Web-based Supplementary Materials for
Exact Confidence Intervals for the Relative Risk and the Odds Ratio

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SUPPLEMENTARY MATERIALS

0.1 *PROOFS*

Proof of Lemma 2. Note two facts: i) $(N_{11}, N_{21}, N_{12}) \sim Multinomial(n, p_{11}, p_{21}, p_{12})$; and

$$\text{ii) } P(L(N_{11}, N_{12}, N_{21}) \leqslant \theta_{p_r} = \frac{p_{11}+p_{12}}{p_{11}+p_{21}}) \geqslant 1-\alpha, \forall\ (p_{11}, p_{12}, p_{21}) \in H_p.$$

Then

$$P(L(N_{11}, N_{21}, N_{12}) \leqslant \frac{p_{11}+p_{21}}{p_{11}+p_{12}}) \geqslant 1-\alpha, \forall\ (p_{11}, p_{12}, p_{21}) \in H_p.$$

This implies that

$$P(\theta_{p_r} = \frac{p_{11}+p_{12}}{p_{11}+p_{21}} \leqslant \frac{1}{L(N_{11}, N_{21}, N_{12})}) \geqslant 1-\alpha, \forall\ (p_{11}, p_{12}, p_{21}) \in H_p,$$

which establishes the first claim of the lemma. The second half is trivial. □

Proof of Lemma 3. Following Theorem 4 in Wang (2010), $L(0, n, 0)$ is equal to the smallest solution of

$$f(\theta_{p_r}) = 1 - \sup_{(p_{11},p_{21}) \in D(\theta_{p_r})} p_{12}^n = 1-\alpha,$$

where p_{12} and $D_{p_r}(\theta_{p_r})$ are given in (8) and (9), respectively. Note that $D_{p_r}(\theta_{p_r})$, in the $p_{11} - p_{21}$ plane, is a triangle when $\theta_{p_r} \geqslant 1$ and the intersection of two triangles when $\theta_{p_r} < 1$. Then the largest value of p_{12}, as a linear function of p_{11} and p_{21}, on set $D_{p_r}(\theta_{p_r})$ occurs at $(p_{11}, p_{21}) = (0, 1/(1+\theta_{p_r}))$, the top vertex of $D_{p_r}(\theta_{p_r})$ (see Figure S1). Thus, at this point, $p_{12} = \theta_{p_r}/(1+\theta_{p_r})$ and

$$f(\theta_{p_r}) = 1 - (\frac{\theta_{p_r}}{1+\theta_{p_r}})^n.$$

The claim (17) is established by solving $f(\theta_{p_r}) = 1-\alpha$. □

Proof of Lemma 4 Note two facts: i) $(N_{11}, N_{21}, N_{12}) \sim Multinomial(n, p_{11}, p_{21}, p_{12})$, and

$$\text{ii) } P(L(N_{11}, N_{21}, N_{12}) \leqslant \frac{(p_{11}+p_{21})(p_{21}+p_{22})}{(p_{11}+p_{12})(p_{12}+p_{22})} = 1/\theta_{p_o}) \geqslant 1-\alpha, \forall\ (p_{11}, p_{12}, p_{21}) \in H_p.$$

Then

$$P(\theta_{p_o} \leqslant \frac{1}{L(N_{11}, N_{21}, N_{12})}) \geqslant 1-\alpha, \forall\ (p_{11}, p_{12}, p_{21}) \in H_p,$$

which establishes the first claim of the lemma. The second claim is trivial. □

Proof of Lemma 5. The first half of the lemma follows

$$P(\theta_{i_r} = \frac{p_1}{p_2} \leqslant U(X,Y)) = P(\frac{p_2}{p_1} \geqslant L_{n_2,n_1}(Y,X)) \geqslant 1-\alpha, \forall \ (p_1,p_2) \in H_i.$$

The second half is trivial. □

0.2 *FIGURES*

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

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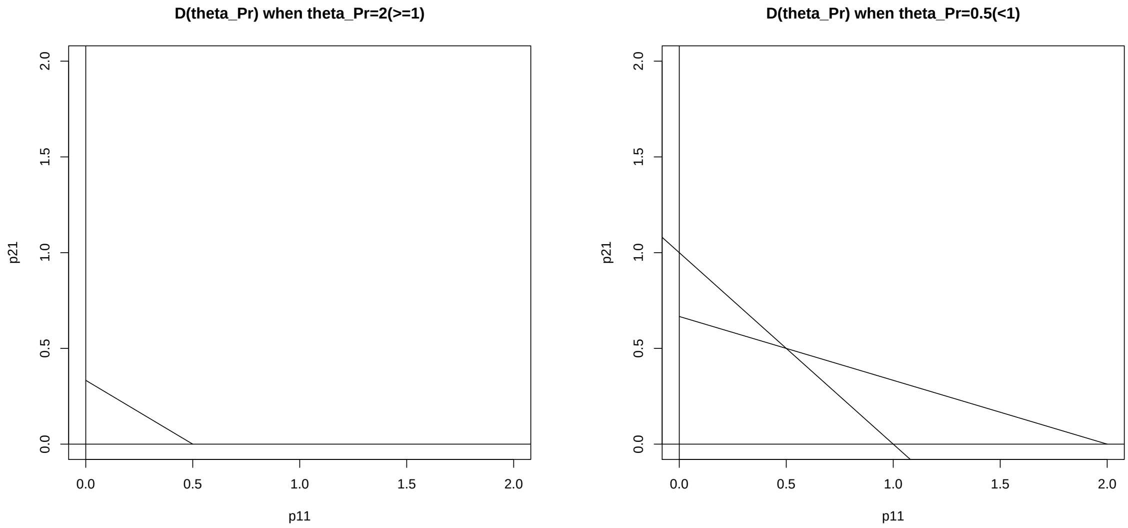
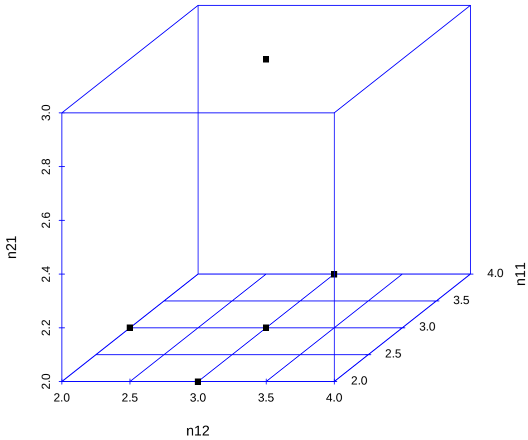


