Web-based Supplementary Materials for "Gene-based Association Analysis for Censored Traits Via Fixed Effect Functional Regressions"

Appendix A. More Simulation Results

In this supplementary section, more simulation results are provided. When some causal variants are rare and some are common, the empirical type I error results are presented in Tables A.1, A.3, and A.5, and the empirical power comparison are presented in Figures A.1, A.3, and A.5. When all causal variants are rare, the empirical type I error results are presented in Tables A.2, A.4, and A.6, and the empirical power comparison are presented in Figures A.2, A.4, and A.6.

Three sets of parameters are used in the simulations:

1. In Tables A.1 and A.2 and Figures A.1 and A.2, the order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$.

2. In Tables A.3 and A.4 and Figures A.3 and A.4, the order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$.

3. In Tables A.5 and A.6 and Figures A.5 and A.6, the order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$.

For the first two sets of parameters, 12 independent randomly seeded simulations were implemented to calculate the empirical type I error rates: 10^5 phenotype-genotype datasets in each simulation were generated to fit the proposed Cox models and to calculate the test statistics and related p-values for each combination of a sample size and a censoring scheme. The total number of simulations is 1.2×10^6 in Tables A.1 and A.2. However, the computation of some simulations can not go through due to the failure of model convergence. Thus, the total number of simulations ranges from 10^6 (or $600,000$) to 1.2×10^6 in Table A.3 (or Table A.4). After the simulations were complete, an empirical type I error rate was calculated as the proportion of the total p-values which were smaller than a given α level.

For the third set of parameters, 24 independent randomly seeded simulations were implemented to calculate the empirical type I error rates: 10^5 phenotype-genotype datasets in each simulation were generated to fit the proposed Cox models and to calculate the test statistics and related p-values for each combination of a sample size and a censoring scheme. However, the computation of some simulations can not go through due to the failure of model convergence. Thus, the total number of simulations ranges from 0 to 2.4×10^6. After the simulations were complete, an empirical type I error rate was calculated as the proportion of the total p-values which were smaller than a given α level. If the total number of simulations is 0, no type I error rate is available.

The empirical power levels in Figures A.1 and A.2 are slightly lower than those of Figures 1 and 2. On the other hand, the empirical power levels in Figures Figures A.3, A.4, A.5, and A.6 are similar to those of Figures 1 and 2. Therefore, the range of parameters $8 \leq K = K_\beta \leq 13$ should be fine to use in fitting the proposed Cox models, in terms of controlling type I error rates well and good power.

Table A.1: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When All Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014)

Sample Size n	**The Censoring Scheme**	**Number of Simulations**	**Nominal Level** α	**Cox FR LRT Statistics**				**Cox SKAT LRT**
				Basis of both GVF and $\beta(u)$		**Basis of beta-Smooth Only**		
				B-sp Basis	**Fourier Basis**	**B-sp Basis**	**Fourier Basis**	
1,000	∞	1,200,000	0.05	0.05296	0.05309	0.05296	0.05309	0.06914
			10^{-3}	0.00111	0.00110	0.00111	0.00110	0.00182
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00021
			10^{-5}	1.166×10^{-5}	1.58×10^{-5}	1.17×10^{-5}	1.58×10^{-5}	2.75×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05322	0.05346	0.05322	0.05346	0.07243
			10^{-3}	0.00114	0.00117	0.00114	0.00117	0.00185
			10^{-4}	0.00012	0.00013	0.00012	0.00013	0.00019
			10^{-5}	1.08×10^{-5}	1.25×10^{-5}	1.08×10^{-5}	1.25×10^{-5}	1.50×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05360	0.05363	0.05360	0.05363	0.07733
			10^{-3}	0.00112	0.00120	0.00112	0.00120	0.00199
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00021
			10^{-5}	1.50×10^{-5}	9.17×10^{-6}	1.50×10^{-5}	9.17×10^{-6}	1.83×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05447	0.05441	0.05447	0.05441	0.08145
			10^{-3}	0.00123	0.00119	0.00123	0.00119	0.00196
			10^{-4}	0.00014	0.00012	0.00014	0.00012	0.00021
			10^{-5}	1.33×10^{-5}	1.67×10^{-5}	1.33×10^{-5}	1.67×10^{-5}	1.83×10^{-5}
2,000	∞	1,200,000	0.05	0.05181	0.05194	0.05181	0.05194	0.06213
			10^{-3}	0.00111	0.00109	0.00111	0.00109	0.00142
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00016
			10^{-5}	1.58×10^{-5}	1.42×10^{-5}	1.58×10^{-5}	1.42×10^{-5}	2.08×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05155	0.05173	0.05155	0.05173	0.06411
			10^{-3}	0.00110	0.00107	0.00110	0.00107	0.00153
			10^{-4}	0.00014	0.00013	0.00014	0.00013	0.00015
			10^{-5}	1.58×10^{-5}	1.17×10^{-5}	1.58×10^{-5}	1.17×10^{-5}	2.33×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05169	0.05193	0.05169	0.05193	0.06785
			10^{-3}	0.00109	0.00106	0.00109	0.00106	0.00156
			10^{-4}	0.00012	0.00010	0.00012	0.00010	0.00016
			10^{-5}	1.92×10^{-5}	1.42×10^{-5}	1.92×10^{-5}	1.42×10^{-5}	2.58×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05232	0.05174	0.05232	0.05174	0.07222
			10^{-3}	0.00113	0.00115	0.00113	0.00115	0.00164
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00017
			10^{-5}	1.25×10^{-5}	1.17×10^{-5}	1.25×10^{-5}	1.17×10^{-5}	1.25×10^{-5}
3,000	∞	1,200,000	0.05	0.05118	0.05148	0.05118	0.05148	0.05969
			10^{-3}	0.00102	0.00106	0.00102	0.00106	0.00137
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00012
			10^{-5}	9.17×10^{-6}	7.50×10^{-6}	9.17×10^{-6}	7.50×10^{-6}	1.67×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05126	0.05120	0.05126	0.05120	0.06066
			10^{-3}	0.00109	0.00103	0.00109	0.00103	0.00139
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00016
			10^{-5}	9.17×10^{-6}	1.08×10^{-5}	9.17×10^{-6}	1.08×10^{-5}	1.83×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05120	0.05131	0.05120	0.05131	0.06287
			10^{-3}	0.00104	0.00104	0.00104	0.00104	0.00145
			10^{-4}	0.00010	0.00009	0.00010	0.00009	0.00018
			10^{-5}	1.17×10^{-5}	1.17×10^{-5}	1.17×10^{-5}	1.17×10^{-5}	1.83×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05135	0.05136	0.05135	0.05136	0.06735
			10^{-3}	0.00104	0.00106	0.00104	0.00106	0.00156
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00016
			10^{-5}	1.25×10^{-5}	1.00×10^{-5}	1.25×10^{-5}	1.00×10^{-5}	1.33×10^{-5}
4,000	∞	1,200,000	0.05	0.05041	0.05061	0.05041	0.05061	0.05753
			10^{-3}	0.00102	0.00106	0.00102	0.00106	0.00127
			10^{-4}	0.00009	0.00010	0.00009	0.00010	0.00014
			10^{-5}	9.17×10^{-6}	1.00×10^{-5}	9.17×10^{-6}	1.00×10^{-5}	1.41×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05076	0.05081	0.05076	0.05081	0.05863
			10^{-3}	0.00105	0.00106	0.00105	0.00106	0.00131
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00014
			10^{-5}	1.25×10^{-5}	5.83×10^{-6}	1.25×10^{-5}	5.83×10^{-6}	1.58×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05095	0.05088	0.05095	0.05088	0.06078
			10^{-3}	0.00101	0.00104	0.00101	0.00104	0.00133
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00012
			10^{-5}	8.33×10^{-6}	7.50×10^{-6}	8.33×10^{-6}	7.50×10^{-6}	1.58×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05133	0.05106	0.05133	0.05106	0.06455
			10^{-3}	0.00105	0.00105	0.00105	0.00105	0.00136
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00014
			10^{-5}	5.00×10^{-6}	6.67×10^{-6}	5.00×10^{-6}	6.67×10^{-6}	1.75×10^{-5}

Table A.2: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When Only Rare Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014).

Sample Size n	**The Censoring Scheme**	**Number of Simulations**	**Nominal Level** α	**Cox FR LRT Statistics**				**Cox SKAT LRT**
				Basis of both GVF and $\beta(u)$		**Basis of beta-Smooth Only**		
				B-sp Basis	**Fourier Basis**	**B-sp Basis**	**Fourier Basis**	
1,000	∞	1,200,000	0.05	0.05393	0.05425	0.05393	0.05425	0.07166
			10^{-3}	0.00120	0.00117	0.00120	0.00117	0.00196
			10^{-4}	0.00013	0.00012	0.00013	0.00012	0.00022
			10^{-5}	1.17×10^{-5}	6.67×10^{-6}	1.17×10^{-5}	6.67×10^{-6}	2.75×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05461	0.05480	0.05461	0.05480	0.07489
			10^{-3}	0.00115	0.00118	0.00115	0.00118	0.00200
			10^{-4}	0.00013	0.00014	0.00013	0.00014	0.00022
			10^{-5}	1.50×10^{-5}	1.92×10^{-5}	1.50×10^{-5}	1.92×10^{-5}	1.58×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05610	0.05561	0.05610	0.05561	0.08054
			10^{-3}	0.00125	0.00123	0.00125	0.00123	0.00216
			10^{-4}	0.00013	0.00014	0.00013	0.00014	0.00022
			10^{-5}	1.33×10^{-5}	1.75×10^{-5}	1.33×10^{-5}	1.75×10^{-5}	2.33×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05917	0.05808	0.05918	0.05808	0.08462
			10^{-3}	0.00134	0.00134	0.00134	0.00134	0.00209
			10^{-4}	0.00012	0.00015	0.00012	0.00015	0.00023
			10^{-5}	1.50×10^{-5}	1.42×10^{-5}	1.50×10^{-5}	1.42×10^{-5}	2.00×10^{-5}
2,000	∞	1,200,000	0.05	0.05196	0.05208	0.05196	0.05208	0.06348
			10^{-3}	0.00104	0.00102	0.00104	0.00102	0.00150
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00016
			10^{-5}	1.58×10^{-5}	9.17×10^{-6}	1.58×10^{-5}	9.17×10^{-6}	2.08×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05254	0.05238	0.05254	0.05238	0.06542
			10^{-3}	0.00109	0.00111	0.00109	0.00111	0.00161
			10^{-4}	0.00012	0.00013	0.00012	0.00013	0.00017
			10^{-5}	1.75×10^{-5}	1.42×10^{-5}	1.75×10^{-5}	1.42×10^{-5}	2.33×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05310	0.05315	0.05310	0.05315	0.06932
			10^{-3}	0.00115	0.00109	0.00115	0.00109	0.00163
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00017
			10^{-5}	1.00×10^{-5}	1.25×10^{-5}	1.00×10^{-5}	1.25×10^{-5}	2.75×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05469	0.05404	0.05469	0.05404	0.07463
			10^{-3}	0.00122	0.00117	0.00122	0.00117	0.00173
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00018
			10^{-5}	1.17×10^{-5}	9.17×10^{-6}	1.17×10^{-5}	9.17×10^{-6}	1.08×10^{-5}
3,000	∞	1,200,000	0.05	0.05182	0.05191	0.05182	0.05191	0.06058
			10^{-3}	0.00106	0.00108	0.00106	0.00108	0.00140
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00015
			10^{-5}	6.67×10^{-6}	8.33×10^{-6}	6.67×10^{-6}	8.33×10^{-6}	1.50×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05151	0.05144	0.05151	0.05144	0.06161
			10^{-3}	0.00104	0.00104	0.00104	0.00104	0.00146
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00015
			10^{-5}	1.50×10^{-5}	1.33×10^{-5}	1.50×10^{-5}	1.33×10^{-5}	2.00×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05230	0.05190	0.05230	0.05190	0.06425
			10^{-3}	0.00112	0.00110	0.00112	0.00110	0.00150
			10^{-4}	0.00010	0.00012	0.00010	0.00012	0.00019
			10^{-5}	1.25×10^{-5}	1.33×10^{-5}	1.25×10^{-5}	1.33×10^{-5}	1.92×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05353	0.05279	0.05353	0.05279	0.06887
			10^{-3}	0.00111	0.00108	0.00111	0.00108	0.00159
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00015
			10^{-5}	1.58×10^{-5}	1.25×10^{-5}	1.58×10^{-5}	1.25×10^{-5}	1.33×10^{-5}
4,000	∞	1,200,000	0.05	0.05109	0.05130	0.05109	0.05130	0.05825
			10^{-3}	0.00104	0.00108	0.00104	0.00108	0.00130
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00014
			10^{-5}	1.58×10^{-5}	1.50×10^{-5}	1.58×10^{-5}	1.50×10^{-5}	1.83×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05117	0.05089	0.05117	0.05089	0.05932
			10^{-3}	0.00102	0.00108	0.00102	0.00108	0.00130
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00015
			10^{-5}	1.42×10^{-5}	1.42×10^{-5}	1.42×10^{-5}	1.42×10^{-5}	1.42×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05171	0.05150	0.05171	0.05150	0.06173
			10^{-3}	0.00103	0.00106	0.00103	0.00106	0.00138
			10^{-4}	0.00010	0.00012	0.00010	0.00012	0.00012
			10^{-5}	1.08×10^{-5}	8.33×10^{-6}	1.08×10^{-5}	8.33×10^{-6}	1.17×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05215	0.05207	0.05215	0.05207	0.06585
			10^{-3}	0.00111	0.00108	0.00111	0.00108	0.00140
			10^{-4}	0.00010	0.00010	0.00010	0.00010	0.00014
			10^{-5}	1.33×10^{-5}	1.50×10^{-5}	1.33×10^{-5}	1.50×10^{-5}	1.58×10^{-5}

