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Supplementary Materials for

Lost in transportation: Information measures and cognitive limits in multilayer navigation

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The PDF file includes:

City	Abbreviation	Nodes	Edges	N	K_{tot}	P-diameter
New York City	NY	433	497	22	161	2
Paris	Pa	299	355	16	78	3
Tokyo	To	217	262	13	56	2
London	Lo	266	308	11	48	2
Madrid	Ma	209	240	12	38	2
Barcelona	Ba	139	165	11	37	2
Moscow	Mo	134	156	11	35	2
Seoul	Se	420	466	12	35	3
Shanghai	Sh	239	264	11	35	3
Mexico City	MC	147	164	11	31	2
Berlin	Ber	170	282	10	29	2
Chicago	Ch	167	222	8	25	2
Osaka	Os	108	123	9	24	2
Beijing	Bei	163	176	13	21	4
Hong Kong	HK	84	87	10	12	4

TABLE S1: **Network characteristics of the largest connected component for the 15 largest metropolitan systems in the world.** The number of routes (N) and connections (K_{tot}), respectively, yield nodes and edges in the dual space. We list cities from most connections to fewest connections between different lines. The number K_{tot} of connections is the key quantity from the perspective of information processing. (See the right panel of Fig. 2.) *P-diameter* indicates the network diameter in dual space. It is equal to 2 for 10 of the 15 networks, and one additionally obtains a value of 2 in Paris if one cuts “3bis” (a four-stop line).

Network	Nodes	Edges	N	K_{tot}
NYC Metro	412	512	20	162
NYC Bus	5306	8435	309	6092
NYC multilayer	5332	8804	330	8461
Paris MRT	629	765	28	162
Paris Bus	3842	5567	277	2708
Paris multilayer	4037	6142	305	4292
Tokyo Metro	217	262	13	56
Tokyo Bus	1359	1663	153	1275
Tokyo multilayer	1422	1908	166	1831

TABLE S2: **Network characteristics of the Bus-Metro multilayer networks.** As in Table S1, we show the number of routes (N) and connections (K_{tot}), and the nodes and edges of the dual space. We note that there is a difference of an order of magnitude between the dimensions of the metro and the bus layers. This is a huge jump in complexity that challenges the ability of people to navigate in multilayer transport networks.

	$C = 1$	$C = 2$	$C = 3$
NYC	85.2%	14.8%	0%
Paris MRT	43.0%	48.5%	8.5%
Tokyo	72.8%	28.2%	0%

TABLE S3: **Structure of simplest paths in three metro systems.** We compare the number of connections in the simplest paths for the metropolitan systems of the three megacities (New York City, Paris, and Tokyo) that we consider in detail. Only Paris has paths with more than 2 connections. A negligible fraction (not shown) of paths have 4 or 5 connections.

	$C = 1$	$C = 2$
NYC	99.4%	0.5%
Paris	86.4%	13.6%
Tokyo	79.9%	20.1%

TABLE S4: **Percentages of paths with $\bar{S} < 8.1$ bits.** For the three megacities that we consider in this paper, about 20% of the trips have an information entropy that is lower than the threshold of 8.1 bits. Such trips predominantly have only a single connection. When there are more, the starting route has a limited number of connections (see Fig. S6 .)

