

Supplementary Figure 1. An example of eigenloci from Epitensor. The first eigenlocus corresponds to RNA-seq transcripts at STC1. The RNA-seq transcripts have coordinated expression patterns with high levels of signals in hESC, TBL, MSC, and IMR90 cells and low levels of signals in NPC cells. The second eigenlocus involves six peaks, i.e. peak i-vi. Peak i-iv and vi correspond to enhancers a-c and e while peak v corresponds to the promoter of STC1. The third eigenlocus has corresponds to the H3K4me3 activity patterns at promoter STC1. The predicted promoter-enhancer and enhancer-enhancer interactions from EpiTensor are highly concordant from Hi-C interactions.

Supplementary Figure 2. An example of exon-exon association identified by EpiTensor.

Supplementary Figure 3. The ROC analysis for the prediction of promoter-exon (a), and exonexon associations (b).

Supplementary Figure 4. Comparison of resolution of Hi-C and EpiTensor interactions. Enhancer A (size=0.2 kb) is more accurately located by EpiTensor than by Hi-C data.

Supplementary Figure 5. (a-c) The ROC analysis for the prediction of promoter-promoter (a), promoter-enhancer (b), and enhancer-enhancer (c) interactions. The prediction results were compared against the ones from ChIA-PET data in K562 cells. **(d-e)** Validation of EpiTensor prediction with eQTL data in GM12878 (d) and HepG2 (e) cells. Percent of eQTLs predicted by EpiTensor was compared with that from random pairs and p-value was computed using binomial test. **(f)** The AUC values for the prediction of promoter-promoter (P-P), promoter-enhancer (P-E), and enhancer-enhancer (E-E) interactions in GM12878, HMEC, HUVEC, IMR90, K562, and NHEK cells. The predictions results were compared against the ones from high-resolution Hi-C data obtained from Rao et al.

Supplementary Figure 6. An example of promoter-enhancer interaction from EpiTensor. The 1st eigenlocus has three peaks (peaks i, ii, and iii). Peak i falls into the promoter of CDC42BPA while peaks ii and iii overlap with enhancers b and d, respectively. These three peaks co-occur in the same eigenlocus because they have similar epigenetic profiles across five cell types. Specifically, peak i has high levels of H3K4me3 in hESC, TBL, and IMR90 cells, but low levels of H3K4me3 in MSC and NPC cells. Peaks ii and iii has high levels of H3K4me1 and H3K27ac in hESC, TBL, and IMR90 cells but not in MSC and NPC cells. This similar multi-cell epigenetic profile is captured by the $1st$ eigenlocus vector in the locus subspace. In contrast, enhancer c has high levels of H3K4me1 in IMR90 cells but not in hSC, TBL, MSC, and NPC cells. Enhancer c does not have similar epigenetic profile as CDC42BPA promoter and their peaks do not co-occur in the same eigenlocus vector. Similarly, enhancer a does not appear in the ^{1st} eigenlocus vector either because it has a different epigenetic profile.

a

Supplementary Figure 7. 3C Validation of distal interactions identified in IMR90 cells. (**a**) Validation of two distal interactions associated with PSMA5 promoter, one with WDR47 promoter (pair i-ii), and the other with TAF13 promoter (pair i-iii). The red and blue lines in 3C signals represent two biological replicates. Each biological replicate is averaged from three technical replicates. (**b**) Validation of four distal interactions associated with a lncRNA promoter, three pairs of interactions with predicted enhancers (pair i-ii, i-iv, and i-v, respective), and the fourth with ITGB1 promoter (pair i-iii). (**c**) Validation of interactions between SNX18 promoter and three predicted enhancers (pair i-ii, i-iii, i-iv, respectively). (**d**) Validation of a distal interaction between KATNBL1 and PGBD4 promoters (pair i-ii). (**e**) Validation of a distal interaction between BHLHE40 promoter and a predicted enhancer (pair i-ii).

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- Hotspot Non-hotspot

Supplementary Figure 8. Comparison of hotspot and non-hotspot promoters in promoterpromoter interactions in terms of H3K4me1, H3K4me3, H3K27ac, H3K36me3, H3K27me3, H3K9me3, and DNaseI-seq profiles. All promoter profiles were centered on TSSs..

Supplementary Figure 9. Comparison of hotspot and non-hotspot promoters in promoterenhancer interactions in terms of H3K4me1, H3K4me3, H3K27ac, H3K36me3, H3K27me3, H3K9me3, and DNaseI-seq profiles. All promoter profiles were centered on TSSs.