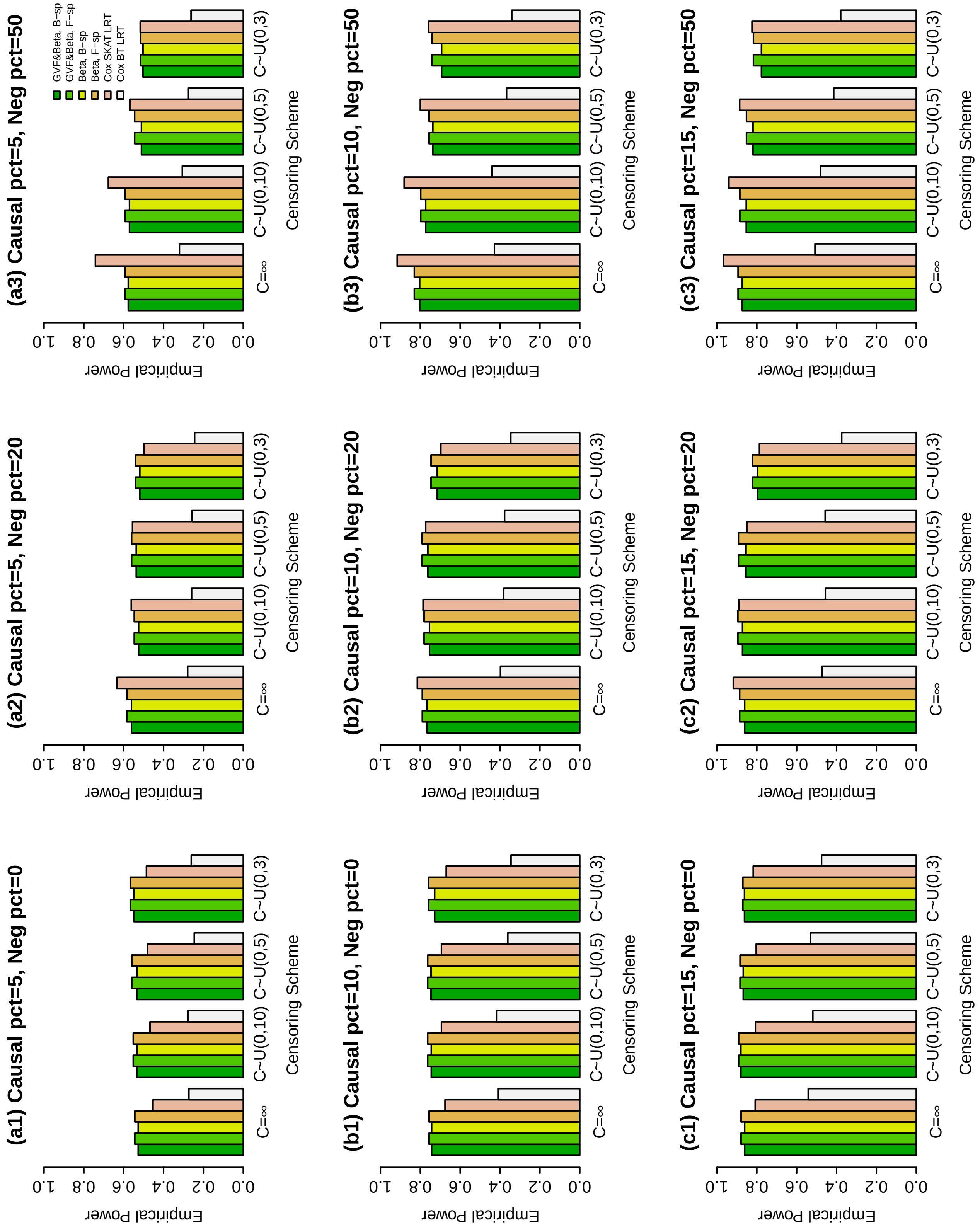

Figure A.1: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$.

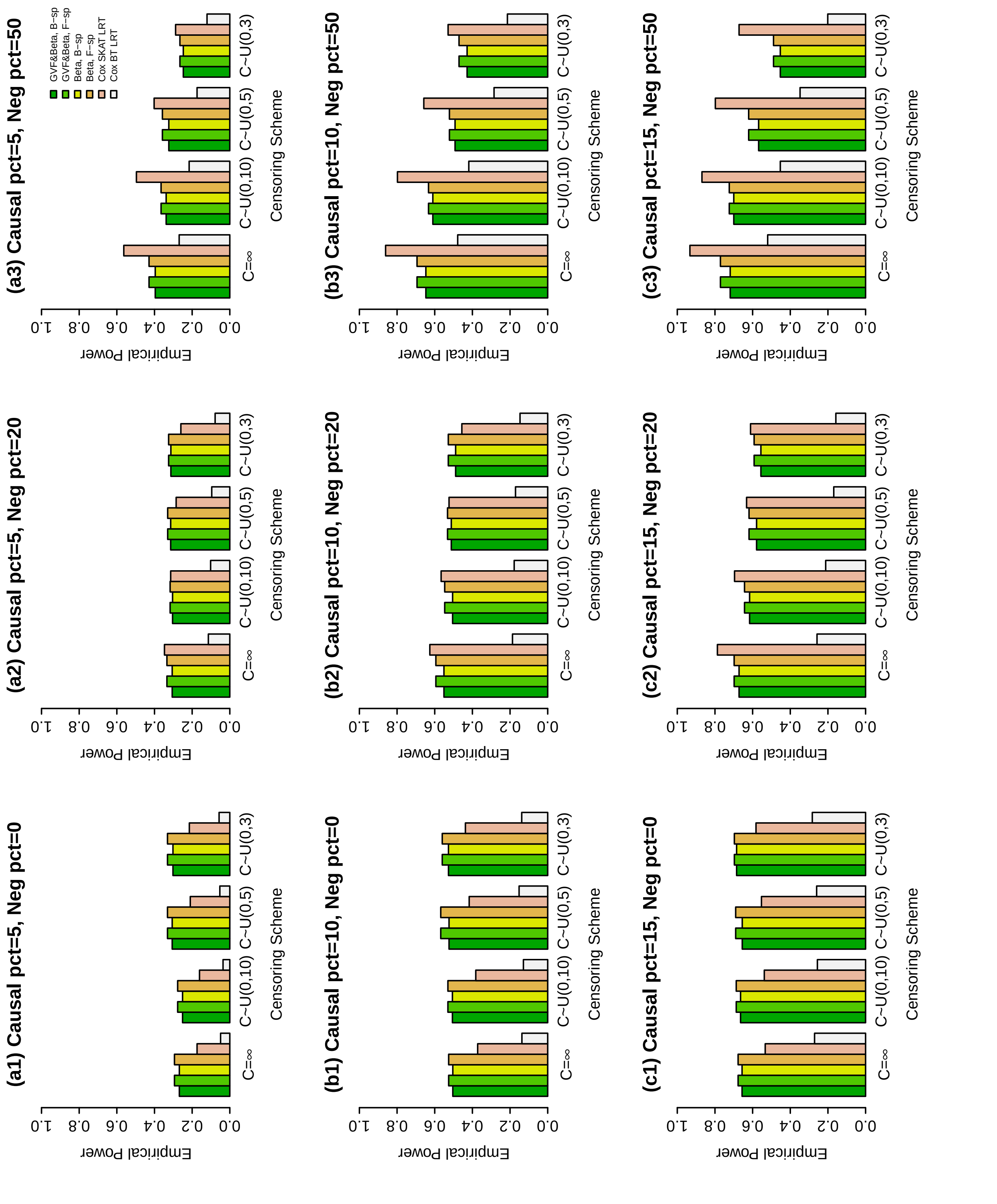

Figure A.2: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$.

Table A.3: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When All Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014)