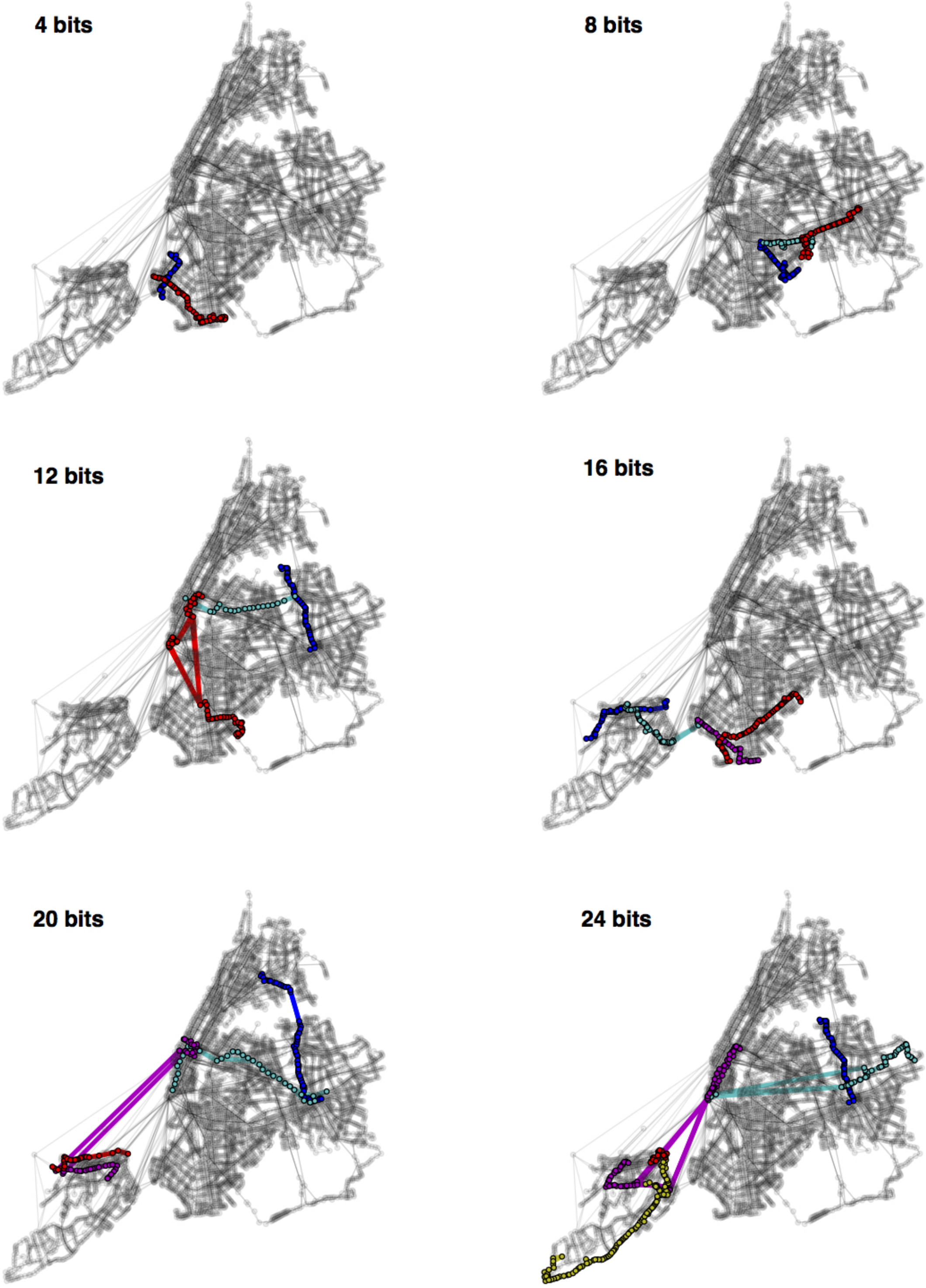

FIG. S1: **Examples of paths with growing $\bar{S}(s,t)$.** For the New York City multilayer transportation network, we show examples with increasing complexity: $\bar{S}(s,t)$ ranges from 4 bits to 24 bits. We color the starting bus line s in blue and the destination bus line t in red.

