Science Advances

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Supplementary Materials for

# Engineering two-photon high-dimensional states through quantum interference

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Published 26 February 2016, *Sci. Adv.* **2**, e1501165 (2016)
DOI: 10.1126/sciadv.1501165

**The PDF file includes:**

Text

Any input state of a 50:50 beamsplitter with one photon incident on each input port can be decomposed into a component that generates coincidence counts and an orthogonal component which does not generate coincidence counts. In order to show this, let us consider a set of photonic modes with creation operators $a_i^\dagger$ and $b_j^\dagger$. These act on the two input paths *A*, *B* and the output paths *A′*, *B′* (Fig. 1). The creation operator $a_i^\dagger$ acting on the vacuum state $|0\rangle$ creates a single photon in mode *i* and path *A* (or *A'*), i.e., $a_i^\dagger|0\rangle = |1_i\rangle_A$; there is an analogous relation for creation operator $b_j^\dagger$. Any input state of the beamsplitter with one photon in each input port can then be written as

$$|\Psi\rangle = \sum_{i,j=1}^{d} \alpha_{ij} |1_i\rangle_A |1_j\rangle_B = \sum_{i,j=1}^{d} \alpha_{ij} a_i^\dagger b_j^\dagger |0\rangle \tag{S1}$$

with complex coefficients $\alpha_{ij}$ that are normalized, i.e., $\sum_{i,j} |\alpha_{i,j}|^2 = 1$. States $|s^+\rangle$ that do not change when the modes in path *A* and path *B* are swapped ($|s^+\rangle \to |s^+\rangle$) are called symmetric, while states $|s^-\rangle$ which acquire with this swap a π phase shift ($|s^-\rangle \to -|s^-\rangle$) are called antisymmetric:

$$\begin{aligned} |s^\pm\rangle &= \sum_{i,j=1}^{d} c_{ij}^{(\pm)} a_i^\dagger b_j^\dagger |0\rangle \\ &= \sum_{i>j=1}^{d} c_{ij}^{(\pm)} \left(a_i^\dagger b_j^\dagger \pm a_j^\dagger b_i^\dagger\right)|0\rangle + \sum_{i=1}^{d} c_{ii}^{(+)} a_i^\dagger b_i^\dagger |0\rangle \end{aligned} \qquad \text{with } c_{ij}^{(\pm)} = \pm c_{ji}^{(\pm)}. \tag{S2}$$

Although the input state in Eq. (S1) is in general neither symmetric nor antisymmetric, it can be expressed by a basis consisting of symmetric states $|s_{ij}^+\rangle$ and antisymmetric states $|s_{ij}^-\rangle$ given by

$$|s_{ij}^+\rangle = \frac{1}{\sqrt{2}}\left(a_i^\dagger b_j^\dagger \pm a_j^\dagger b_i^\dagger\right)|0\rangle \qquad \text{for } i \neq j, \tag{S3}$$

$$\text{and } |s_{ii}^+\rangle = a_i^\dagger b_i^\dagger |0\rangle, \; |s_{ii}^-\rangle = 0. \tag{S4}$$

This can be easily seen from inserting the identities. $a_i^\dagger b_j^\dagger |0\rangle \equiv \left(|s_{ij}^+\rangle + |s_{ij}^-\rangle\right)/\sqrt{2}$ and $a_i^\dagger b_i^\dagger |0\rangle \equiv |s_{ii}^+\rangle$ into Eq. (S1):

$$\begin{aligned} |\Psi\rangle &= \sum_{i\neq j} \frac{\alpha_{ij}}{\sqrt{2}}\left(|s_{ij}^+\rangle + |s_{ij}^-\rangle\right) + \sum_i \alpha_{ii} |s_{ii}^+\rangle \\ &= \frac{1}{\sqrt{2}} \sum_{i>j} \left(\alpha_{ij} + \alpha_{ji}\right)|s_{ij}^+\rangle + \left(\alpha_{ij} - \alpha_{ji}\right)|s_{ij}^-\rangle + \sum_i \alpha_{ii} |s_{ii}^+\rangle \quad , \\ &= |s^+\rangle + |s^-\rangle \end{aligned} \tag{S5}$$

where the symmetric part $\left|s^{+}\right\rangle$ and the antisymmetric part $\left|s^{-}\right\rangle$ are given by Eq. (S2) with $c_{ij}^{(\pm)} = \left(\alpha_{ij} \pm \alpha_{ji}\right)/2$ . Both vectors are unnormalized.