Sample Size n	**The Censoring Scheme**	**Number of Simulations**	**Nominal Level** α	**Cox FR LRT Statistics**				**Cox SKAT LRT**
				Basis of both GVF and $\beta(u)$		**Basis of beta-Smooth Only**		
				B-sp Basis	**Fourier Basis**	**B-sp Basis**	**Fourier Basis**	
1,000	∞	1,200,000	0.05	0.05415	0.05442	0.05415	0.05442	0.06914
			10^{-3}	0.00116	0.00115	0.00116	0.00115	0.00182
			10^{-4}	0.00013	0.00012	0.00013	0.00012	0.00021
			10^{-5}	1.17×10^{-5}	9.17×10^{-6}	1.17×10^{-5}	9.17×10^{-6}	2.75×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05475	0.05471	0.05475	0.05471	0.07243
			10^{-3}	0.00119	0.00119	0.00119	0.00119	0.00185
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00019
			10^{-5}	1.33×10^{-5}	9.17×10^{-6}	1.33×10^{-5}	9.17×10^{-6}	1.50×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05555	0.05485	0.05556	0.05485	0.07733
			10^{-3}	0.00122	0.00116	0.00122	0.00116	0.00199
			10^{-4}	0.00013	0.00011	0.00014	0.00011	0.00021
			10^{-5}	1.67×10^{-5}	1.17×10^{-5}	1.75×10^{-5}	1.17×10^{-5}	1.83×10^{-5}
	$U(0,3)$	1,100,000	0.05	0.05758	0.05629	0.05761	0.05629	0.08144
			10^{-3}	0.00131	0.00126	0.00131	0.00126	0.00195
			10^{-4}	0.00015	0.00014	0.00015	0.00014	0.00021
			10^{-5}	1.91×10^{-5}	1.45×10^{-5}	1.91×10^{-5}	1.45×10^{-5}	1.91×10^{-5}
2,000	∞	1,200,000	0.05	0.05263	0.05252	0.05263	0.05252	0.06213
			10^{-3}	0.00112	0.00110	0.00112	0.00110	0.00142
			10^{-4}	0.00011	0.00013	0.00011	0.00013	0.00016
			10^{-5}	9.17×10^{-6}	1.42×10^{-5}	9.17×10^{-6}	1.42×10^{-5}	2.08×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05220	0.05215	0.05220	0.05215	0.06411
			10^{-3}	0.00112	0.00108	0.00112	0.00108	0.00153
			10^{-4}	0.00012	0.00013	0.00012	0.00013	0.00015
			10^{-5}	1.75×10^{-5}	1.33×10^{-5}	1.75×10^{-5}	1.33×10^{-5}	2.33×10^{-5}
	$U(0,5)$	1,100,000	0.05	0.05298	0.05271	0.05298	0.05271	0.06786
			10^{-3}	0.00115	0.00107	0.00115	0.00107	0.00155
			10^{-4}	0.00013	0.00010	0.00013	0.00010	0.00016
			10^{-5}	1.36×10^{-5}	1.27×10^{-5}	1.36×10^{-5}	1.27×10^{-5}	2.72×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05381	0.05294	0.05382	0.05294	0.07222
			10^{-3}	0.00121	0.00113	0.00121	0.00113	0.00164
			10^{-4}	0.00014	0.00012	0.00014	0.00012	0.00017
			10^{-5}	1.33×10^{-5}	1.33×10^{-5}	1.33×10^{-5}	1.33×10^{-5}	1.25×10^{-5}
3,000	∞	1,200,000	0.05	0.05152	0.05148	0.05152	0.05148	0.05969
			10^{-3}	0.00107	0.00106	0.00107	0.00106	0.00137
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00012
			10^{-5}	1.00×10^{-5}	7.50×10^{-6}	1.00×10^{-5}	7.50×10^{-6}	1.67×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05188	0.05144	0.05188	0.05144	0.06066
			10^{-3}	0.00105	0.00106	0.00105	0.00106	0.00139
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00016
			10^{-5}	5.83×10^{-6}	1.08×10^{-5}	5.83×10^{-6}	1.08×10^{-5}	1.83×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05194	0.05131	0.05194	0.05131	0.06287
			10^{-3}	0.00109	0.00105	0.00109	0.00105	0.00145
			10^{-4}	0.00009	0.00011	0.00009	0.00011	0.00018
			10^{-5}	9.17×10^{-6}	1.00×10^{-5}	9.17×10^{-6}	1.00×10^{-5}	1.83×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05273	0.05195	0.05273	0.05195	0.06735
			10^{-3}	0.00108	0.00110	0.00108	0.00110	0.00156
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00016
			10^{-5}	1.58×10^{-5}	1.17×10^{-5}	1.58×10^{-5}	1.17×10^{-5}	1.33×10^{-5}
4,000	∞	1,200,000	0.05	0.05087	0.05103	0.05087	0.05103	0.05753
			10^{-3}	0.00104	0.00106	0.00104	0.00106	0.00127
			10^{-4}	0.00010	0.00009	0.00010	0.00009	0.00014
			10^{-5}	1.00×10^{-5}	1.33×10^{-5}	1.00×10^{-5}	1.33×10^{-5}	1.42×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05109	0.05106	0.05109	0.05106	0.05863
			10^{-3}	0.00105	0.00108	0.00105	0.00108	0.00131
			10^{-4}	0.00014	0.00012	0.00014	0.00012	0.00014
			10^{-5}	8.33×10^{-6}	6.67×10^{-6}	8.33×10^{-6}	6.67×10^{-6}	1.58×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05148	0.05124	0.05148	0.05124	0.06078
			10^{-3}	0.00106	0.00106	0.00106	0.00106	0.00133
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00012
			10^{-5}	8.33×10^{-6}	7.50×10^{-6}	8.33×10^{-6}	7.50×10^{-6}	1.58×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05198	0.05153	0.05198	0.05153	0.06455
			10^{-3}	0.00108	0.00109	0.00108	0.00109	0.00136
			10^{-4}	0.00010	0.00010	0.00010	0.00010	0.00014
			10^{-5}	8.33×10^{-6}	7.50×10^{-6}	8.33×10^{-6}	7.50×10^{-6}	1.75×10^{-5}

Table A.4: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When Only Rare Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014).

Sample Size n	**The Censoring Scheme**	**Number of Simulations**	**Nominal Level** α	**Cox FR LRT Statistics**				**Cox SKAT LRT**
				Basis of both GVF and $\beta(u)$		**Basis of beta-Smooth Only**		
				B-sp Basis	**Fourier Basis**	**B-sp Basis**	**Fourier Basis**	
1,000	∞	1,100,000	0.05	0.05602	0.05629	0.05602	0.05629	0.07169
			10^{-3}	0.00126	0.00128	0.00126	0.00128	0.00195
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00022
			10^{-5}	9.09×10^{-6}	1.72×10^{-5}	9.09×10^{-6}	1.72×10^{-5}	2.63×10^{-5}
	$U(0,10)$	900,000	0.05	0.05738	0.05675	0.05739	0.05675	0.07505
			10^{-3}	0.00125	0.00127	0.00125	0.00127	0.00194
			10^{-4}	0.00013	0.00014	0.00013	0.00014	0.00021
			10^{-5}	1.78×10^{-5}	1.89×10^{-5}	1.78×10^{-5}	1.89×10^{-5}	1.56×10^{-5}
	$U(0,5)$	800,000	0.05	0.05950	0.05819	0.05951	0.05819	0.08059
			10^{-3}	0.00136	0.00129	0.00136	0.00129	0.00219
			10^{-4}	0.00016	0.00015	0.00016	0.00015	0.00022
			10^{-5}	2.25×10^{-5}	1.63×10^{-5}	2.25×10^{-5}	1.63×10^{-5}	2.38×10^{-5}
	$U(0,3)$	600,000	0.05	0.06461	0.06237	0.06476	0.06237	0.08430
			10^{-3}	0.00162	0.00154	0.00163	0.00154	0.00205
			10^{-4}	0.00018	0.00016	0.00018	0.00016	0.00020
			10^{-5}	1.67×10^{-5}	2.00×10^{-5}	1.67×10^{-5}	2.00×10^{-5}	2.00×10^{-5}
2,000	∞	1,200,000	0.05	0.05305	0.05299	0.05305	0.05299	0.06348
			10^{-3}	0.00109	0.00108	0.00109	0.00108	0.00150
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00016
			10^{-5}	1.17×10^{-5}	9.17×10^{-6}	1.17×10^{-5}	9.17×10^{-6}	2.08×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05369	0.05344	0.05369	0.05344	0.06542
			10^{-3}	0.00117	0.00118	0.00117	0.00118	0.00161
			10^{-4}	0.00014	0.00011	0.00014	0.00011	0.00017
			10^{-5}	2.00×10^{-5}	1.08×10^{-5}	2.00×10^{-5}	1.08×10^{-5}	2.33×10^{-5}
	$U(0,5)$	1,100,000	0.05	0.05498	0.05432	0.05498	0.05432	0.06926
			10^{-3}	0.00123	0.00115	0.00123	0.00115	0.00163
			10^{-4}	0.00013	0.00012	0.00013	0.00012	0.00017
			10^{-5}	1.27×10^{-5}	1.27×10^{-5}	1.27×10^{-5}	1.27×10^{-5}	2.81×10^{-5}
	$U(0,3)$	100,000	0.05	0.05778	0.05608	0.05778	0.05608	0.07419
			10^{-3}	0.00126	0.00116	0.00126	0.00116	0.00159
			10^{-4}	0.00017	0.00011	0.00017	0.00011	0.00012
			10^{-5}	1.00×10^{-5}	2.00×10^{-5}	1.00×10^{-5}	2.00×10^{-5}	0
3,000	∞	1,200,000	0.05	0.05243	0.05223	0.05243	0.05223	0.06058
			10^{-3}	0.00110	0.00108	0.00110	0.00108	0.00140
			10^{-4}	0.00013	0.00010	0.00013	0.00010	0.00015
			10^{-5}	1.08×10^{-5}	5.83×10^{-6}	1.08×10^{-5}	5.83×10^{-6}	1.50×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05232	0.05189	0.05232	0.05189	0.06161
			10^{-3}	0.00109	0.00109	0.00109	0.00109	0.00146
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00015
			10^{-5}	1.42×10^{-5}	1.17×10^{-5}	1.42×10^{-5}	1.17×10^{-5}	2.00×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05333	0.05256	0.05333	0.05256	0.06425
			10^{-3}	0.00111	0.00110	0.00111	0.00110	0.00150
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00019
			10^{-5}	1.67×10^{-5}	1.25×10^{-5}	1.67×10^{-5}	1.25×10^{-5}	1.92×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05563	0.05398	0.05564	0.05398	0.06887
			10^{-3}	0.00119	0.00117	0.00119	0.00117	0.00159
			10^{-4}	0.00011	0.00011	0.00011	0.00011	0.00015
			10^{-5}	1.17×10^{-5}	2.50×10^{-5}	1.17×10^{-5}	2.50×10^{-5}	1.33×10^{-5}
4,000	∞	1,200,000	0.05	0.05161	0.05148	0.05161	0.05148	0.05825
			10^{-3}	0.00109	0.00111	0.00109	0.00111	0.00130
			10^{-4}	0.00012	0.00013	0.00012	0.00013	0.00014
			10^{-5}	1.67×10^{-5}	1.83×10^{-5}	1.67×10^{-5}	1.83×10^{-5}	1.83×10^{-5}
	$U(0,10)$	1,200,000	0.05	0.05168	0.05131	0.05168	0.05131	0.05932
			10^{-3}	0.00103	0.00107	0.00103	0.00107	0.00130
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00015
			10^{-5}	1.25×10^{-5}	1.67×10^{-5}	1.25×10^{-5}	1.67×10^{-5}	1.42×10^{-5}
	$U(0,5)$	1,200,000	0.05	0.05270	0.05227	0.05270	0.05227	0.06173
			10^{-3}	0.00110	0.00108	0.00110	0.00108	0.00138
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00012
			10^{-5}	8.33×10^{-6}	7.50×10^{-6}	8.33×10^{-6}	7.50×10^{-6}	1.17×10^{-5}
	$U(0,3)$	1,200,000	0.05	0.05379	0.05326	0.05379	0.05326	0.06585
			10^{-3}	0.00114	0.00112	0.00114	0.00112	0.00140
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00014
			10^{-5}	1.58×10^{-5}	1.08×10^{-5}	1.58×10^{-5}	1.08×10^{-5}	1.58×10^{-5}