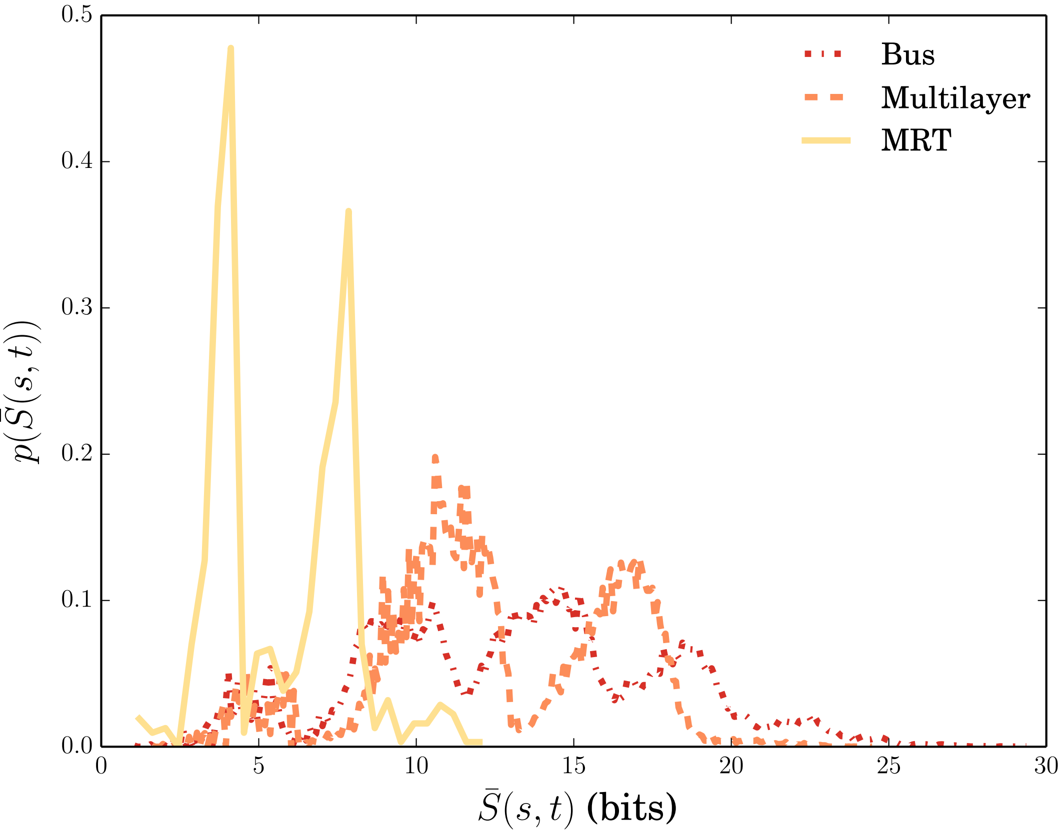

FIG. S2: **Entropy distribution for MRT layer, bus layer, and complete multilayer transportation network in Paris.** For the multilayer network, we restrict the distribution to trips whose origin and destination are each in the bus layer. We see that the effect of multiplexity on the bus layer is to shift the peaks to the right, and we also obtain larger peaks for smaller values of C (see Fig. S3).

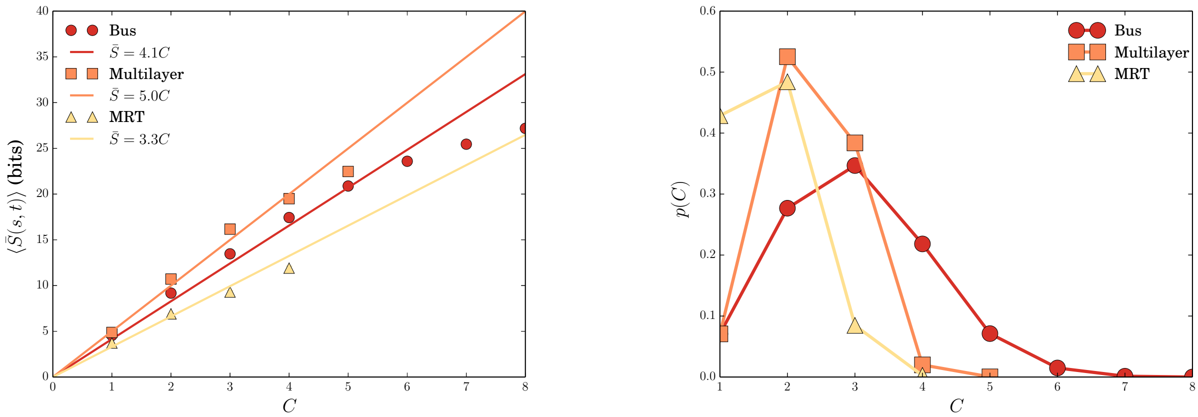

FIG. S3: **Effects of multiplexity. (Left)** The values of $\bar{S}$ grow with C. The growth is largest for the multilayer transportation network (in the sense that it has the highest value of $\langle k \rangle$), smaller for the bus layer (where the lines have fewer connections), and smallest for the MRT layer. **(Right)** Conversely, the mean path length is smaller for the bus monolayer network than for the multilayer network, in which the bus service interacts with the (longer-range) lines in the MRT layer.

