Calculation of the maximal angle of twist during stag beetle biting

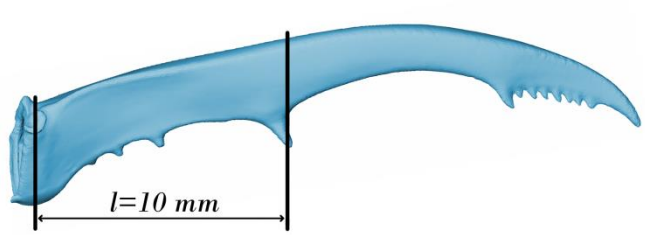
The angle of twist of an object with a uniform cross-section due to a torque T is:

$$\theta = angle\ of\ twist = \frac{T.l}{J.G}$$

Because the male stag beetle jaws have no uniform cross-section, this formula can only give a rough estimation of its angle of twist.

The necessary parameters are calculated as follows:

$l = 0.010\ m$ (see figure below)



$J = torsion\ constant = 1{,}2.10^{-12}m^4$ (minimal J of the jaw base, which is indicated on the figure above)

$G = shear\ modulus \approx \frac{E}{2(1+v)} = \frac{5{,}1GPa}{2(1+0{,}3)} = 2{,}0GPa$ (E is the Young's modulus and v the Poisson ratio)

$T = applied\ torque = \vec{r} \times \vec{F} = |r|.|F| = 0{,}0026m.\frac{6{,}9N}{2} = 0{,}0089Nm$ (*F*: bite reaction force on the large tooth of one jaw, see Goyens et al., 2014, Journal of Experimental Biology; *r*: moment arm from slice centroid to the force application point. We assume a right angle between both vectors to arrive at a maximal value.)

Using these values for l, J, G and T, the angle of twist θ is:

$\theta = angle\ of\ twist = \frac{T.l}{J.G} = \frac{0{,}0089Nm.0{,}010m}{1{,}2.10^{-12}.m^4.2{,}0.10^9Pa}$ = 0.037 rad = 2.1°

Because the bite force vector F is usually not directed normally to the moment arm r, the applied torque will be less than calculated above. Hence, the calculated angle of twist θ should be interpreted as an upper limit.