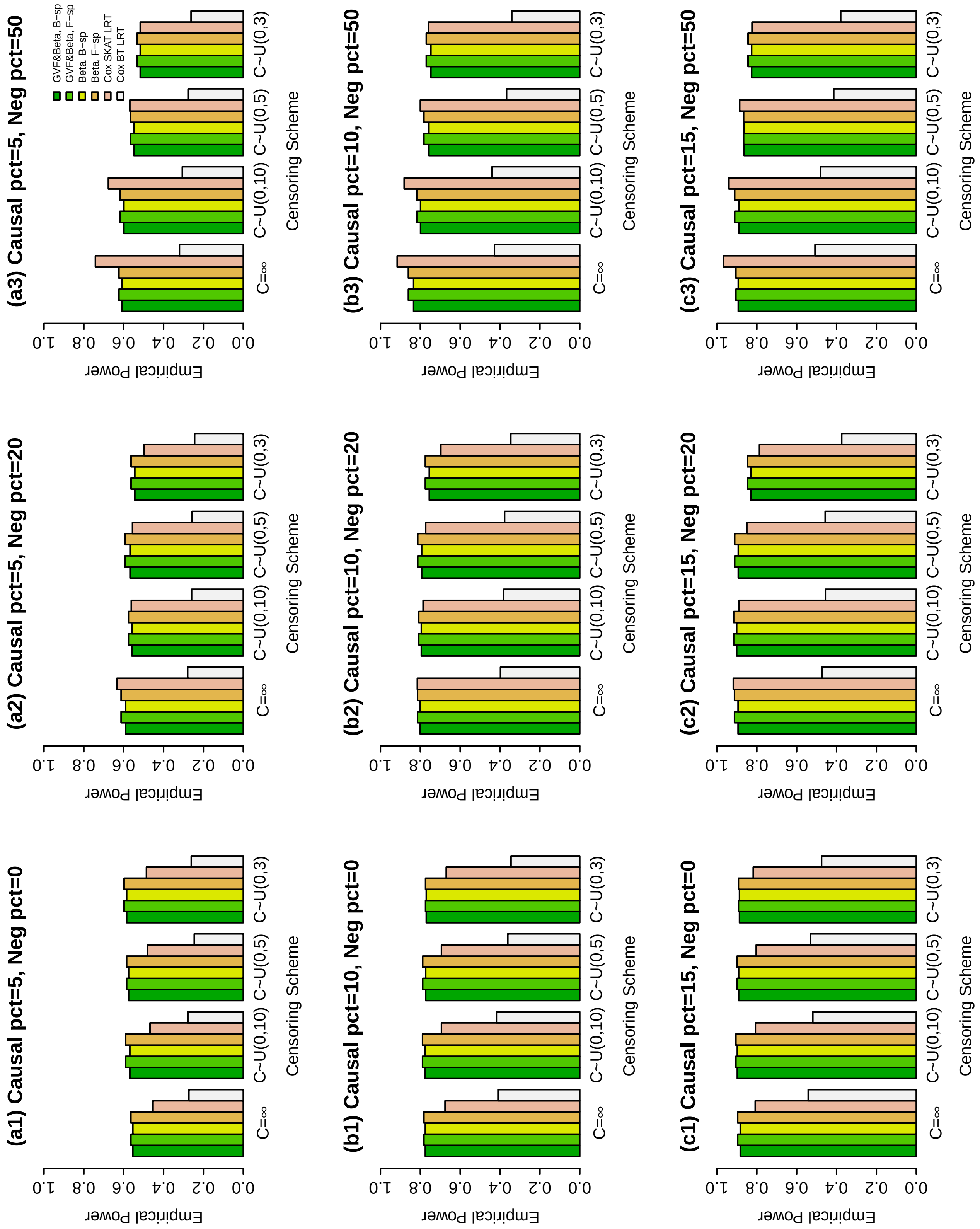

Figure A.3: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$.

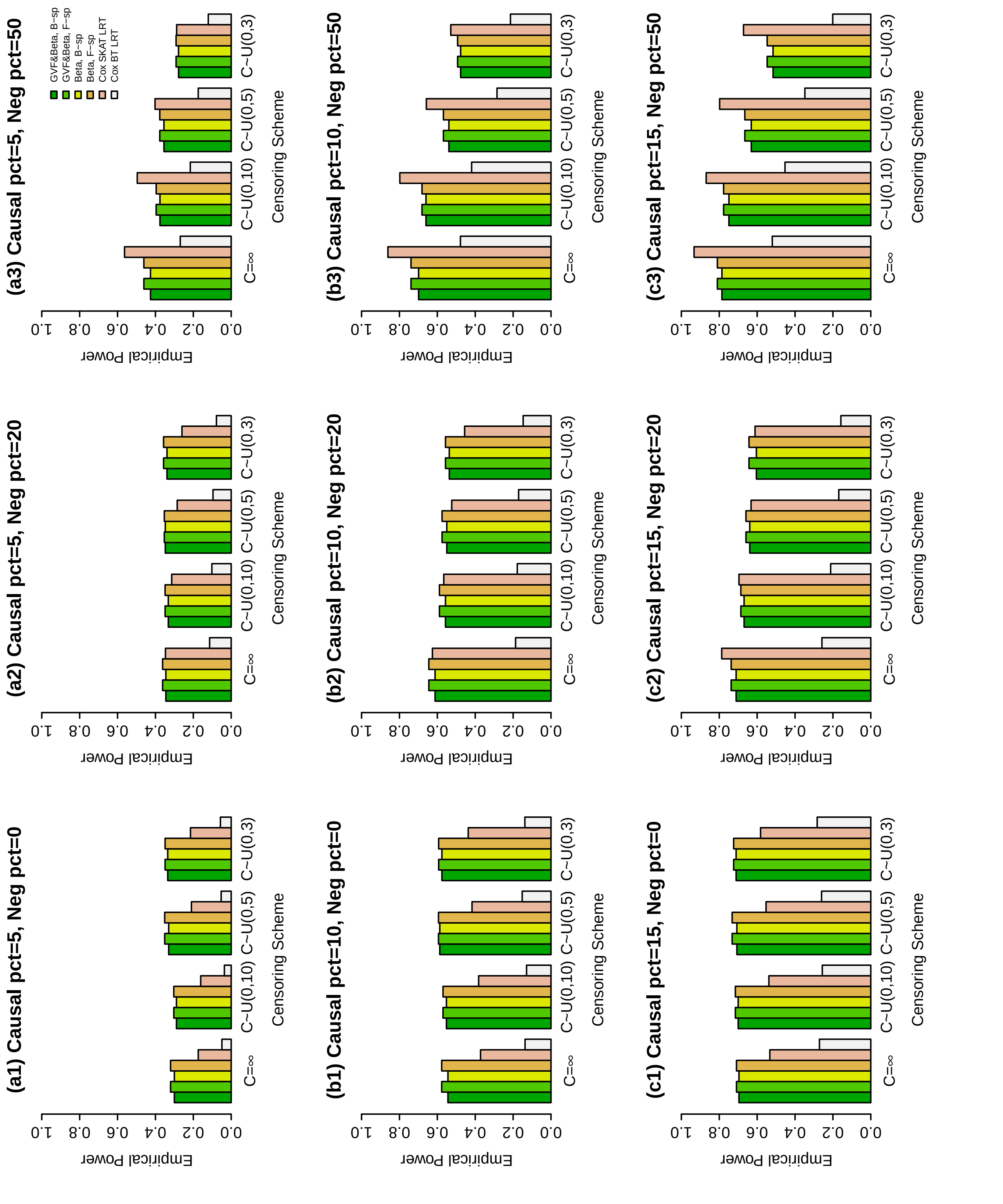

Figure A.4: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$.

Table A.5: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When All Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014)

Sample Size n	The Censoring Scheme	Number of Simulations	Nominal Level α	Cox FR LRT Statistics: Basis of both GVF and $\beta(u)$		Cox FR LRT Statistics: Basis of beta-Smooth Only		Cox SKAT LRT
				B-sp Basis	Fourier Basis	B-sp Basis	Fourier Basis	
1,000	∞	900,000	0.05	0.05748	0.05719	0.05752	0.05719	0.06950
			10^{-3}	0.00127	0.00127	0.00127	0.00127	0.00185
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00021
			10^{-5}	1.00×10^{-5}	1.00×10^{-5}	1.00×10^{-5}	1.00×10^{-5}	2.67×10^{-5}
	$U(0,10)$	100,000	0.05	0.05906	0.05809	0.05911	0.05809	0.07252
			10^{-3}	0.00136	0.00142	0.00137	0.00142	0.00187
			10^{-4}	0.00015	0.00013	0.00015	0.00013	0.00015
			10^{-5}	2.00×10^{-5}	1.00×10^{-5}	2.00×10^{-5}	1.00×10^{-5}	1.00×10^{-5}
2,000	∞	1,800,000	0.05	0.05390	0.05347	0.05390	0.05347	0.06203
			10^{-3}	0.00116	0.00116	0.00116	0.00116	0.00146
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00016
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00016
			10^{-5}	1.88×10^{-5}	1.38×10^{-5}	1.88×10^{-5}	1.38×10^{-5}	2.11×10^{-5}
	$U(0,10)$	800,000	0.05	0.05463	0.05405	0.05464	0.05405	0.06406
			10^{-3}	0.00121	0.00122	0.00121	0.00122	0.00152
			10^{-4}	0.00012	0.00010	0.00012	0.00010	0.00016
			10^{-5}	1.25×10^{-5}	8.75×10^{-6}	1.25×10^{-5}	8.75×10^{-6}	1.75×10^{-5}
	$U(0,5)$	400,000	0.05	0.05610	0.05516	0.05614	0.05516	0.06826
			10^{-3}	0.00119	0.00115	0.00119	0.00115	0.00150
			10^{-4}	0.00013	0.00012	0.00013	0.00012	0.00018
			10^{-5}	1.00×10^{-5}	0	1.00×10^{-5}	0	2.25×10^{-5}
3,000	∞	2,200,000	0.05	0.05270	0.05233	0.05270	0.05233	0.05952
			10^{-3}	0.00112	0.00110	0.00112	0.00110	0.00137
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00013
			10^{-5}	1.27×10^{-5}	1.36×10^{-5}	1.27×10^{-5}	1.36×10^{-5}	1.86×10^{-5}
	$U(0,10)$	2,100,000	0.05	0.05289	0.05256	0.05289	0.05256	0.06040
			10^{-3}	0.00113	0.00111	0.00113	0.00111	0.00136
			10^{-4}	0.00011	0.00010	0.00011	0.00010	0.00014
			10^{-5}	1.14×10^{-5}	1.19×10^{-5}	1.14×10^{-5}	1.19×10^{-5}	1.38×10^{-5}
	$U(0,5)$	1,900,000	0.05	0.05411	0.05309	0.05412	0.05309	0.06313
			10^{-3}	0.00117	0.00113	0.00117	0.00113	0.00145
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00016
			10^{-5}	1.00×10^{-5}	1.10×10^{-5}	1.00×10^{-5}	1.10×10^{-5}	1.36×10^{-5}
	$U(0,3)$	600,000	0.05	0.05641	0.05476	0.05646	0.05476	0.06780
			10^{-3}	0.00119	0.00110	0.00119	0.00110	0.00155
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00017
			10^{-5}	1.00×10^{-5}	5.00×10^{-6}	1.00×10^{-5}	5.00×10^{-6}	1.33×10^{-5}
4,000	∞	2,200,000	0.05	0.05188	0.05172	0.05188	0.05172	0.05752
			10^{-3}	0.00107	0.00106	0.00107	0.00106	0.00127
			10^{-4}	0.00011	0.00012	0.00011	0.00012	0.00015
			10^{-5}	1.40×10^{-5}	1.50×10^{-5}	1.40×10^{-5}	1.50×10^{-5}	1.13×10^{-5}
	$U(0,10)$	2,400,000	0.05	0.05217	0.05182	0.05217	0.05182	0.05861
			10^{-3}	0.00114	0.00113	0.00114	0.00113	0.00132
			10^{-4}	0.00013	0.00011	0.00013	0.00011	0.00014
			10^{-5}	8.75×10^{-6}	1.62×10^{-5}	8.75×10^{-6}	1.62×10^{-5}	1.67×10^{-5}
	$U(0,5)$	1,600,000	0.05	0.05282	0.05225	0.05283	0.05225	0.06079
			10^{-3}	0.00110	0.00107	0.00110	0.00107	0.00134
			10^{-4}	0.00010	0.00012	0.00010	0.00012	0.00014
			10^{-5}	1.25×10^{-5}	1.18×10^{-5}	1.25×10^{-5}	1.18×10^{-5}	1.56×10^{-5}
	$U(0,3)$	1,400,000	0.05	0.05435	0.05318	0.05437	0.05318	0.06459
			10^{-3}	0.00124	0.00120	0.00125	0.00120	0.00138
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00014
			10^{-5}	1.14×10^{-5}	1.07×10^{-5}	1.14×10^{-5}	1.07×10^{-5}	1.85×10^{-5}

Table A.6: **Empirical Type I Error Rates of the Cox FR LRT Statistics and Cox SKAT LRT, When Only Rare Variants in 6 kb Regions Were Used to Generate Genotype Data**. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$. The results of Cox SKAT LRT were from R SeqMeta package by Chen et al. (2014).

