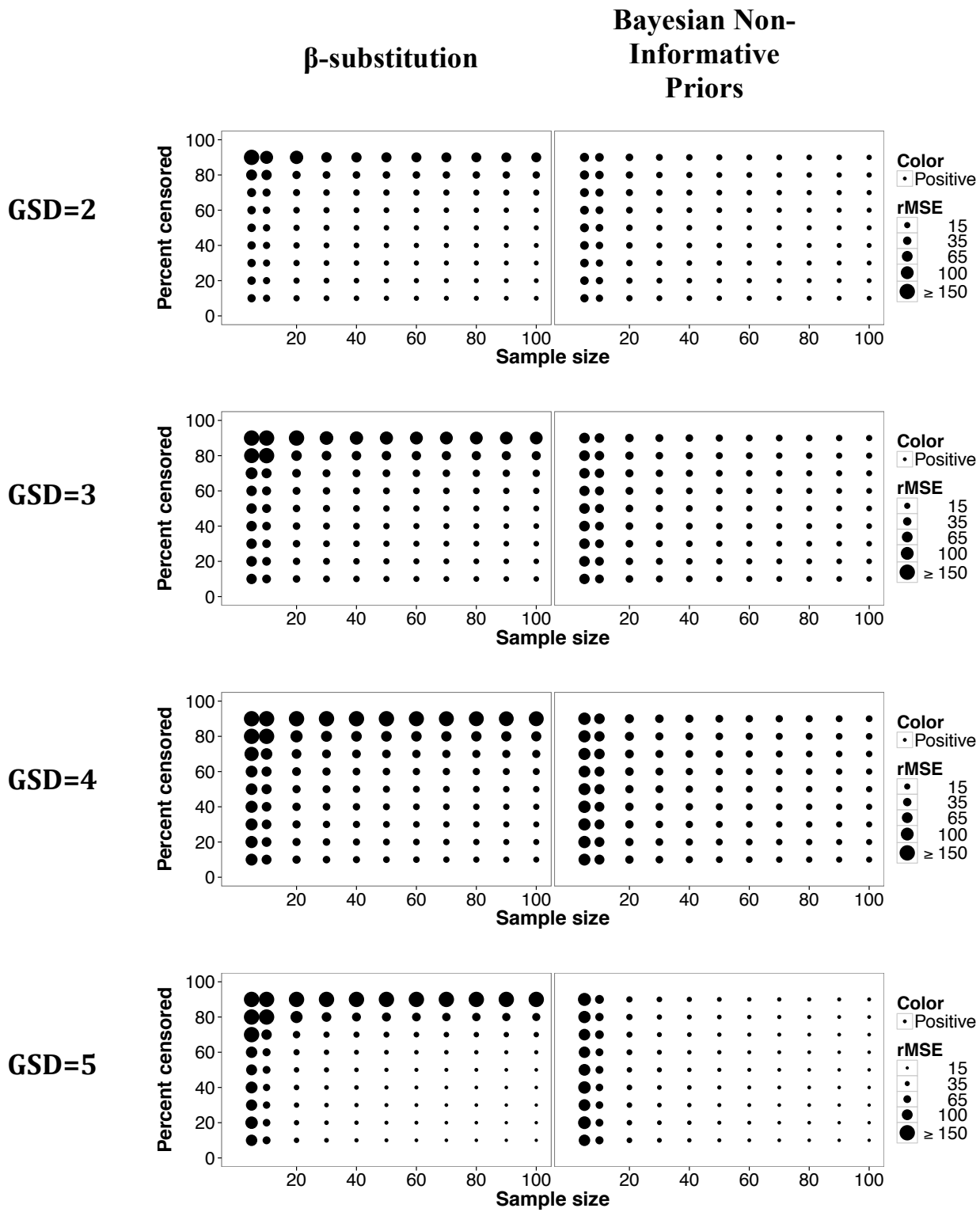


Figure 6: Relative rMSE in the estimate of the GM of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