Note that there are $d(d-1)/2$ mutually orthogonal symmetric basis states, $\left|s_{ij}^{+}\right\rangle$ with $i \neq j$ and the same number of mutually orthogonal antisymmetric states $\left|s_{ij}^{-}\right\rangle$. However, there are also *d* symmetric basis states with $i = j$, completing the total number of basis states to $d^2$, the dimension of Hilbert space of the input states with one *d*-level photon in each of two paths.

Under the action of a 50:50 beam splitter inducing the transformation

$$\begin{aligned} a_i^\dagger b_j^\dagger &\rightarrow \frac{1}{2}\left(a_i^\dagger + b_i^\dagger\right)\left(a_j^\dagger - b_j^\dagger\right) \\ &= \frac{1}{2}\left(a_i^\dagger a_j^\dagger + a_j^\dagger b_i^\dagger - a_i^\dagger b_j^\dagger - b_i^\dagger b_j^\dagger\right) \end{aligned}, \qquad \text{(S6)}$$

the symmetric part of the input state $\left|\Psi\right\rangle$ is mapped onto a superposition of states with two photons in one of the paths:

$$\begin{aligned} \left|s^{+}\right\rangle &= \frac{1}{2}\sum_{i>j}\left(\alpha_{ij} + \alpha_{ji}\right)\left(a_i^\dagger b_j^\dagger + a_j^\dagger b_i^\dagger\right)\left|0\right\rangle + \sum_i \alpha_{ii} a_i^\dagger b_i^\dagger \left|0\right\rangle \\ &\rightarrow \frac{1}{2}\sum_{i\geq j}\left(1 - \frac{\delta_{ij}}{2}\right)\left(\alpha_{ij} + \alpha_{ji}\right)\left(a_i^\dagger a_j^\dagger - b_i^\dagger b_j^\dagger\right)\left|0\right\rangle \\ &= \frac{1}{2}\sum_{i>j}\left(\alpha_{ij} + \alpha_{ji}\right)\left(\left|1_i, 1_j\right\rangle_{A'}\left|0\right\rangle_{B'} - \left|0\right\rangle_{A'}\left|1_i, 1_j\right\rangle_{B'}\right) + \frac{1}{\sqrt{2}}\sum_{i=0}^{d}\alpha_{ii}\left(\left|2_i\right\rangle_{A'}\left|0\right\rangle_{B'} - \left|0\right\rangle_{A'}\left|2_i\right\rangle_{B'}\right) \end{aligned}. \qquad \text{(S7)}$$

On the other hand, the antisymmetric part $\left|s^{-}\right\rangle$ of the input state does not change under the action of the beamsplitter:

$$\begin{aligned} \left|s^{-}\right\rangle &= \frac{1}{2}\sum_{i>j}\left(\alpha_{ij} - \alpha_{ji}\right)\left(a_i^\dagger b_j^\dagger - a_j^\dagger b_i^\dagger\right)\left|0\right\rangle \\ &\rightarrow \frac{1}{2}\sum_{i\geq j}\left(\alpha_{ij} - \alpha_{ji}\right)\left(a_j^\dagger b_i^\dagger - a_i^\dagger b_j^\dagger\right)\left|0\right\rangle \\ &= \frac{1}{2}\sum_{i>j}\left(\alpha_{ij} - \alpha_{ji}\right)\left(\left|1_i\right\rangle_{A'}\left|1_j\right\rangle_{B'} - \left|1_j\right\rangle_{A'}\left|1_i\right\rangle_{B'}\right) \end{aligned}, \qquad \text{(S8)}$$

Hence, the symmetric part $\left|s^{+}\right\rangle$ produces no component with single photons in the output ports and thus no coincidence counts while the antisymmetric part stays invariant ( $\left|s^{-}\right\rangle \rightarrow \left|s^{-}\right\rangle$) and thus yields only coincidence counts. When conditioning on coincidence counts in the output ports of the beamsplitter, it thus acts as a filter that projects onto the antisymmetric component $\left|s^{-}\right\rangle$. The success probability $p$ for this projection is given by the square of the norm of the antisymmetric part, i.e., $p=\left\langle s^{-} \middle| s^{-}\right\rangle=\sum_{i>j}\left|\alpha_{ij}-\alpha_{ji}\right|^{2}/2$ .