Hotspot Non-hotspot

Supplementary Figure 10. Comparison of hotspot and non-hotspot enhancers in promoterenhancer interactions in terms of H3K4me1, H3K4me3, H3K27ac, H3K36me3, H3K27me3, H3K9me3, and DNaseI-seq profiles. All enhancer profiles were centered on the center of candidate enhancers.

Hotspot Non-hotspot

Supplementary Figure 11. Comparison of hotspot and non-hotspot enhancers in enhancerenhancer interactions in terms of H3K4me1, H3K4me3, H3K27ac, H3K36me3, H3K27me3, H3K9me3, and DNaseI-seq profiles. All enhancer profiles were centered on the center of candidate enhancers.

a

Hotspot promoters in promoter-enhancer interactions

Supplementary Figure 12. Characterization of interaction hotspot promoters in promoterenhancer interactions. (**a**) Comparison of hotspots and non-hotspots in terms of chromatin accessibility, histone modifications, gene expression, TF binding, and motif enrichment. Pvalues calculated using Wilcox test are denoted: $*$ $P \le 0.001$; $*$ 0.001 < $P \le 0.05$; \Box $P > 0.05$. (**b)** GO term analysis of hotspot promoters.

Supplementary Figure 13. Characterization of interaction hotspot enhancers in promoterenhancer interactions. (**a**) Comparison of hotspots and non-hotspots in terms of chromatin accessibility, histone modifications, TF binding, and motif enrichment. P-values calculated using Wilcox test are denoted: ** $P \le 0.001$; * $0.001 < P \le 0.05$; $\Box P > 0.05$. (**b**) GREAT analysis of hotspot enhancers.

Supplementary Figure 14. Characterization of interaction hotspot enhancers in enhancerenhancer interactions. (**a**) Comparison of hotspots and non-hotspots in terms of chromatin accessibility, histone modifications, TF binding, and motif enrichment. P-values calculated using Wilcox test are denoted: ** $P \le 0.001$; * $0.001 < P \le 0.05$; $\Box P > 0.05$. (**b**) GREAT analysis of hotspot enhancers.

Supplementary Table 1 List of six core chromatin modifications and RNA-seq data in 121 cell types

Stran							Lengt
d	Primer		Chromosome coordinates			Sequence $(5'-3)$	h
	HiCN1-H-		10996572		10996574		
	Anchor	chr1:	$\overline{0}$	$\overline{}$	$\boldsymbol{0}$	CTGGACTCCATAAACCACGGA	21
			10956758		10956760		
	HiCN1-H-1	chr1:	1 10957009		$\mathbf{0}$ 10957011	TCTGTTGTTCAGAGGTGTGC	20
	HiCN1-H-2	chr1:	3		5	TGGGAGCCTGTATAAATGTTTCA	23
			10958101		10958103		
	HiCN1-H-3	chr1:	2		1	ACTGCACCTGGCTCGATATT	20
			10958240		10958242		
	HiCN1-H-4	chr1:	8		6	CTTGGGCTCGCAATGTTGG	19
			10958293		10958294		
	HiCN1-H-5	chr1:	θ	\equiv	9	GGGCGGCATATGAAATGCTT	20
			10959346		10959348		
	HiCN1-H-6	chr1:	4	\equiv	3	AGTGGTGGTGCTTTGTTCTC	20
	HiCN1-H-7	chr1:	10960073 7	$\overline{}$	10960075 7	TAGGCAACACACCACTATGCC	21
$\overline{}$			10960559		10960561		
	HiCN1-H-8	chr1:	4	\blacksquare	5	GAATAATGCTGTGTTGGAAGGC	22
			10962500		10962502		
$\boldsymbol{+}$	HiCN1-H-9	chr1:	3	$\overline{}$	2	GAAAGCAGGAGCTGGATGGT	20
			10962513		10962515		
÷,	HiCN1-H-10	chr1:	4	\blacksquare	3	AGACGTCTACACCCTGGGAT	20
			10996922		10996924		
$\boldsymbol{+}$	HiCN1-IC-F	chr1:	2		1	CTTAAATCGCGGGTGGTTGC	20
			10996945		10996947		
\overline{a}	HiCN1-IC-R	chr1:	9	$\overline{}$	8	ACCGTGGAGGAACCCTAAGA	20
	HiCP1-H- Anchor	chr1:	32400038	\Box	32400058	GAGAGGAGCAGGTGCATACAT	21
$\ddot{}$	HiCP1-H-1	chr1:	32464186	\overline{a}	32464206	TGCAGACCAACCATGCTGATA	21
	HiCP1-H-2	chr1:	32468909		32468929		21
$\qquad \qquad +$				$\overline{}$		TTCTACAGTGAGGTCCACAGC	
$\! + \!$	HiCP1-H-3	chr1:	32473194	$\bar{}$	32473214	GGGATGCAATTGGCTTTCTGG	21
$\! + \!$	HiCP1-H-4	chr1:	32477638	$\frac{1}{2}$	32477662	TACGTGTTTTGTTTTCATTTCTGTG	25
$^{+}$	HiCP1-H-5	chr1:	32487016	$\bar{\mathcal{L}}$	32487038	AGAACCTATTGGCTTTGTGCTCT	23
$^{+}$	HiCP1-H-6	chr1:	32488684		32488704	ACACCAAGCATGTAGGAGAGC	21
$\boldsymbol{+}$	HiCP1-H-7	chr1:	32496011		32496031	GCAGTATCCCAAGGTAAGGCA	21
$\begin{array}{c} + \end{array}$	HiCP1-H-8	chr1:	32499290	\Box	32499312	GGAAAGTGACTAAGGTGGGTCTC	23
$\boldsymbol{+}$	HiCP1-IC-F	chr1:	32402503		32402522	CGACTCCGGACAGACAAACA	20
\overline{a}	HiCP1-IC-R	chr1:	32402730	$\bar{}$	32402749	GCCCCTGAGAGCTAACGAAA	20
	HiCP10-H-	chr10					
$\overline{}$	Anchor		33293318	\Box	33293337	TCAGGAAGAGGGAAAGGACT	20
		chr10					
	HiCP10-H-1		33225735	\Box	33225757	TCTTCTTGTGGAGGGACAGATAC	23
	HiCP10-H-2	chr10	33233791	$\overline{}$	33233814	GTCTCTGGTTTATCATCCTCATGC	24
		chr10					
	HiCP10-H-3		33233958	\Box	33233979	CAAGGAATAGGAGGTTTGCTCA	22
		chr10					
	HiCP10-H-4		33237577		33237599	CTCCGTCTTTTATCTGGAACCCA	23
		chr10				GGAGATGAAGAGGCAATAAAGAC	
	HiCP10-H-5		33241680	$\overline{}$	33241704	AG	25