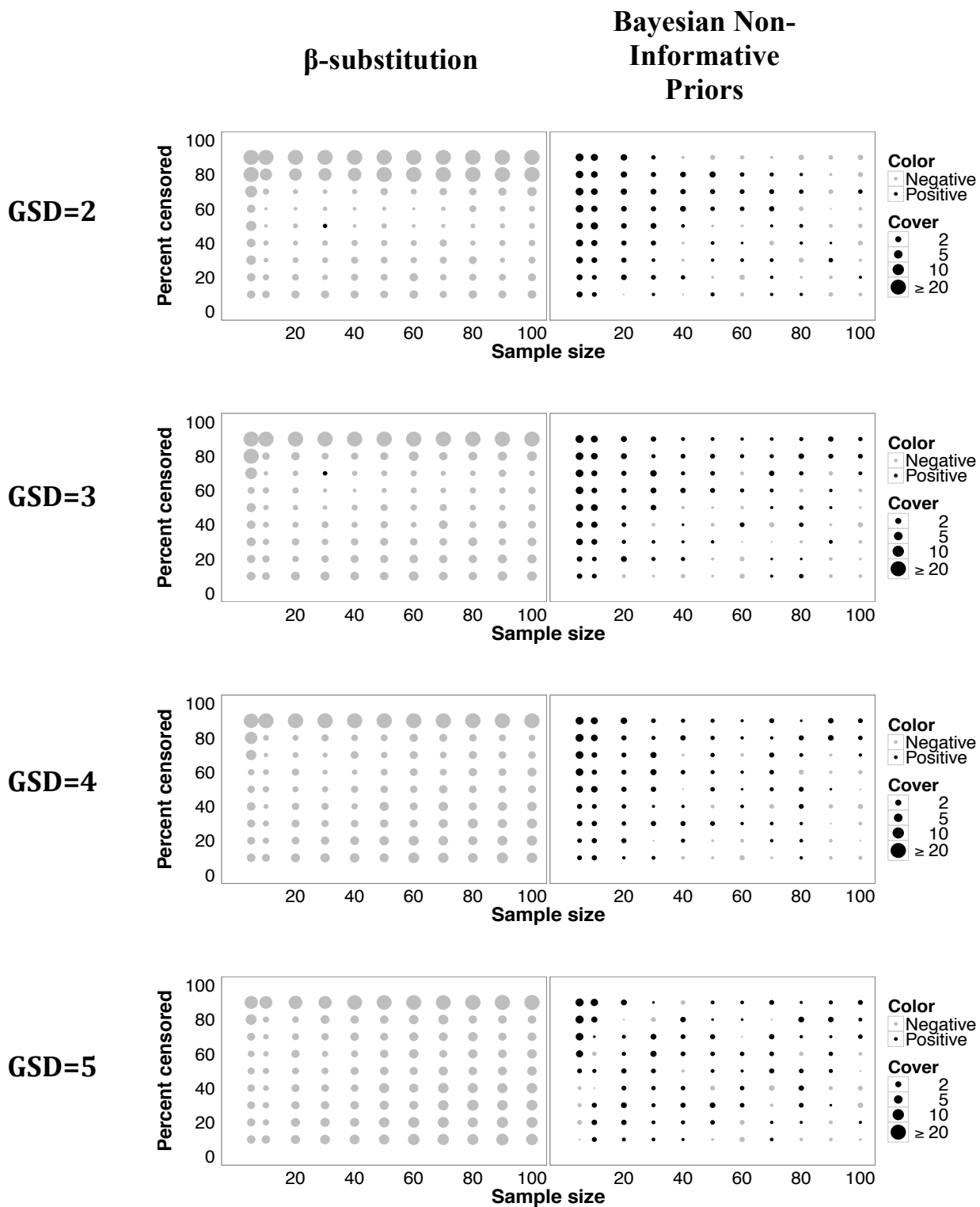


Figure 7: Coverage probabilities (in percent) for the AM of a lognormal distribution and large gap multiple LODs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. The size of the circle represents difference between the actual coverage probability minus the target 95%. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.