Sample Size n	**The Censoring Scheme**	**Number of Simulations**	**Nominal Level** α	**Cox FR LRT Statistics**				**Cox SKAT LRT**
				Basis of both GVF and $\beta(u)$		**Basis of beta-Smooth Only**		
				B-sp Basis	**Fourier Basis**	**B-sp Basis**	**Fourier Basis**	
1,000	∞	100,000	0.05	0.06052	0.06091	0.06060	0.06091	0.07265
			0.01	0.01358	0.01313	0.01360	0.01313	0.017
			10^{-3}	0.00154	0.00156	0.00155	0.00156	0.00184
			10^{-4}	0.00014	0.00014	0.00015	0.00014	0.00024
			10^{-5}	0	0	0	0	1.00×10^{-5}
1,500	∞	400,000	0.05	0.05758	0.05677	0.05758	0.05677	0.06718
			0.01	0.01242	0.01204	0.01242	0.01204	0.01525
			10^{-3}	0.00136	0.00130	0.00136	0.00130	0.00166
			10^{-4}	0.00015	0.00015	0.00015	0.00015	0.00017
			10^{-5}	7.50×10^{-6}	1.00×10^{-5}	7.50×10^{-6}	1.00×10^{-5}	1.50×10^{-5}
2,000	∞	1,300,000	0.05	0.05535	0.05486	0.05535	0.05486	0.06322
			0.01	0.01160	0.01132	0.01160	0.01132	0.01379
			10^{-3}	0.00124	0.00114	0.00124	0.00114	0.00154
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00016
			10^{-5}	1.00×10^{-5}	1.69×10^{-5}	1.00×10^{-5}	1.69×10^{-5}	2.00×10^{-5}
2,500	∞	1,700,000	0.05	0.05473	0.05421	0.05473	0.05421	0.06183
			0.01	0.01144	0.01108	0.01144	0.01108	0.01340
			10^{-3}	0.00117	0.00120	0.00117	0.00120	0.00150
			10^{-4}	0.00013	0.00013	0.00013	0.00013	0.00017
			10^{-5}	1.76×10^{-5}	1.00×10^{-5}	1.76×10^{-5}	1.00×10^{-5}	1.35×10^{-5}
	$U(0, 10)$	600,000	0.05	0.05497	0.05451	0.05497	0.05451	0.06253
			0.01	0.01143	0.01124	0.01143	0.01124	0.01343
			10^{-3}	0.00125	0.00123	0.00125	0.00123	0.00143
			10^{-4}	0.00014	0.00015	0.00014	0.00015	0.00014
			10^{-5}	1.67×10^{-5}	1.67×10^{-5}	1.67×10^{-5}	1.67×10^{-5}	1.83×10^{-5}
3,000	∞	1,900,000	0.05	0.05393	0.05334	0.05393	0.05334	0.06031
			0.01	0.01102	0.01088	0.01102	0.01088	0.01291
			10^{-3}	0.00117	0.00113	0.00117	0.00113	0.00139
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00014
			10^{-5}	8.42×10^{-6}	1.05×10^{-5}	8.42×10^{-6}	1.05×10^{-5}	1.63×10^{-5}
	$U(0, 10)$	1,000,000	0.05	0.05388	0.05329	0.05388	0.05329	0.06133
			0.01	0.01121	0.01095	0.01121	0.01095	0.01296
			10^{-3}	0.00121	0.00117	0.00121	0.00117	0.00142
			10^{-4}	0.00012	0.00013	0.00012	0.00013	0.00015
			10^{-5}	1.40×10^{-5}	2.40×10^{-5}	1.40×10^{-5}	2.40×10^{-5}	1.40×10^{-5}
	$U(0, 5)$	200,000	0.05	0.05474	0.05466	0.05474	0.05466	0.06376
			0.01	0.01139	0.01139	0.01140	0.01139	0.01373
			10^{-3}	0.00119	0.00126	0.00119	0.00126	0.00154
			10^{-4}	0.00016	0.00013	0.00016	0.00013	0.00018
			10^{-5}	2.00×10^{-5}	1.50×10^{-5}	2.00×10^{-5}	1.50×10^{-5}	1.50×10^{-5}
4,000	∞	2,100,000	0.05	0.05271	0.05262	0.05271	0.05262	0.05809
			0.01	0.01076	0.01074	0.01076	0.01074	0.01218
			10^{-3}	0.00111	0.00107	0.00111	0.00107	0.00128
			10^{-4}	0.00012	0.00011	0.00012	0.00011	0.00015
			10^{-5}	1.33×10^{-5}	1.14×10^{-5}	1.33×10^{-5}	1.14×10^{-5}	1.47×10^{-5}
	$U(0, 10)$	1,500,000	0.05	0.05325	0.05289	0.05325	0.05289	0.05914
			0.01	0.01101	0.01074	0.01101	0.01074	0.01244
			10^{-3}	0.00111	0.00111	0.00111	0.00111	0.00131
			10^{-4}	0.00010	0.00011	0.00010	0.00011	0.00014
			10^{-5}	1.26×10^{-5}	1.53×10^{-5}	1.26×10^{-5}	1.53×10^{-5}	1.93×10^{-5}
	$U(0, 5)$	600,000	0.05	0.05456	0.05328	0.05456	0.05328	0.06154
			0.01	0.01103	0.01093	0.01103	0.01093	0.01265
			10^{-3}	0.00122	0.00113	0.00122	0.00113	0.00134
			10^{-4}	0.00012	0.00012	0.00012	0.00012	0.00014
			10^{-5}	1.83×10^{-5}	2.00×10^{-5}	1.83×10^{-5}	2.00×10^{-5}	2.00×10^{-5}

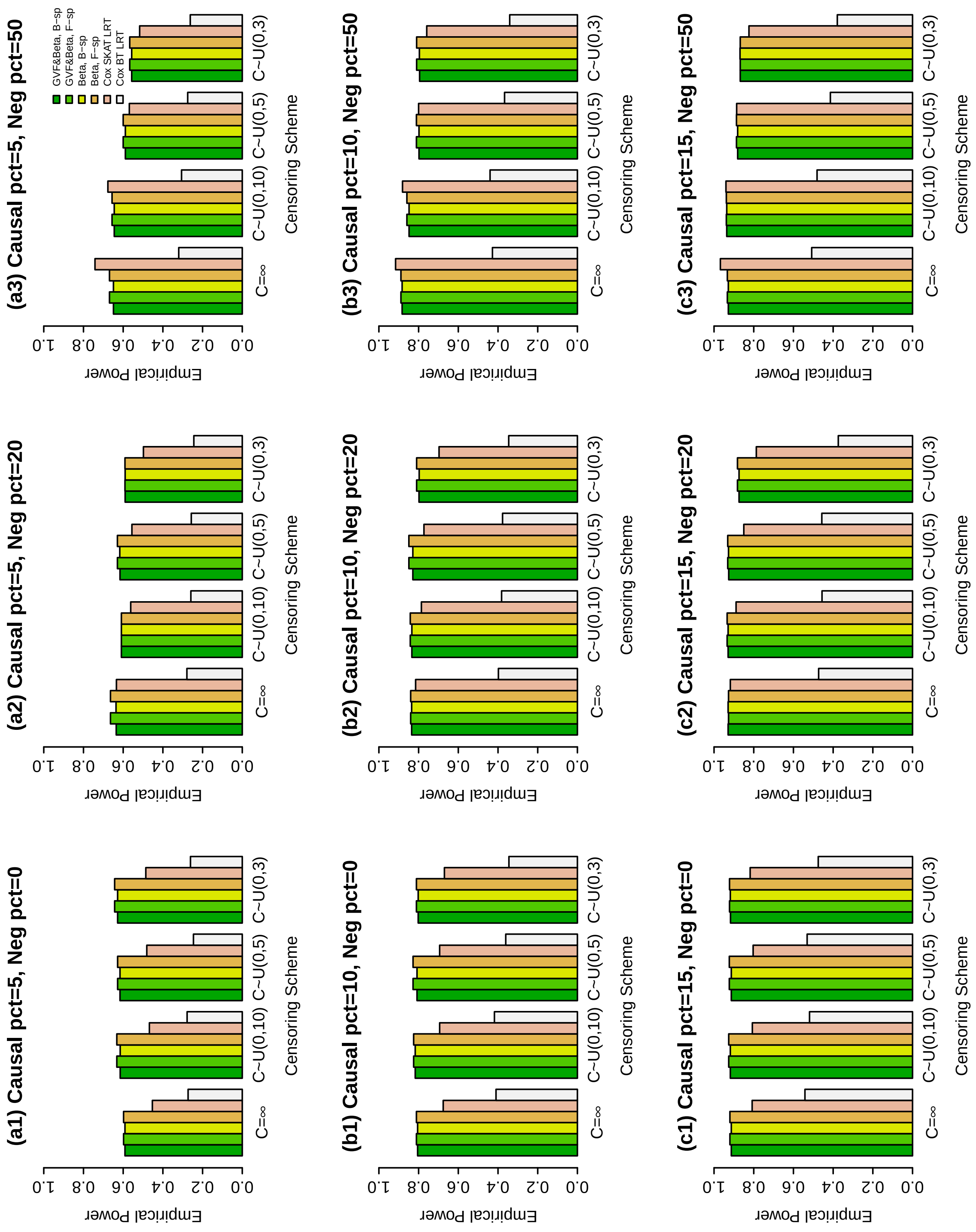

Figure A.5: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$.