Supplementary Table 4 Primer sequences for 3C experiments

Supplemental Note 1 -Tensor Basics

1. Introduction

Often the input for machine learning problems is inherently multi-dimensional. For example, gray-scale video sequences are three-dimensional: column, row, and time [1, 2]. Color video sequences are four-dimensional, as there is an additional color dimension [3, 4]. Data in social networks are usually organized in three dimensions: time, authors, and keywords [5]. These multidimensional data can be formally represented as tensors, the elements of which are addressed by more than two indices. The number of indices used in the description defines the order of the tensor and each index defines one of the so-called "mode". Tensors can be viewed as an extension of matrices, vectors, and scalars. A matrix is a second-order tensor, a vector a firstorder tensor, and a scalar zeroth-order tensor. Following the notations used in [6, 7], we denote tensors by calligraphic letters, matrices by uppercase boldface letters, and vectors by lowercaseboldface letters.

Figure 1. Slices of a $3rd$ -order tensor.

Mathematically, an N^{th} -order tensor is denoted as A with mode $I_1 \times I_2 \times ... \times I_N$. The *n*-th mode of A is of size I_n . We denote the index of a single element within a tensor by subscripts (e.g. $A_{i_1i_2...i_N}$). We use colon notation to denote the full range of a given index. Consider a 3rd-

order tensor, specifying a single index yields a slice. $A_{i,:,:}$ yields the *i* th horizontal slice, $A_{i,j,:}$ the *j* th lateral slice, and $A_{n,k}$ the *k* th frontal slice (Fig. 1). Similarly, specifying two indices in a 3rd –order tensor yields a vector. $A_{i,j,k}$ yields a column vector, $A_{i,j,k}$ yields a row vector, and $A_{i,j,k}$ yields a tube vector.

2. Matricization of a tensor

Matricization of a tensor refers to the process of rearranging the elements of a tensor to form a matrix. It is also known as an "unfolding" operation and is defined to a specific mode. For order three, the matricization process is visualized in Fig. 2. In general, we have the following definition.

Let A be a tensor of mode $I_1 \times I_2 \times ... \times I_N$, and suppose we wish to rearrange it to be a matrix $\mathbf{A}_{(n)}$ with the size of $I_n \times (I_{n+1}I_{n+1} \dots I_N I_1 I_2 \dots I_{n-1})$. $\mathbf{A}_{(n)}$ contains the element $a_{i_1 i_2 \dots i_N}$ of A at the position with row number i_n and column number equal to

Figure 2. Matricizing a $3rd$ -order tensor.