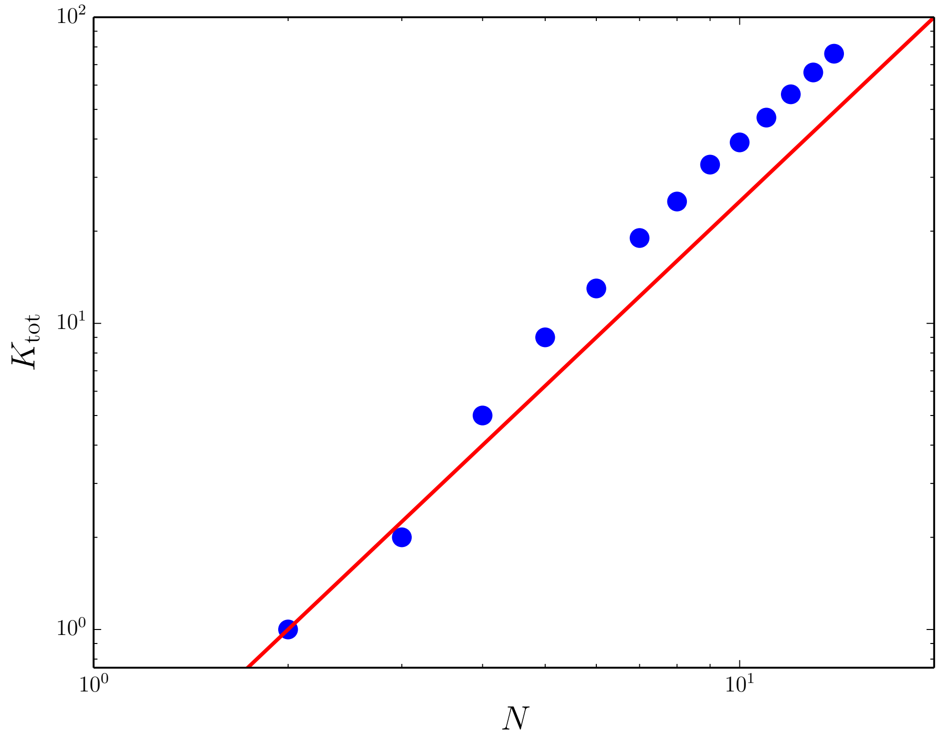

FIG. S4: **Growth of the Paris metropolitan network.** Letting the network grow with its historical progression from Line 1 to Line 14, we see that the number $K_{\rm tot}$ of connections in the dual space (blue dots) grows similarly to a lattice (red line), which has $(N/2)^2$ intersections.

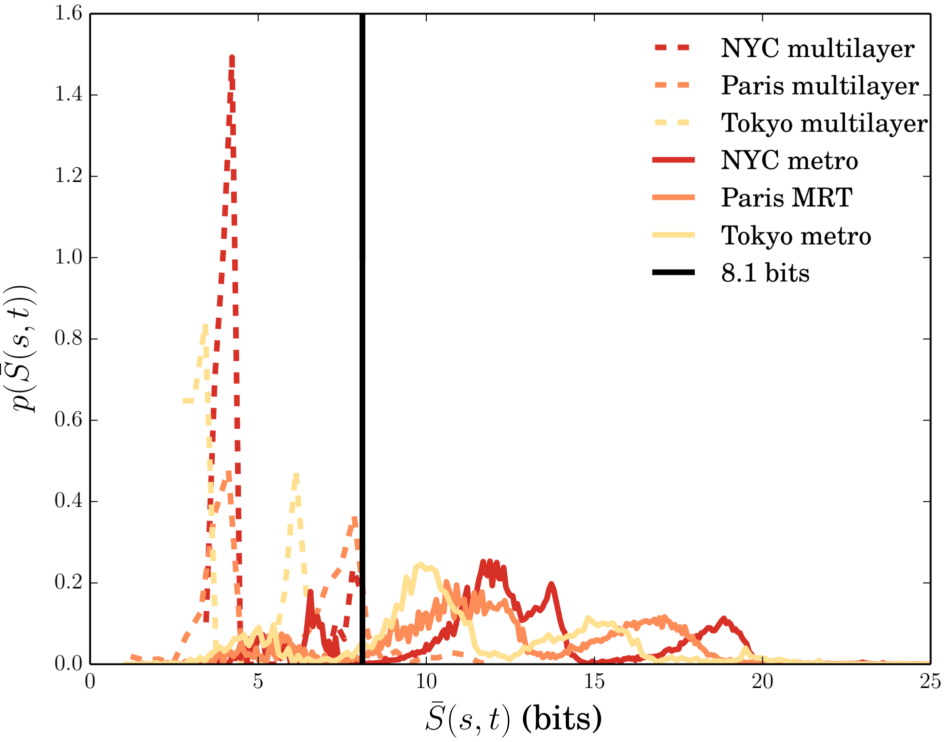

FIG. S5: **Information entropy of multilayer transportation networks**. This figure represents the probability density distributions of $\bar{S}(s,t)$. In Fig. 4 of the main manuscript, we show the associated cumulative distributions. Similar to Fig. 4, we associate one layer to bus routes and another to metro lines. The solid curves are associated with multilayer networks that include a metro layer for New York City, Paris, and Tokyo. The dashed curves are associated with all possible paths in a metro layer. We observe that every distribution is characterized by a peak structure, and every peak is associated to a number C of connections.