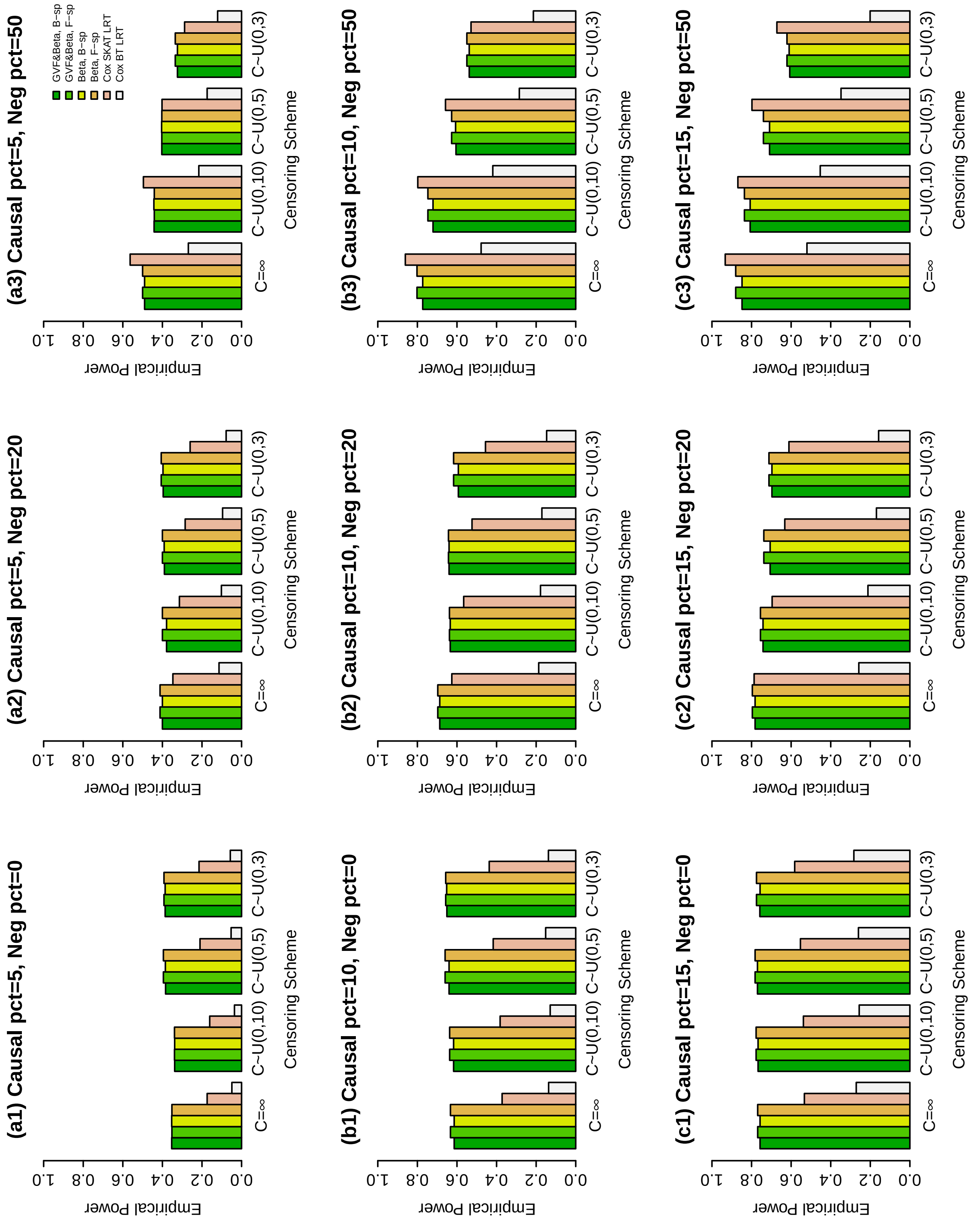

Figure A.6: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$.

Web-based Supplementary Materials for "Gene-based Association Analysis for Censored Traits Via Fixed Effect Functional Regressions"

Appendix B. More Simulation Results

In this **Supplementary Materials**, Appendix B, three more settings were considered: 30%, 40%, and 50% of variants were causal in the 6 kb subregion. When some causal variants are rare and some are common, $c = \log(1.75), \log(1.50)$, and $\log(1.25)$ if 30%, 40%, and 50% of the variants were causal, respectively. When all causal variants are rare, $c = \log(2.5), \log(2.0)$, and $\log(1.5)$ if 30%, 40%, and 50% of the variants were causal, respectively.

In Figures B.1, B.2, B.3, B.4, B.5, B.6, Figures B.7, and B.8, the range of parameters K and K_β are from 6 to 13 as those in the main text and **Supplementary Materials**, Appendix A. If some causal variants are rare and some are common, the Cox FR LRT statistics have higher or similar power as Cox SKAT LRT. If all causal variants are rare, the Cox FR LRT statistics have higher or similar power as Cox SKAT LRT except when 50%/50% causal variants had negative/positive effects. The Cox FR LRT statistics have higher power than Cox BT LRT in all the figures.

The power levels of Cox SKAT LRT are higher than those of Cox BT LRT except when 50% variants are causal and all causal variants had positive effects [graph (c1) in Figures B.1, B.2, B.3, B.4, B.5, B.6, B.7, and B.8]. This is consistent with the results of Chen et al. (2014).

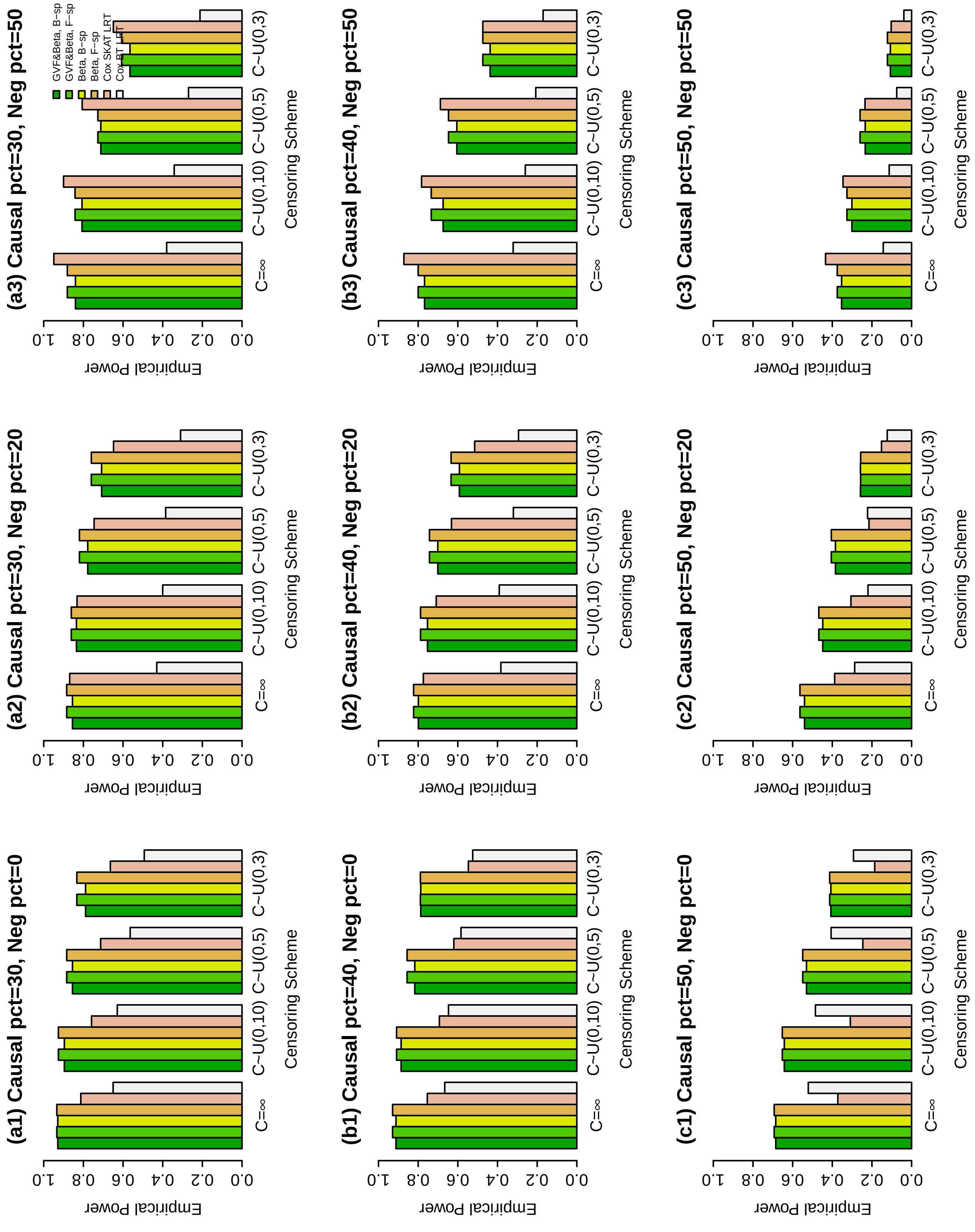

Figure B.1: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$.

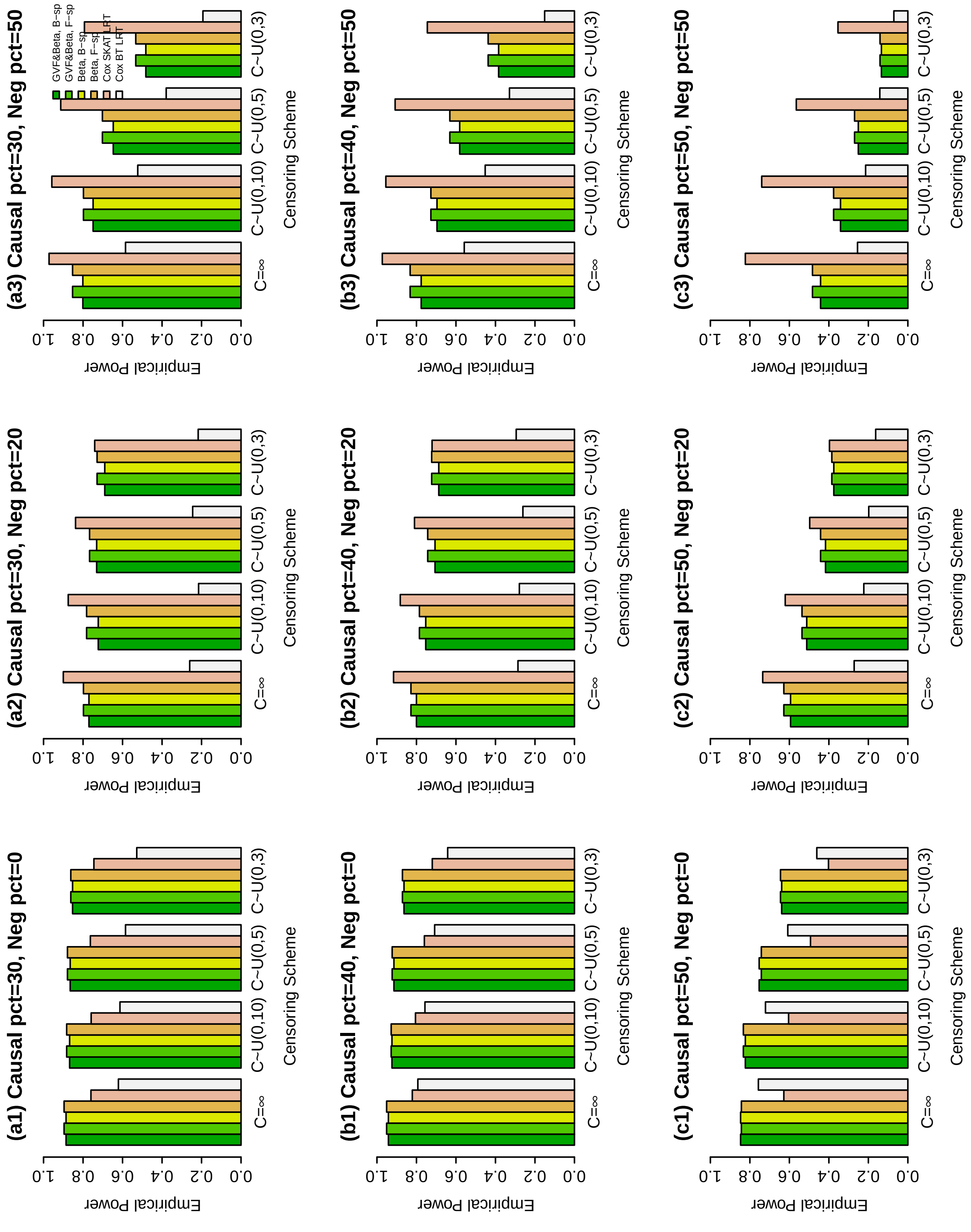

Figure B.2: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 6$; the number of Fourier basis functions was $K = K_\beta = 7$.