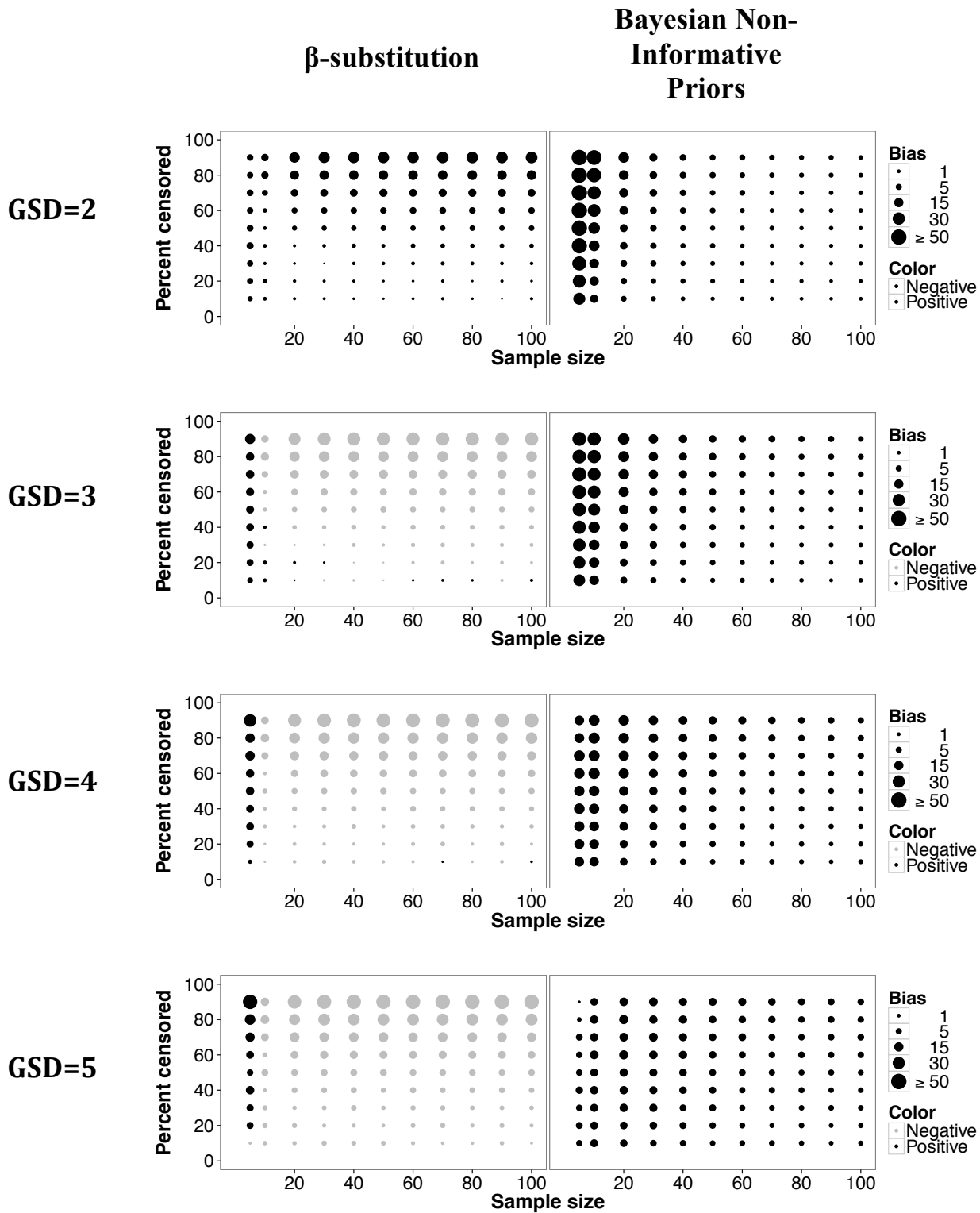


Figure 8: Relative bias in the estimate of the GSD of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

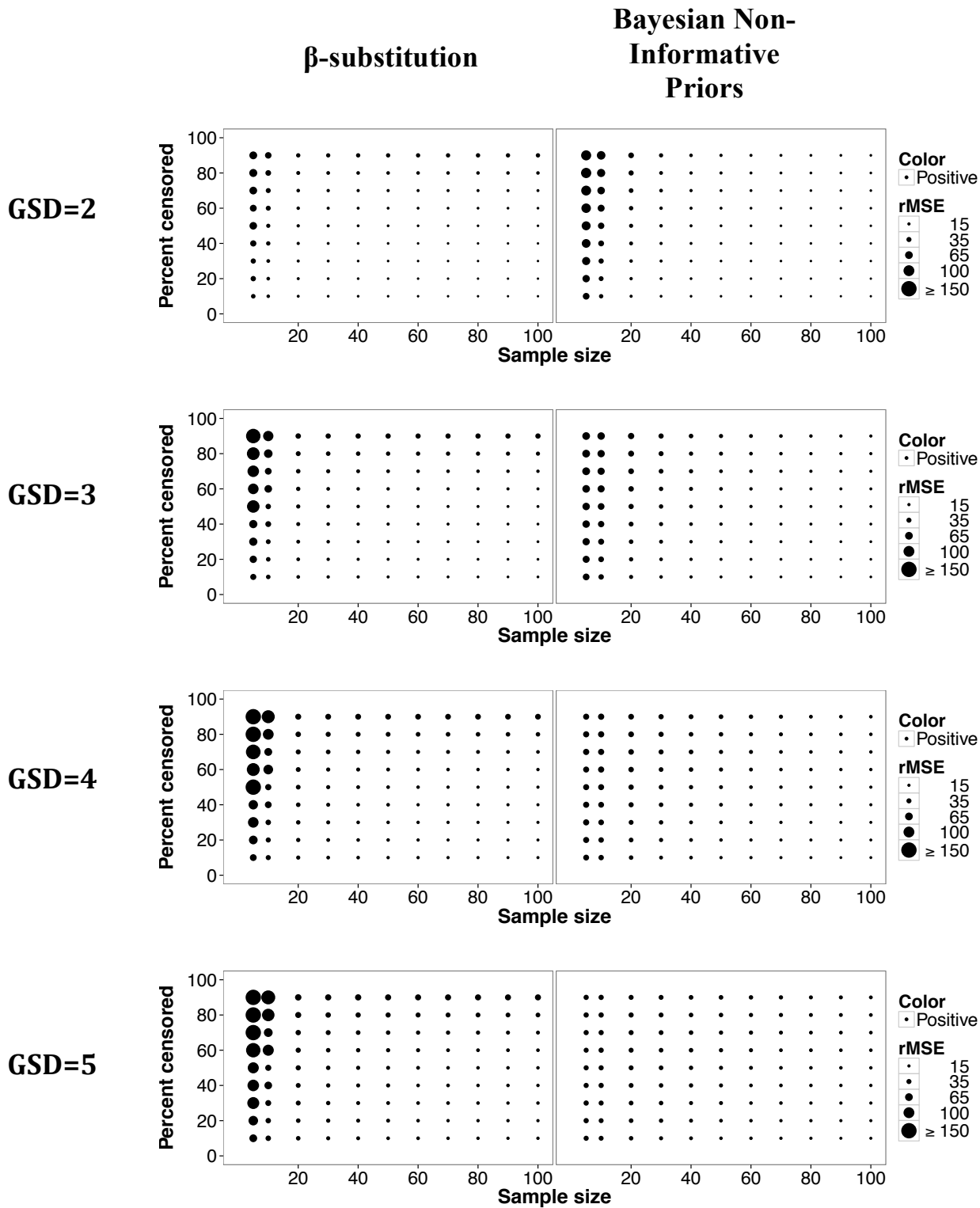


Figure 9: Relative rMSE in the estimate of the GSD of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

