**Appendix 1** Technical details of model [posted as supplied by author]

Using the simulation environment provided by the statistical software WinBUGS 1.4.3.[1 2] a Multilevel Poisson regression model is fitted to the Hospital Episode Statistics dataset. The hierarchical structure of the data has individuals in age-sex group *i*, within ward *j*, within district *k.* An offset term is included to allow for the size of the population in each ward, age and sex group. So we are modelling the log of the ratio of the number of admissions for total joint replacement ( $O_i$ ) to the number of people in the population ( $E_i$ ), given as:

$$\log\left( O_i / E_i \right) = \beta_0 CONS + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + e_i$$

From the English Longitudinal Study of Ageing dataset we have the predicted log-rate of need ( $\text{y\_need}_\text{i}$ ) (and associated standard errors) for each age, sex, ward group in England. We control for the log-rate of need as an additional covariate in the Multilevel Poisson regression model fitted to the Hospital Episode Statistics dataset. The model compares the log of the rate ratio of provision to need in each age, sex, ward group, producing equity rate ratios by socio-demographic, hospital and distance variables.

The estimate of need is given a normal likelihood function:

$$\text{y\_need}_\text{i} \sim N(\mu\text{\_need}_i, \sigma^2\text{\_need}_\text{i})$$

where *i* = 1,…,79690.

The fixed-effects Poisson regression model is expressed as:

$$\log\left( O_i / E_i \right) = \log(\mu\text{\_need}_i) + \beta_0 CONS + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + e_i$$

This can be re-written as:

$$\log\left( \frac{\mu\text{\_prov}_i}{\mu\text{\_need}_i} \right) = \beta_0 CONS + \beta X_i^T + e_i$$

where $\left( \frac{\mu\text{\_prov}_i}{\mu\text{\_need}_i} \right)$ is the log of the rate ratio of provision to need for each age-sex-ward group *i*. *CONS* is the constant term, $X_i^T$ the vector of explanatory variables, and $e_i$ is the individual level residual constrained to a Normal distribution.

In this dataset we have a nested hierarchical structure, of observations in age-sex group *i*, within ward *j*, within district *k*. A random-intercepts Multilevel model is used to control for evidence of clustering in the data, by allowing the overall rate of provision to need to vary across wards and districts. Overdispersion that remains after controlling for clustering in the data is also accounted for in the regression model, by adjusting the standard errors to account for this missing information. We began by fitting a multivariable model including the socio-demographic variables alone, testing for evidence of important interactions. We then looked at the effect of hospital and distance variables, by fitting a full model including all variables and using backwards selection to exclude variables that do not improve model fit. Random-slopes models are then individually fitted for each socio-demographic variable to see if their effects vary across districts. Overall rates of provision to need are produced for each of the 354 districts in England, together with estimates of uncertainty. In WinBUGS, model fit is assessed using the deviance information criteria (DIC), with lower DIC representing a better fitting model after taking account of model complexity[3].

*Implementation*

Each variable in the model is given a prior distribution. In a Bayesian analysis, the distribution of the model parameters is obtained by combining a *likelihood* function with the *prior* distribution for the parameters to obtain the *posterior* distribution. In this case flat non-informative priors are used, so the posterior density is dominated by the likelihood function and, hence provides very similar inference to likelihood based classical methods. Markov Chain Monte Carlo (MCMC) methods are implemented using Gibbs Sampling in WinBUGS to evaluate the posterior distribution, which simulates a new value for each parameter in the model from its conditional distribution assuming the current values of the other parameters are the true values. Convergence was assessed using the Brooks-Gelman-Rubin diagnostic tool[4] and considered adequate after 10000 iterations in all models. For each model, a further sample of 5000 simulations was then run on which the results are based. These iterations form a posterior distribution for each model variable. The model estimates are obtained by calculating the mean, standard error, 2.5% and 97.5% centile of the posterior distribution of each variable. These are equity rate ratios (RR), their standard errors and 95% credible intervals.


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