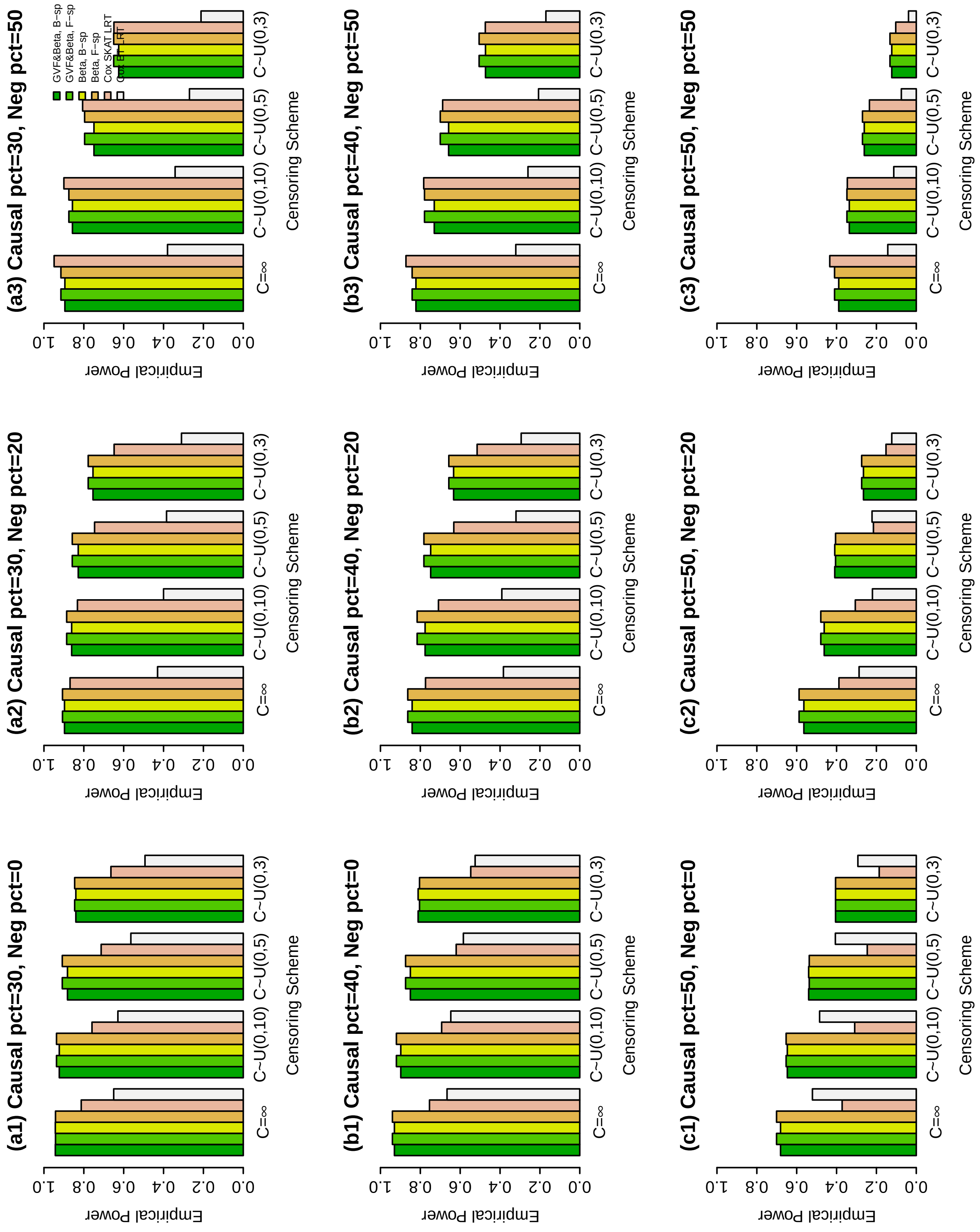

Figure B.3: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$.

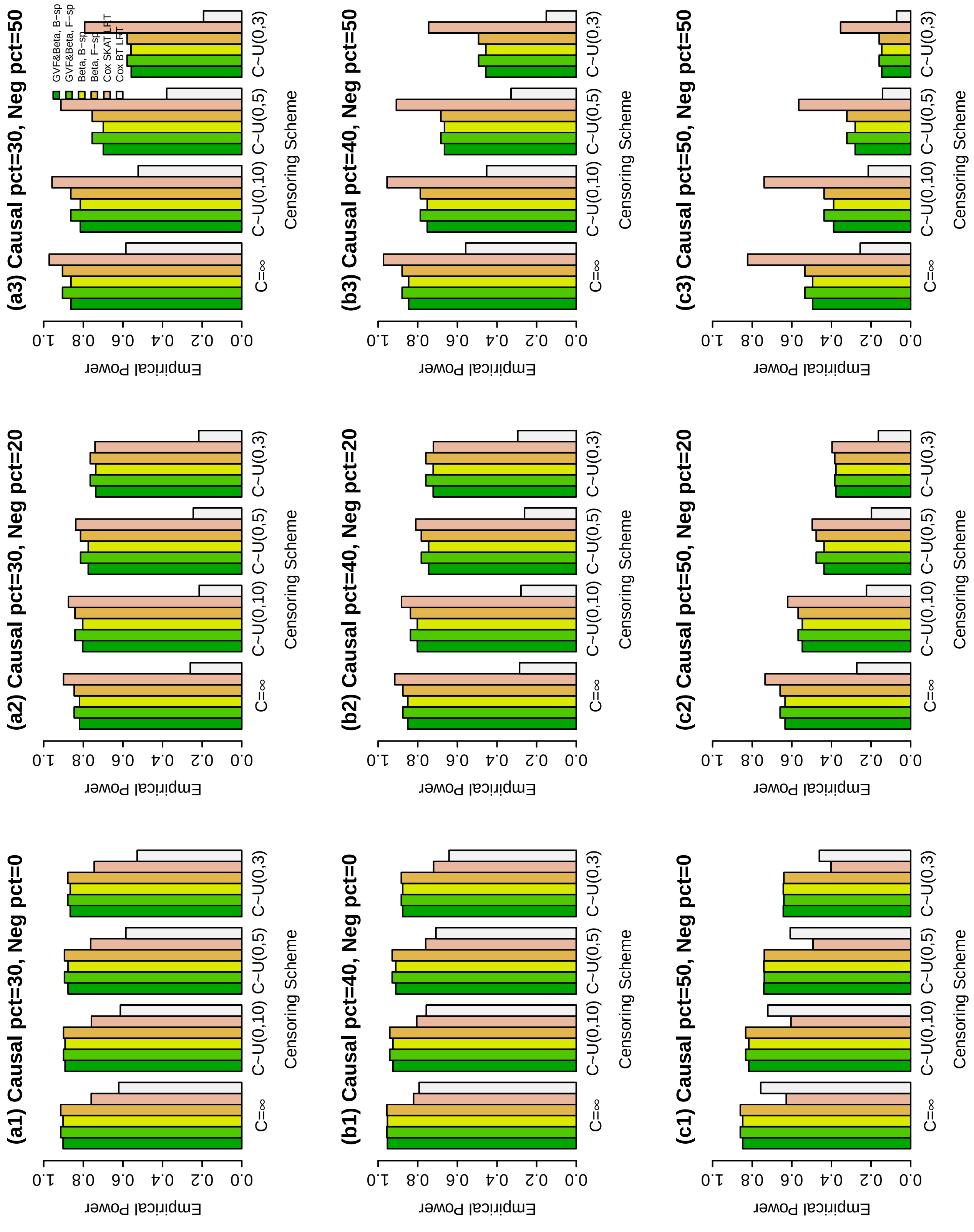

Figure B.4: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct $=0$, All Causal Variants Had Positive Effects; When Neg pct $=20$, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct $=50$, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 8$; the number of Fourier basis functions was $K = K_\beta = 9$.

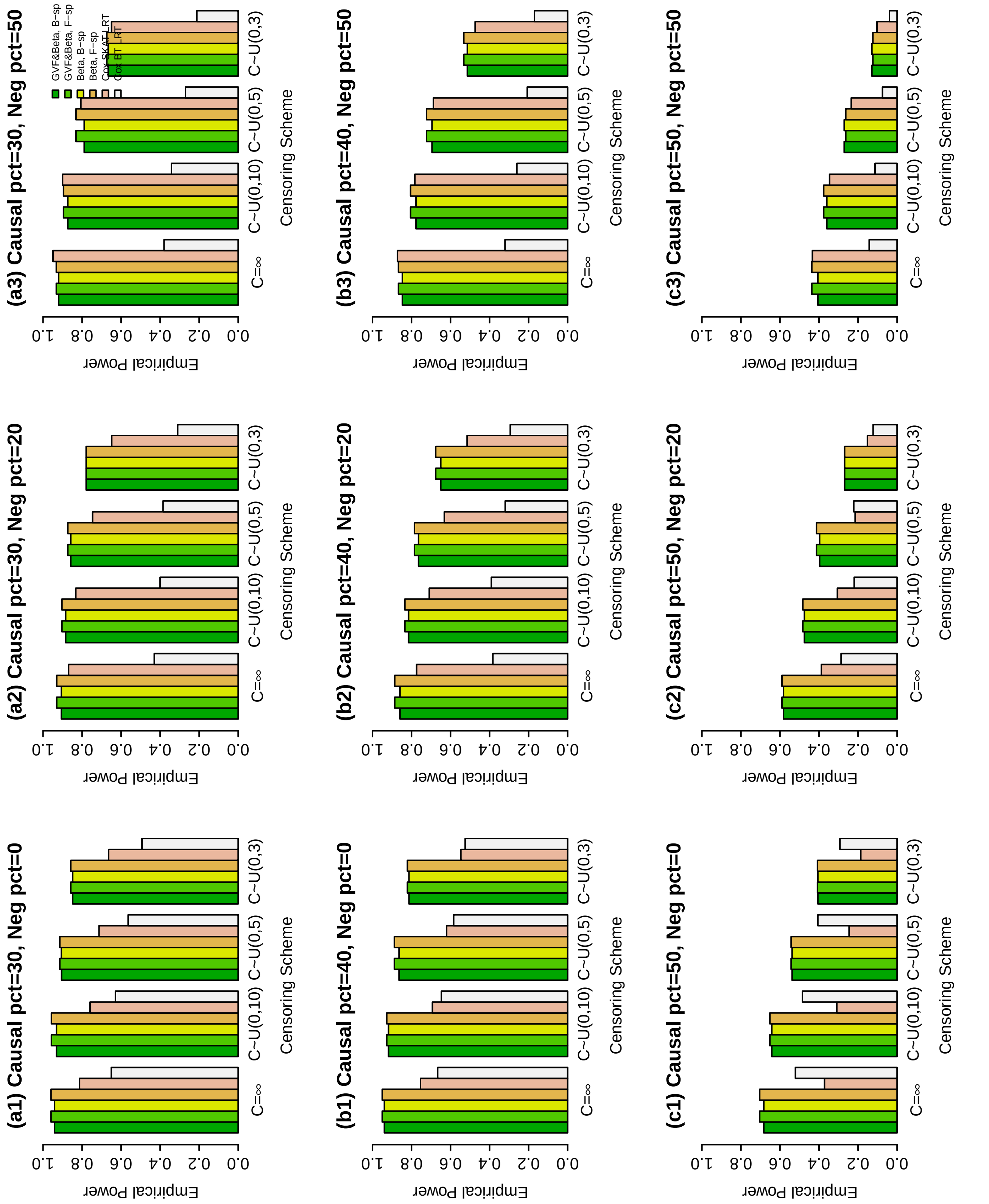

Figure B.5: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 10$; the number of Fourier basis functions was $K = K_\beta = 11$.