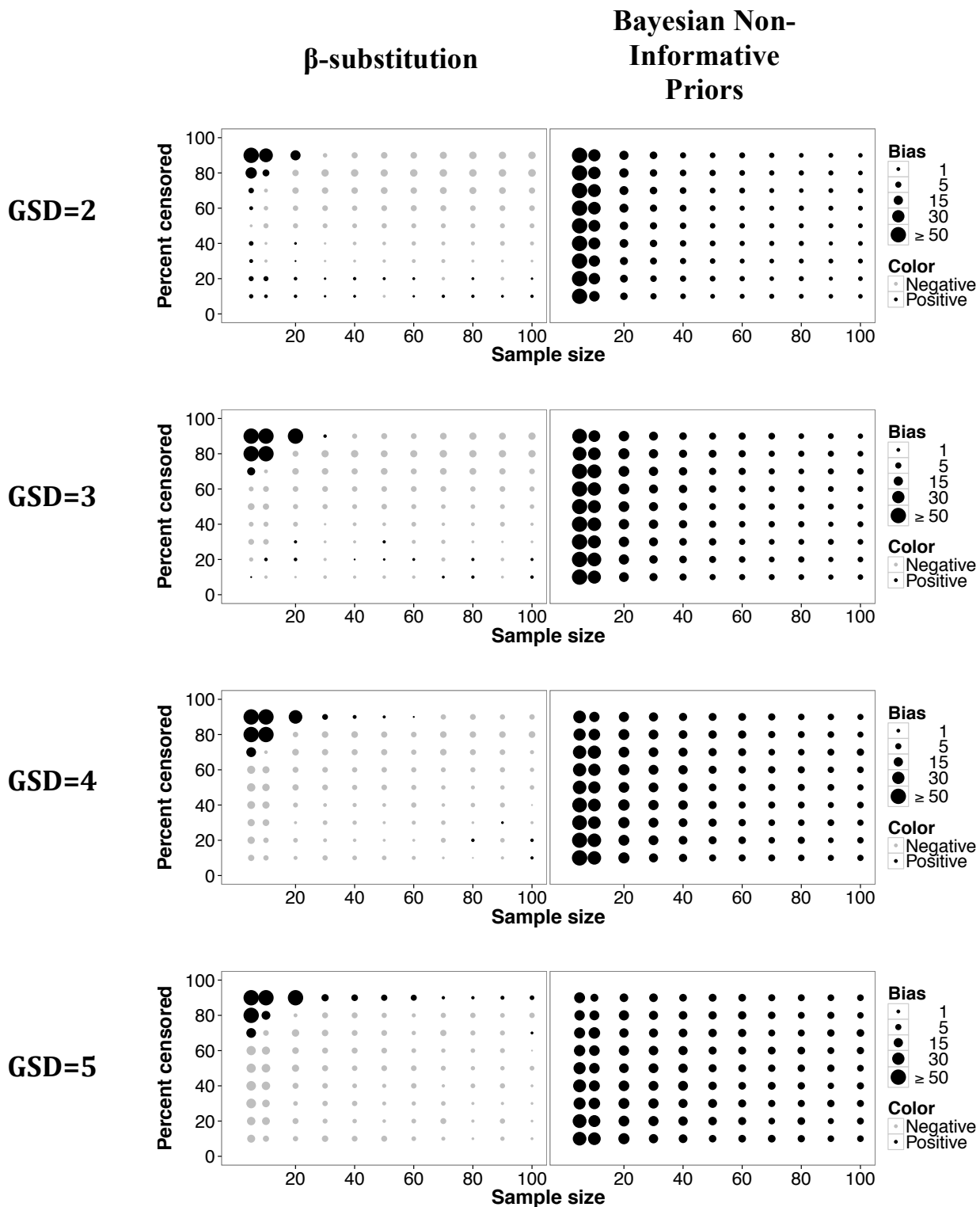


Figure 10: Relative bias in the estimate of the 95<sup>th</sup> percentile of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

