JOURNAL OF MICROBIOLOGY & BIOLOGY EDUCATION, March 2016, p. 137-142
DOI: http://dx.doi.org/10.1128/jmbe.v17i1.1066

JMBE Journal of Microbiology & Biology Education

Supplemental Materials

for

Equations of the End: Teaching Mathematical Modeling Using the Zombie Apocalypse

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Table of Contents

(Total pages 7)

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Appendix 1: Model descriptions.

Model Descriptions:

The basic Susceptible-Zombie-Removed (SZR) model shown in Figure 1A is a direct adaptation of the classic Susceptible-Infected-Removed/Recovered model used in mathematical epidemiology. The behavior of the model is governed by the following system of equations, which are presented in their differential form, but which may be readily adapted to more approachable difference equations.

$$\frac{dS}{dt} = -\beta\frac{SZ}{N}$$
$$\frac{dZ}{dt} = \beta\frac{SZ}{N} - \gamma Z \quad \text{(Supplemental Equation 1)}$$
$$\frac{dR}{dt} = \gamma Z$$

In this model susceptible (S) individuals mix with and are infected by zombies (Z) at a rate β, which in turn decay and are removed at a rate γ. At any value of $\frac{\beta}{\gamma} > 1$, an epidemic of zombieism is possible. This value, generally, is known as the basic reproductive number (R_0), the average number of infections caused by a single infected zombie in an entirely susceptible population. With more complex models, the formula for R_0 is correspondingly more complex. The methods for determining R_0 well documented elsewhere. The addition of a latent period, α, in-between initial exposure is one common extension of the basis SIR model, and in the zombie epidemic framework, is described in equation form below.

$$\frac{dS}{dt} = -\beta\frac{SZ}{N}$$
$$\frac{dE}{dt} = \beta\frac{SZ}{N} - \alpha E \quad \text{(Supplemental Equation 2)}$$
$$\frac{dZ}{dt} = \alpha E - \gamma Z$$
$$\frac{dR}{dt} = \gamma Z$$

Finally, the model used in the *White Zed* software is a somewhat more complex extension of this basic model form, in that it allows for immunity via vaccination, and the removal of zombies due to interaction with susceptible individuals as well as the transmission of disease, representing those individuals fighting or otherwise incapacitating the zombies, removing their ability to transmit. Note that this model

is distinct from that depicted in Figure 1B, though their overall intentions are similar. The equations for the *White Zed* model are:

$$\frac{dS}{dt} = -\beta\frac{SZ}{N} - \sigma S$$
$$\frac{dV}{dt} = \sigma S$$
$$\frac{dE}{dt} = \beta\frac{SZ}{N} - \alpha E \qquad \text{(Supplemental Equation 3)}$$
$$\frac{dZ}{dt} = \alpha E - \gamma Z - \delta\frac{SZ}{N}$$
$$\frac{dR}{dt} = \gamma Z + \delta\frac{SZ}{N}$$

By setting α, σ, or δ to zero, the latent period, vaccination, or zombie-survivor interaction that results in a removed zombie can be effectively removed, and by setting all three to zero, the model becomes equivalent to the SZR model.

Appendix 2: Example worksheet.

An Introduction to Infectious Disease Dynamics

Section 1: Basic SIR Model

Launch the *White Zed* (http://cartwrig.ht/apps/whitezed/) website on your machine, and ensure that it is working – as you change any of the sliders presented, the graph at the top of the page should dynamically update. For the beginning of this tutorial, you may want to click *Help* at the bottom of the graph, in order to turn on text descriptions of the controls.

Set everything in the *Infection Parameters* section to 0, save for β, which is set by default to 1. Adjust the *Initial Population Makeup* section to have 10,000 Susceptible, and 1 Zombie – the first infected individual to appear in a small, isolated town. Observe this outbreak – how long does it take for the population to begin dropping dramatically? Are there any survivors in this scenario? What does increasing or decreasing the value for β do? How about the initial number of zombies? Explore possible values for β, while clicking on the + or – symbol beneath the graph to change the scale. Is there any way to prevent the town from being overrun in this scenario? Why, or why not?

The model you have been working with is known as an "SI" model, as it has compartments for only Susceptible and Infected individuals (in this case, zombies). Now let's add a new category class, Removed (R) to represent the natural decay of zombies in the environment, as they are trapped in ditches and dead ends, attacked by predators, or simply fall apart and no longer present a threat. Do this by setting

the γ parameter in the model to something other than zero – start with 0.1. Remember that γ is a rate, and is the inverse of number of days a zombie remains active. Thus, if a zombie lasts for 10 days, $\gamma = 10^{-1} = 0.1$. Higher values of gamma indicate a shorter duration. What does this do to the shape of the outbreak? Are there any survivors?

Section 2. The Basic Reproduction Number

As we have seen in the last section, it is difficult to save our hypothetical town from being overrun with zombies. But it is not impossible to do so. The first means by which an outbreak may be prevented is if the disease itself is less transmissible, or infected individuals remain capable of infecting for a shorter period of time. Change β to 0.25 and keep γ at 0.1. Notice that there are survivors in this scenario, represented by people who are still susceptible after the last zombie has decayed. The lower β, and thus the less transmissible the disease, the larger number of survivors they are. Similarly, try setting β back to 1, and raising γ – what does this due to the epidemic? Can you suggest a setting where γ might be considerably higher than it is in our first scenario? What does this suggest about the dynamics of a disease in different settings, even when the basic biology of the pathogen remains the same?

The average number of infections caused by a single infected zombie in an entirely susceptible population is known as the basic reproduction number (R_0). In order for a disease to cause an epidemic in the population, the value of this number must be above 1. The formula for R_0 will vary depending on what type of model is

being used, but in the case of the SIR model, $R_0 = \frac{\beta}{\gamma}$. Relate this to the observations you made about the behavior of the model previously.

Section 3. Preventing Outbreaks

While changes in β and γ may alter the severity of an outbreak, in many ways these are fundamental properties of the disease, as well as the location and population it is spreading in. What if we wish to take more active steps to prevent the outbreak? Suppose someone has developed a vaccine against the zombie infection. There are two ways to model vaccination – first, as a pre-existing population of people immune to infection. Set β to 1, and γ to 0.25. Try changing the *Initial Population Makeup* sliders so that there are 7500 Susceptible individuals, and 2500 Vaccinated individuals. What does this do to the epidemic? Does it prevent it completely? Is there a ratio of Susceptible to Vaccinated individuals that does?

This ratio of Susceptible to Immune individuals that prevents the establishment of a disease in the population is known as a critical fraction, and for the SIR model we have been, the percentage of the population that has to be immune equal to $1 - \frac{1}{R_0}$. In this case, as R_0 is equal to 4, that means 75% of the population must be vaccinated in order to prevent an outbreak. Consider our initial case, where γ = 0.10. Now that critical threshold is 90% of the population vaccinated. Look up the basic reproduction number for Measles, Pertussis or Rubella. Why might groups advocating against vaccinating children be considered a major threat to public health?

In some cases however, such as when vaccine-induced immunity does not last very long, or with newly emerging infections, vaccination cannot take place in advance, but is instead introduced in response to an outbreak. Return the population to 10,000 Susceptible individuals. Begin increasing the σ parameter, the rate at which people are vaccinated. As with γ, this is the inverse of the average number of days before someone is vaccinated. This, a value of 0.05 implies a 20 day wait, while 0.10 implies a 10 day wait. Alter these values – what does this do to the epidemic? Are there values where the epidemic can be halted entirely? How might the planning and concerns about how to administer a vaccination program differ based on whether or not the vaccine can be deployed ahead of time?