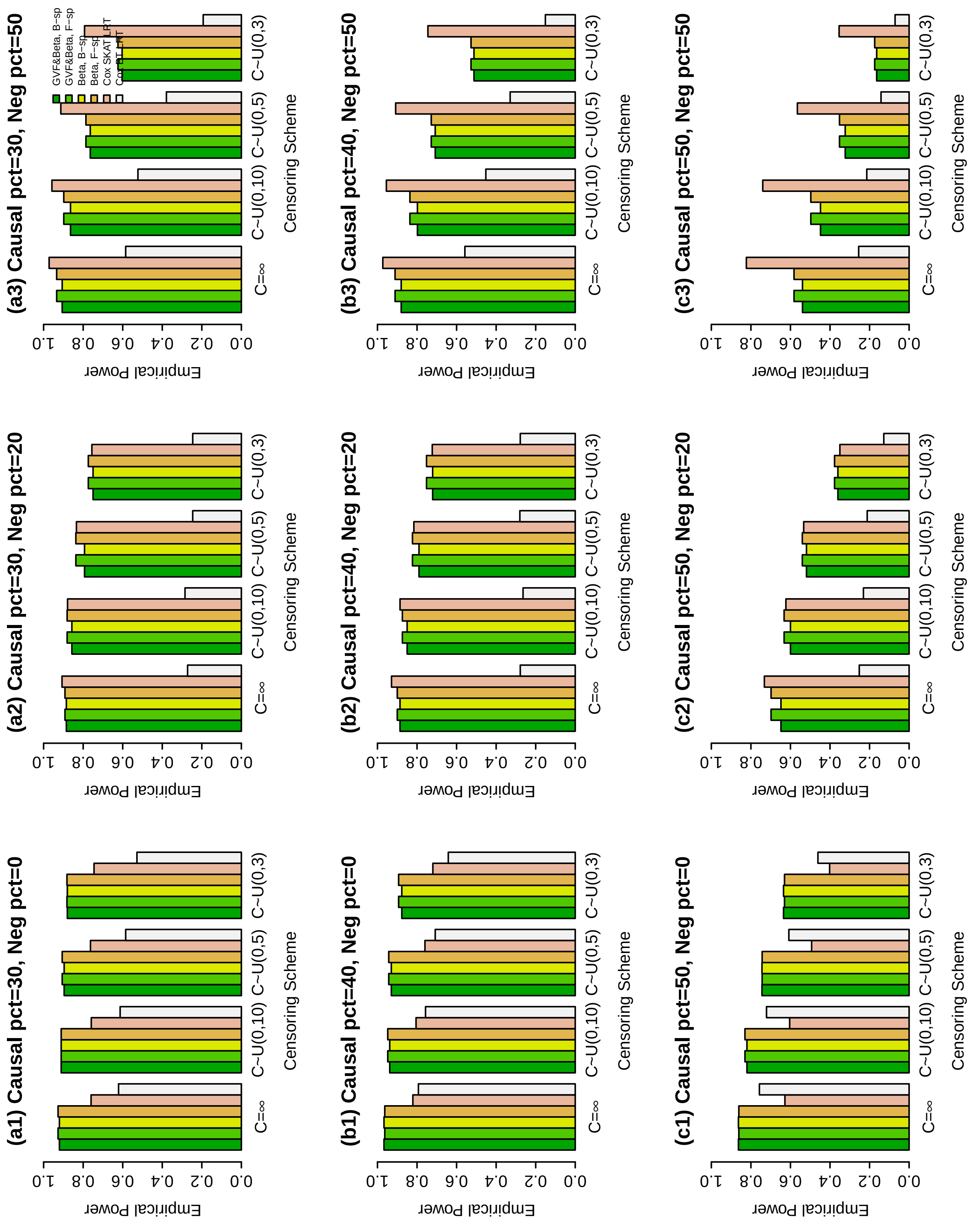

Figure B.6: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 10$; the number of Fourier basis functions was $K = K_\beta = 11$.

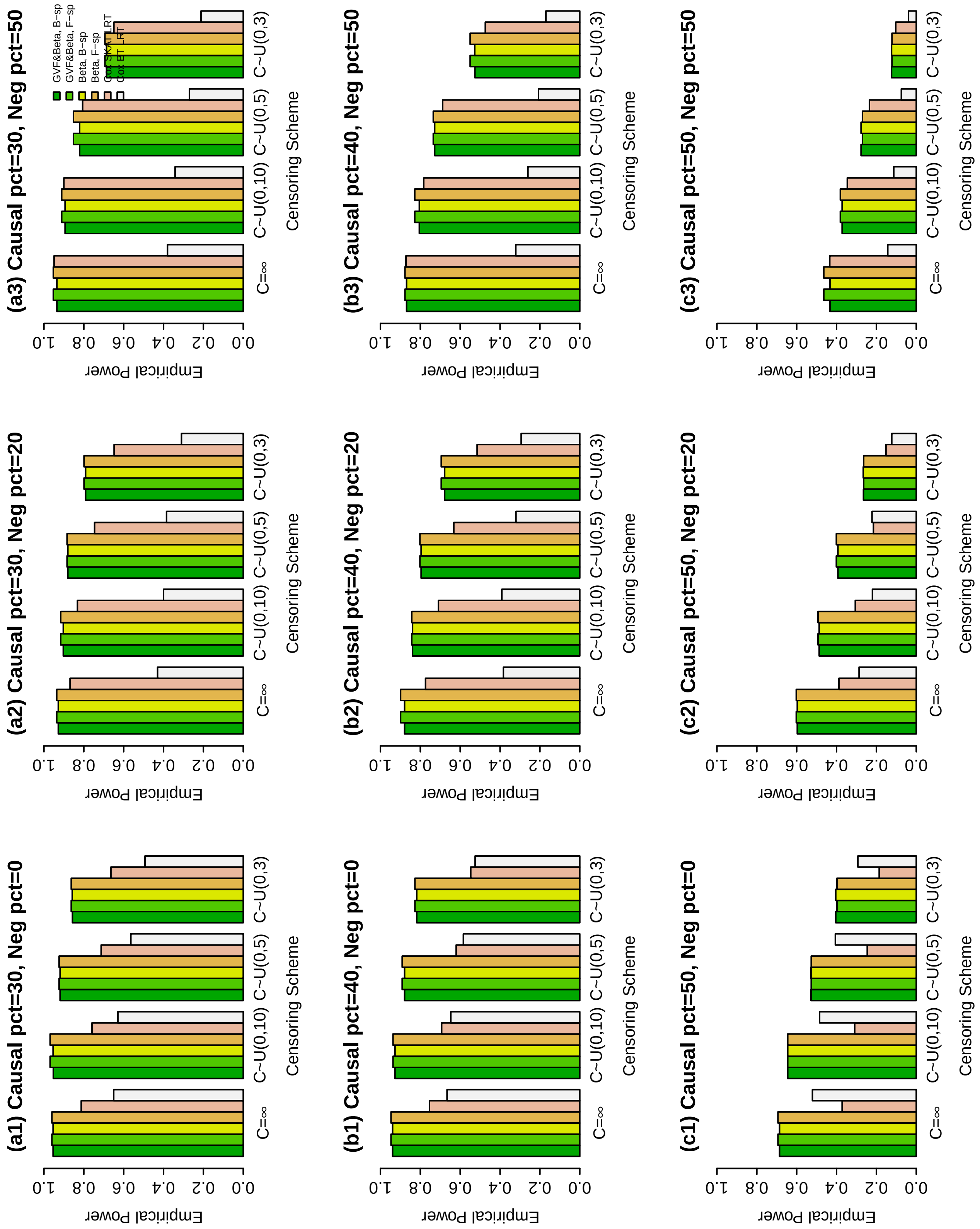

Figure B.7: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When Some Causal Variants are Rare and Some are Common and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$.

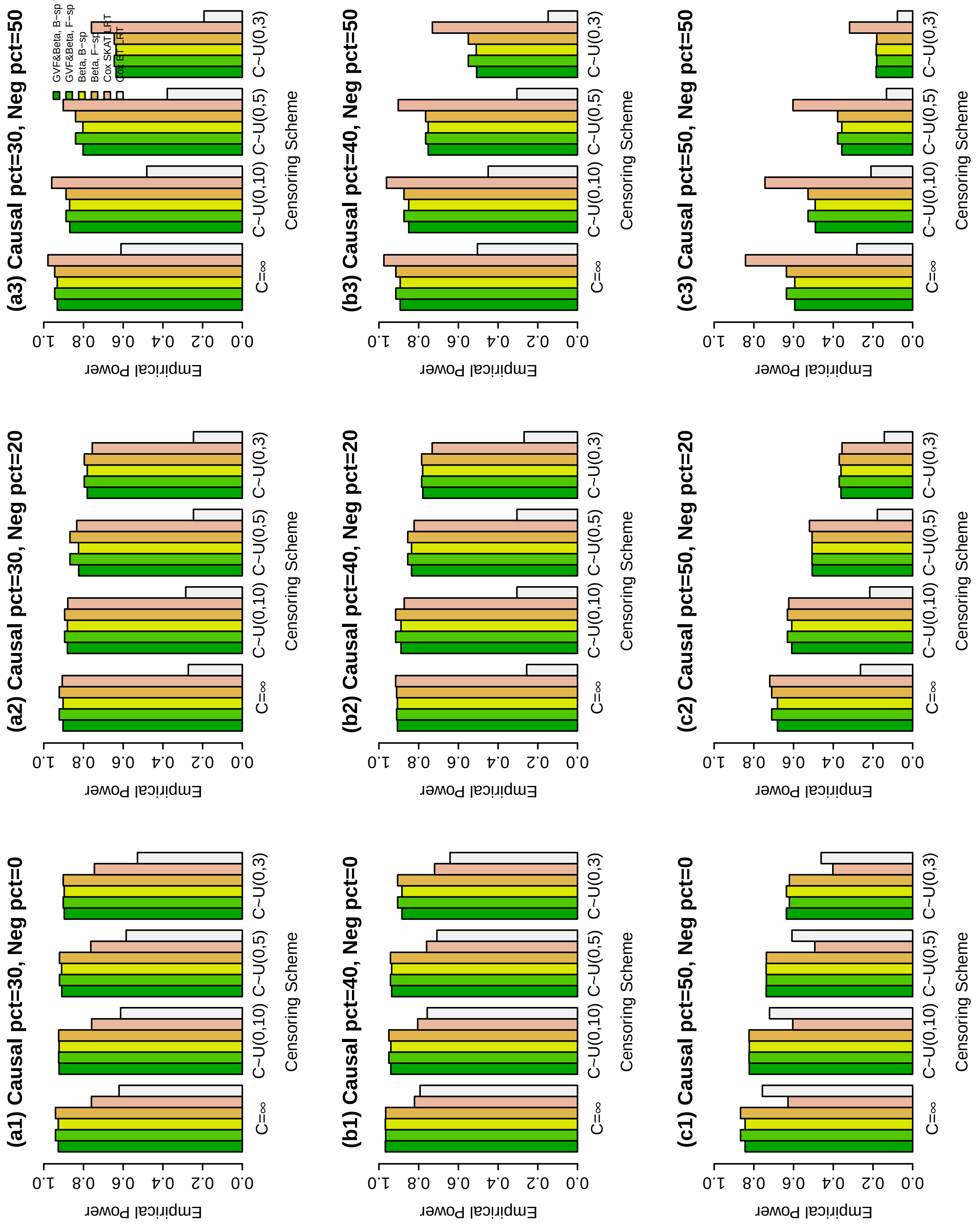

Figure B.8: **The Empirical Power of the Cox FR LRT Statistics of the Cox Models (4) and (5) and Cox SKAT LRT and Cox BT LRT by Chen et al. (2014) at $\alpha = 0.001$, When All Causal Variants are Rare and a Sample Size of 2,000.** When Neg pct = 0, All Causal Variants Had Positive Effects; When Neg pct = 20, 20%/80% Causal Variants Had Negative/Positive Effects; When Neg pct = 50, 50%/50% Causal Variants Had Negative/Positive Effects. The order of B-spline basis was 4, and the number of B-spline basis functions was $K = K_\beta = 12$; the number of Fourier basis functions was $K = K_\beta = 13$.