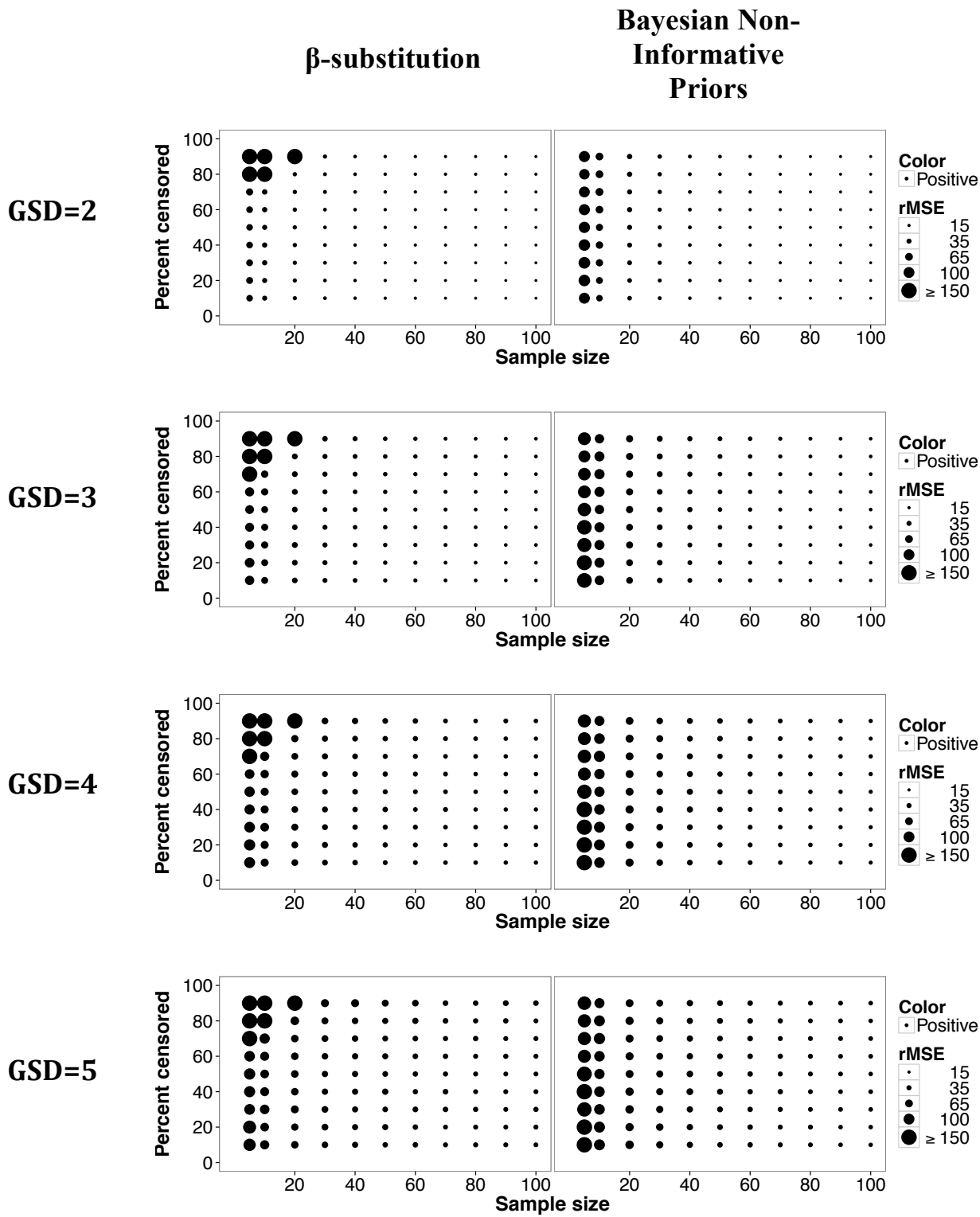


Figure 11: Relative rMSE in the estimate of the 95<sup>th</sup> percentile of a lognormal distribution and large gap multiple LODs for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%.

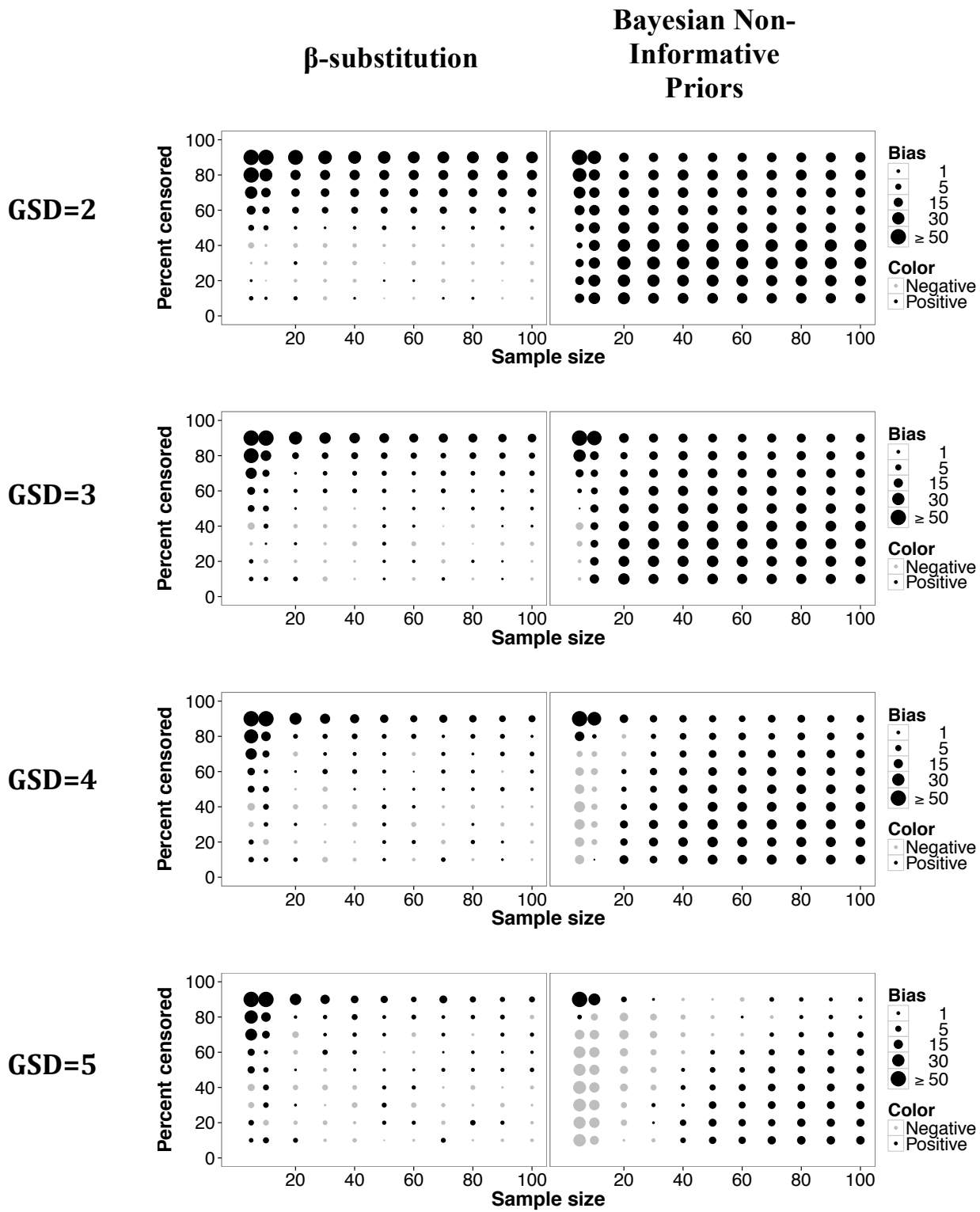


Figure 12: Relative bias in the estimate of the AM of a mixed distribution (GMs=1 and 10) and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.

