A Continuous Linear Optimal Transport Approach for Pattern Analysis in Image Datasets

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1 Supplemntary material

1.1 Independent Component Analysis (ICA):

Given a dataset of images, $I_1, ..., I_N$ and a template image I_0 (which can be calculated from the dataset as described in the paper), the continuous linear optimal transport (LOT), provides a linear embedding for images. We have shown in the paper that for the i'th image I_i the corresponding representation in the LOT embedding is $u_i\sqrt{I_0}$.

In the paper we used two differenet linear techniques namely the Principal Component Analysis (PCA) and a variation of the Linear Discriminant Analysis (LDA) to explore and model different modes of variations in the dataset. However, due to the invertible nature of the LOT one can utilize any statistical technique (linear or nonlinear) in the LOT space and visualize the outcome in the image space.

Here, as suggested by one of the reviewers we demonstrate the results of applying Independent Component Analysis (ICA) to the understudy datasets in the LOT space. Figure 1 shows the top PCA and ICA modes for the underlying datasets. Note that the inferred ICA modes are sorted relative to the captured data variations (variance of the projection of the data points on the corresponding direction).

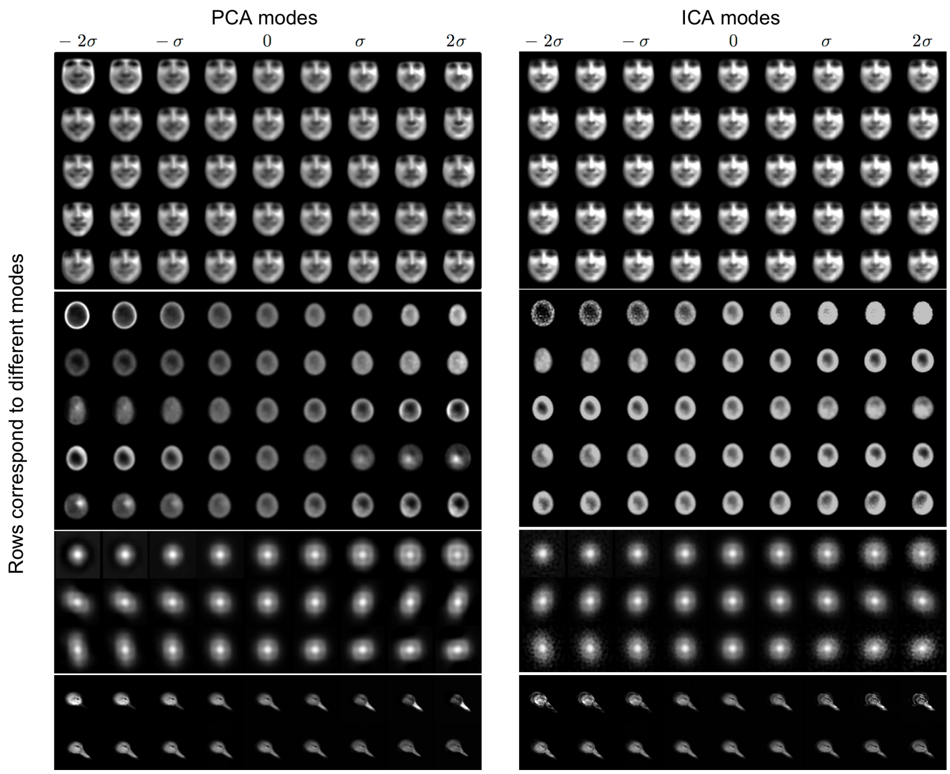

Figure 1: PCA and ICA modes for each dataset.

	Accuracy%			
	Neutral vs. Smiling	Spiral vs. Elliptical	Male vs. Female	Mallard vs. Gadwall
WND-CHRM Features	71.94% ± 1.5	99.85% ± 0.03	71.98% ± 1.31	99.37% ± 0.61
Continuous LOT	83.04% ± 0.8	77.82% ± 0.35	82.45% ± 0.8	85.26% ± 1.35

Table 1: Accuracy table for classification of facial expressions, gender in facial expression dataset, galaxy dataset, and bird type (mallard vs. gadwall) using a Linear SVM classifier with 10-fold cross validation in feature space and and continuous LOT-space.

WND-CHRM		Label	
		FA	FTC
Test	FA	15	12
	FTC	3	17
(a)	Accuracy	68.09%	

Continous LOT		Label	
		FA	FTC
Test	FA	27	0
	FTC	0	20
(b)	Accuracy	100%	

Table 2: Accuracy table for patient classification in the thyroid nuclei dataset using K-NN on the first LDA direction in (a) feature space, and (b) continuous LOT-space.

1.2 Feature-based methods:

When it comes to understanding/discovering the discriminant information in a set of images, most image processing tools and computer vision techniques rely on extracting numerical features from images. Feature-based methods, such as Bag-of-features and its numerous variations, are extensively used for learning discriminant information between different classes [1, 2]. These methods are employed in different arenas including but not limited to large-scale image retrieval, scene classification, histopathology image classification, cancer detection from nuclear morphology, etc. and have shown promising results. Despite not being 'generative', as mentioned earlier, feature-based methods have been successfully applied. Recently, Shamir et al. developed a multi-purpose pattern recognition software based on their previously introduced algorithm WND-CHRM [3], which computes up to $\sim$ 2700 features from each image covering many different types of shape and texture-related features described earlier. Below we show that the WND-CHRM feature-set is able to perform very well in certain discrimination tasks. In the computational results shown below, the WND-CHRM framework was used to calculate the aforementioned features. After feature extraction, and $z - score$ normalization, a feature selection scheme based on the Fisher score, as described in [3], was used to select the most discriminant features utilizing training data only. The selected subset of features is used as input to our classification scheme. Finally, the obtained classification accuracy is compared to that of our proposed method.

We have compared our continuous LOT approach to a feature-based method as explained in [3]. For the feature based method we used the multi-purpose pattern recognition software provided by Shamir et al. [3], which computes up to $\sim$ 2700 features from each image covering many different types of shape and texture-related features. Tables 1 and 2 show the results for 10-fold cross validated classification accuracy on all the datasets for these two methods. It can be seen that the calssification results are data dependent and for two of the datasets, namely the Galaxy and the Bird datasets, the feature-based method achieves better performance while for the rest the datasets the continuous LOT outperforms the latter.

References

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