3. Norm and inner product of a tensor

Let A and B be two tensors of mode $I_1 \times I_2 \times ... \times I_N$, the inner product $\langle A, B \rangle$ is given by

$$
\langle A, B \rangle \stackrel{\text{def}}{=} \sum_{i_1} \sum_{i_2} \cdots \sum_{i_N} A_{i_1 i_2 \ldots i_N} B_{i_1 i_2 \ldots i_N}^*
$$

in which * denotes the complex conjugate.

Using the definition of inner product, the Frobenius-norm of a tensor A is given by

$$
\|\mathbf{A}\|_{F} \stackrel{\text{def}}{=} \sqrt{\langle \mathbf{A}, \mathbf{A} \rangle} = \sqrt{\sum_{i_1} \sum_{i_2} \cdots \sum_{i_N} \mathbf{A}_{i_1 i_2 \ldots i_N}^2}
$$

4. Tensor multiplication: the *n***-mode product**

The *n*-mode product defines multiplication of a tensor by a matrix in mode *n*. Before we introduce the definition of *n*-mode product, let us first take a look at the matrix product $\mathbf{F} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^T$, where **F** is $J_1 \times J_2$, Σ is $I_1 \times I_2$, **U** is $J_1 \times I_1$, and **V** is $J_2 \times I_2$. In this matrix multiplication, the columns of Σ is multiplied by U and the rows of Σ is multiplied by V. Let us denote column multiplication by \times_1 and row multiplication by \times_2 , $\mathbf{F} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^T$ can be rewritten as $\mathbf{F} = \sum x_1 \mathbf{U} x_2 \mathbf{V}$. In general, we have the following definition.

Let A be a tensor of $I_1 \times I_2 \times ... \times I_N$ and U a matrix of $J_n \times I_n$, the *n*-mode product $A \times n$ U is a $(I_1 \times I_2 \times ... \times I_{n-1} \times J_n \times I_{n+1} \times ... \times I_N)$ -tensor given by

$$
\left(\mathbf{A} \times_{n} \mathbf{U}\right)_{i_{1}i_{2}...i_{n-1}j_{n}i_{n+1}...i_{N}} \stackrel{\text{def}}{=} \sum_{i_{n}} \mathbf{A}_{i_{1}i_{2}...i_{n-1}i_{n}i_{n+1}...i_{N}} \mathbf{U}_{j_{n}i_{n}}.
$$

The *n*-mode product satisfies the following properties.

- a. Let A be a tensor of $I_1 \times I_2 \times ... \times I_N$, U a matrix of $J_m \times I_m$, and V a matrix of $J_n \times I_n$, $A \times_{m} \mathbf{U} \times_{n} \mathbf{V} = (A \times_{m} \mathbf{U}) \times_{n} \mathbf{V} = (A \times_{n} \mathbf{V}) \times_{m} \mathbf{U}$
- b. Let A be a tensor of $I_1 \times I_2 \times ... \times I_N$, U a matrix of $J_n \times I_n$, and V a matrix of $K_n \times J_n$, $(A \times_n \mathbf{U}) \times_n \mathbf{V} = A \times_n (\mathbf{V} \cdot \mathbf{U})$

5. Tensor decomposition

A matrix A of size $I_1 \times I_2$ is a two-mode mathematical object that has two associated vector spaces, a row space and a column space. Singular value decomposition (SVD) orthogonalizes these two spaces and decomposes the matrix as $A = U_1 \Sigma U_2^T$, the product of an orthogonal column-space represented by the left matrix \mathbf{U}_1 of the size $I_1 \times J_1$, a singular value matrix Σ of the size $J_1 \times J_2$, and an orthogonal row space represented by the right matrix U_2 of the size $I_2 \times J_2$. In terms of the mode-n product defined before, this matrix product can be rewritten as $\mathbf{A} = \sum \times_{1} \mathbf{U}_{1} \times_{2} \mathbf{U}_{2}$.

By extension, an order $N > 2$ tensor A is an *N*-mode matrix comprising *N* spaces. "*N*-mode" SVD is an extension of SVD that orthogonalizes these *N* spaces as follows

$$
A = S \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \dots \times_N \mathbf{U}_N
$$

as illustrated in Figure 3 for the case $N = 3$. Tensor S, known as the core tensor, governs the interaction between the mode matrices U_n , for $n = 1, ..., N$.

Figure 3. Decomposition of a $3rd$ –order tensor.

Supplementary References

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