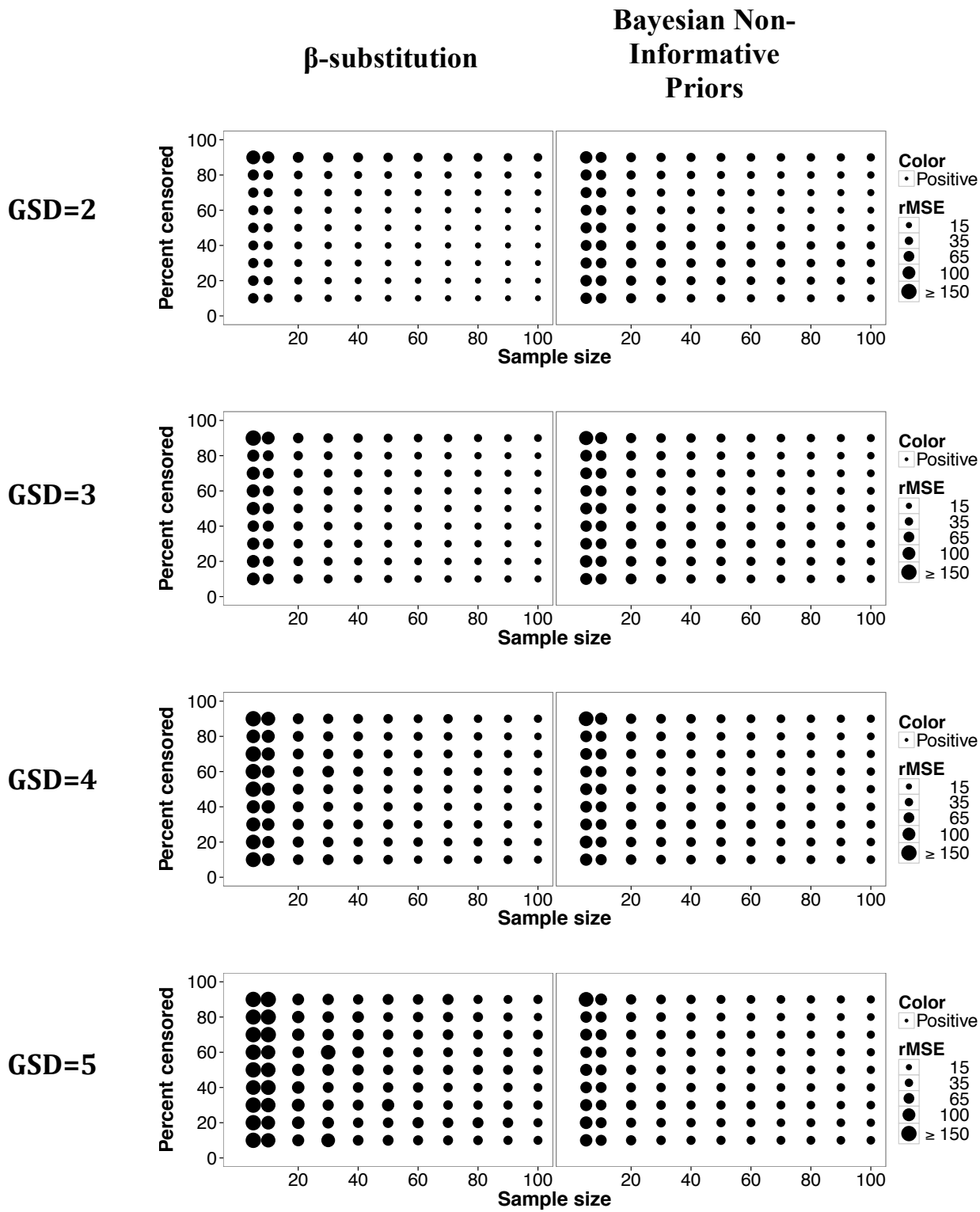


Figure 13: Relative rMSE in the estimate of the AM of a mixed distribution (GMs =1 and 10) and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.