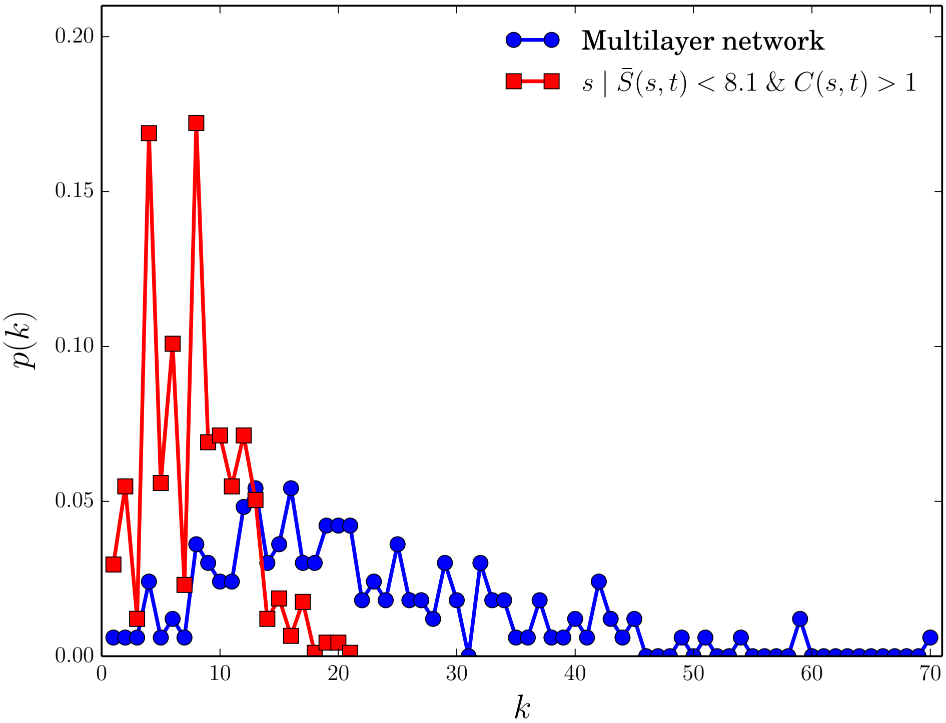

FIG. S6: **Dual-space degree of low-information starting points for paths with $C = 2$ in the Tokyo multilayer network.** As one can see in Table S4, most of the trips below the cognitive limit have $C = 1$ connections. The trips below the cognitive threshold with $C > 1$ connections are characterized by a low connectivity of the origin route. We show this feature for the Tokyo network. We see that $C = 2$ for 20.1% of the trips that are below the threshold of 8.1 bits. For this fraction of trips $s \to t$, the degrees k in the dual space of origin routes s (red squares) are small in comparison to the degrees of all routes in the whole multilayer network (blue circles).

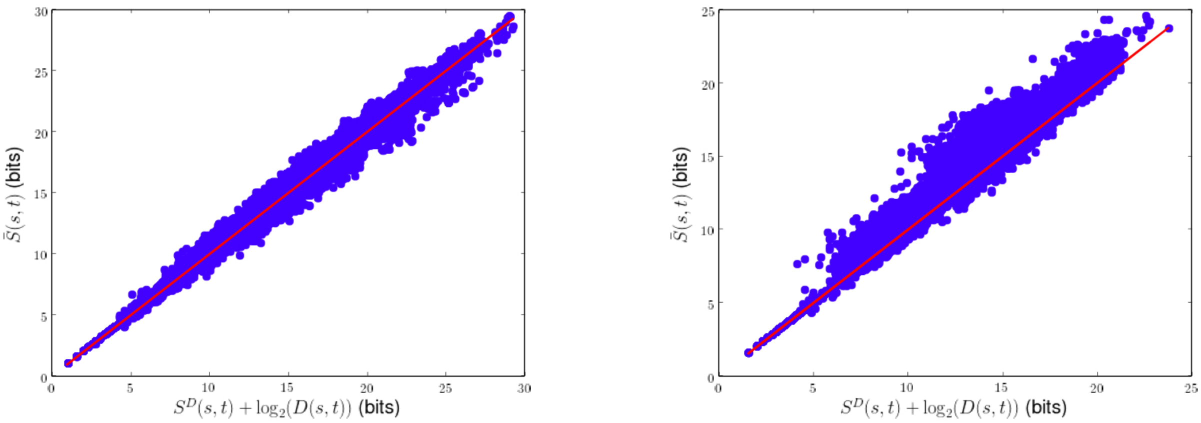

FIG. S7: **Empirical validation of Eq. 7.** Comparing the (left) bus monolayer network to the (right) multilayer network in Paris, we note that including the metro-rail-tramway (MRT) layer yields larger fluctuations. The mean square deviation is 0.34 bits for the bus layer and 0.82 bits of the multilayer network that contains both bus and MRT modes. This suggests that for the same route pair (s, t), different paths become optimal for different origins i and destinations j.