Figure S1. Set $D_{p_r}(\theta_{p_r})$ is a triangle (left) when $\theta_{p_r}(=2) \geqslant 1$, and is the intersection of two right triangles (right) when $\theta_{p_r}(=0.5) < 1$.



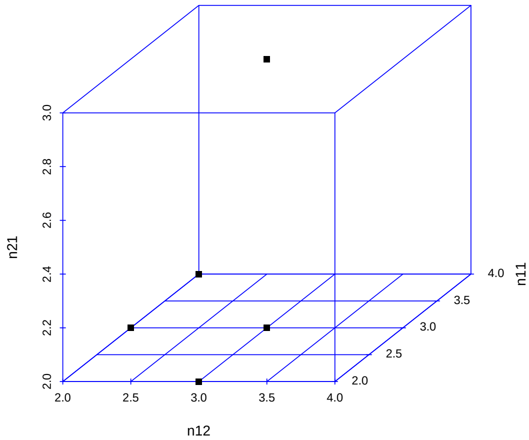


Figure S2. Points $\underline{x}_p = (3, 3, 2)$, A, B, C and D (left). Points $\underline{x}_p$, B, C, D and E (right).

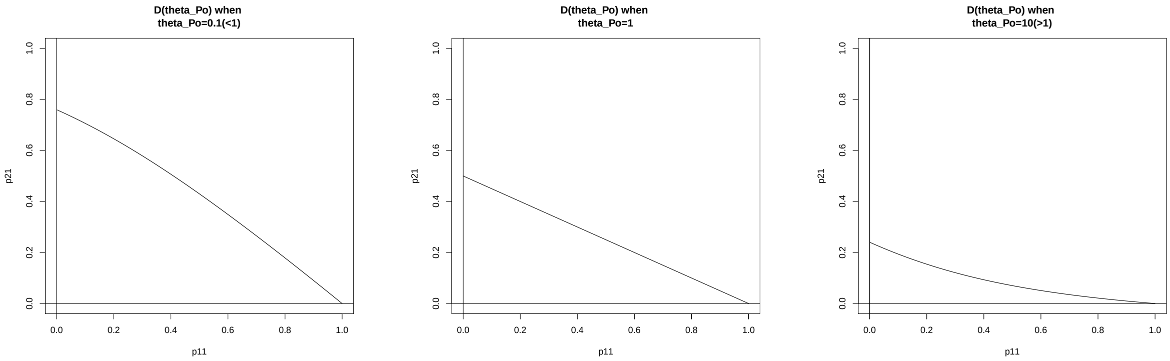


Figure S3. Set $D_{p_o}(\theta_{p_o})$, a right curved triangle with $(p_{11}, p_{21}) = (0, 0)$ as one of its vertices for $\theta_{p_o} = 0.1 < 1$(left),$= 1$(middle), $= 10 > 1$(right).

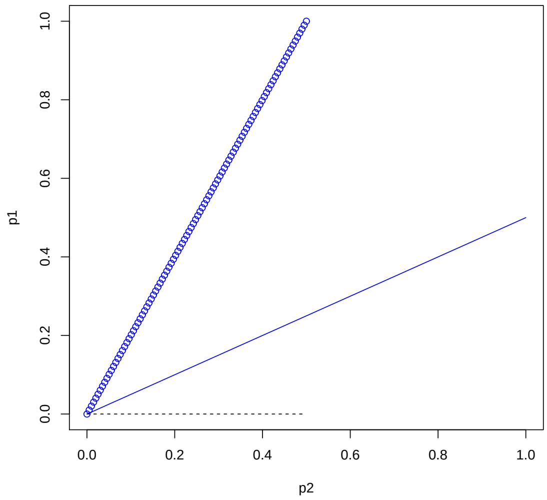


Figure S4. The parameter space H_{i_r} (the unit square), the sets of $\theta_{i_r} = 0.5$ (the solid line) and $\theta_{i_r} = 2$ (the circle line), and $D_{i_r}(2) = [0, 0.5]$ on the p_2- axis (the dashed line). Note $\theta_{i_r} \in [0, +\infty)$ and $p_2 \in D_{i_r}(\theta_{i_r})$.