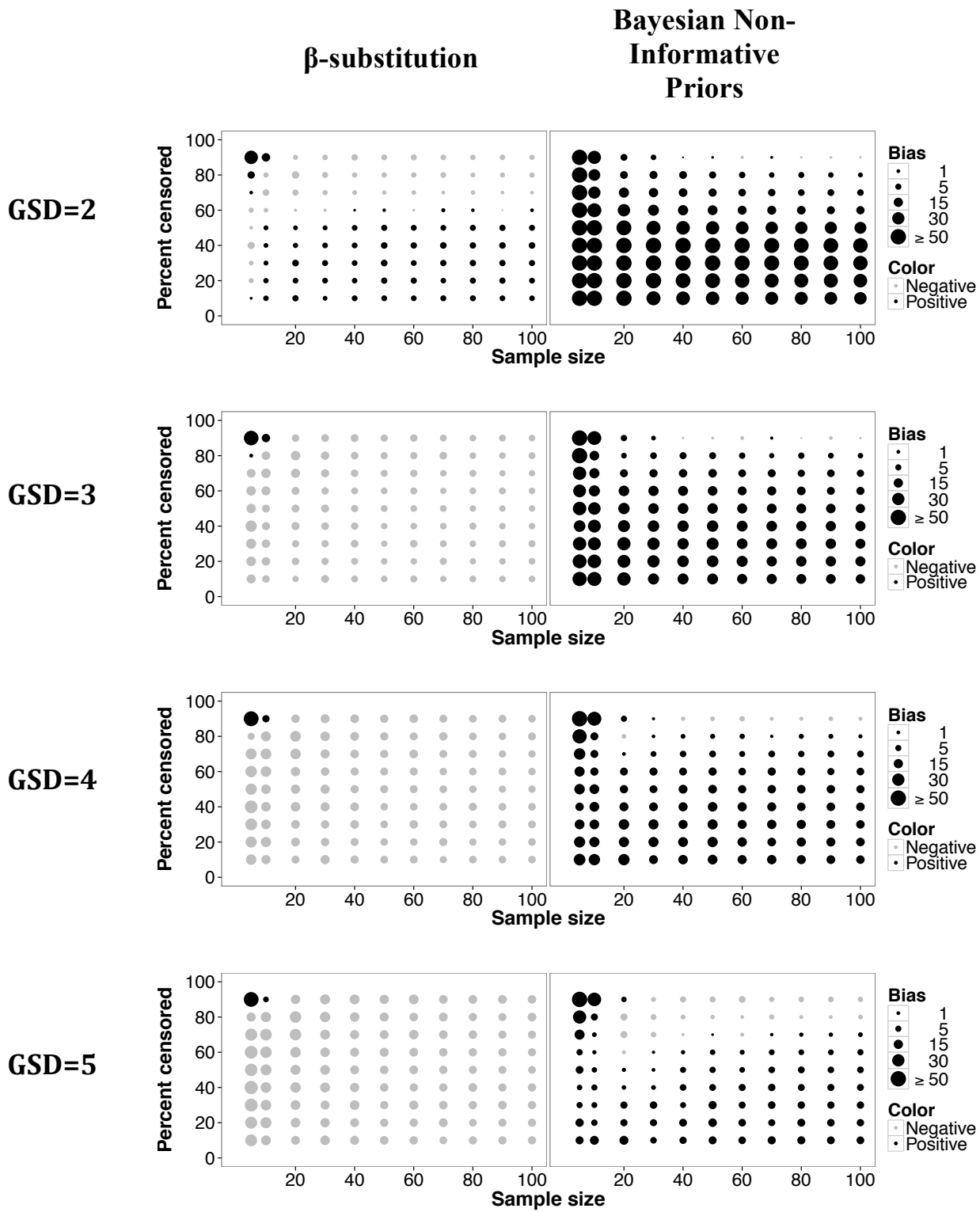


Figure 14: Relative bias in the estimate of the 95<sup>th</sup> percentile of a mixed distribution (GMs = 1 and 10) and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.

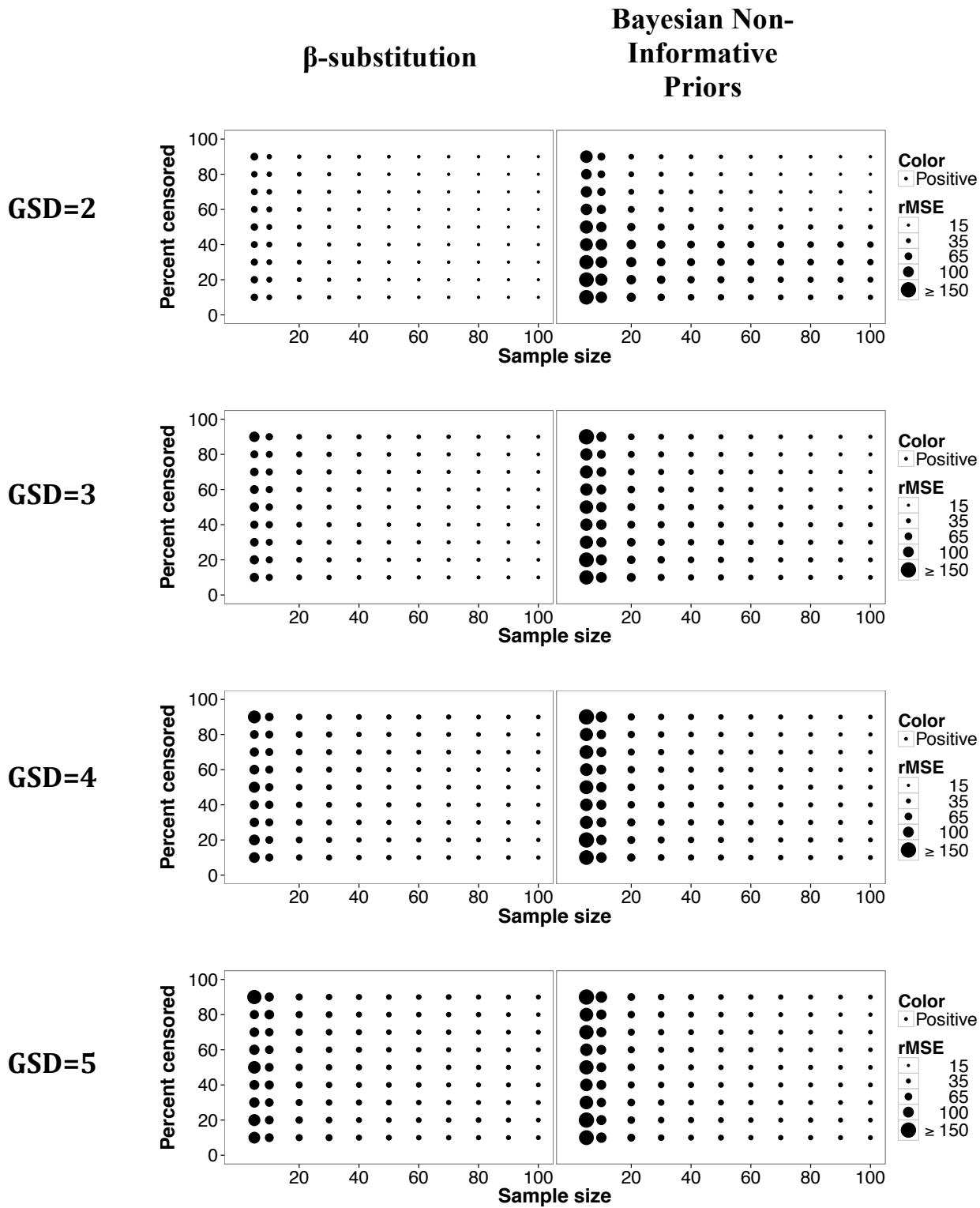


Figure 15: Relative rMSE in the estimate of the 95<sup>th</sup> percentile of a mixed distribution (GMs = 1 and 10) and a single LOD for different sample sizes, percent censoring, and GSDs for the  $\beta$ -substitution method and the Bayesian method with non-informative priors.



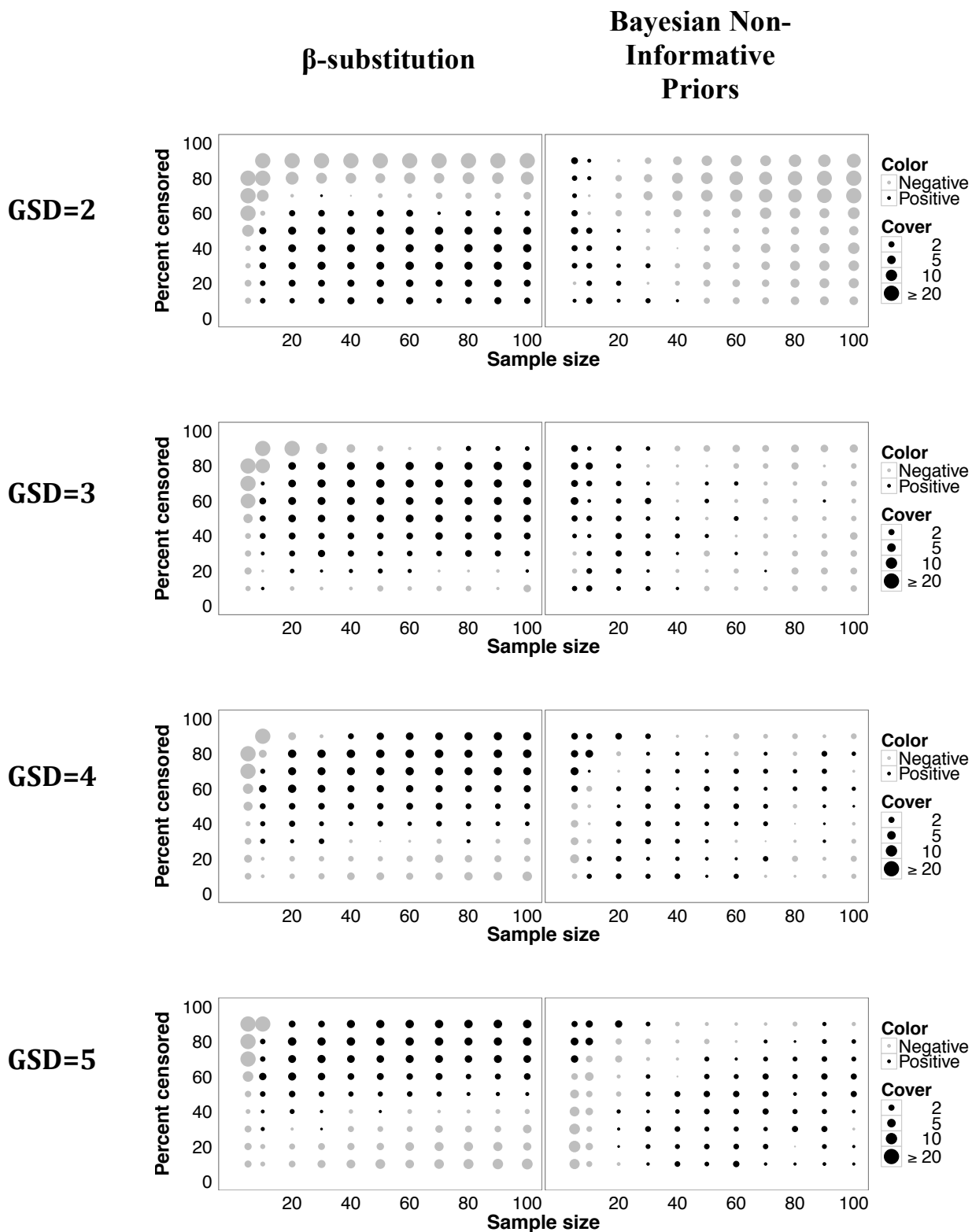


Figure 16: Coverage probabilities (in percent) for the AM of a mixed distribution (GMs =1 and 10) and a single LOD for the  $\beta$ -substitution method and the Bayesian method with non-informative priors. The size of the circle represents difference between the actual coverage probability minus the target 95%. When  $p_1$  is at 90%, the expected level of censoring is actually about 60%