Erratum: Corepressive interaction and clustering of degrade-and-fire oscillators [Phys. Rev. E 84, 051916 (2011)]

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Lemma 2 in the paper claims the existence of a norm $\|\cdot\|$ in $\mathbb{R}^{K-1}$ for which the map $R^{*,K_{\rm per}}_{\{n_k\}} = R^{*}_{\{n_{k+K_{\rm per}}\}} \circ \cdots \circ R^{*}_{\{n_{k+1}\}} \circ R^{*}_{\{n_k\}}$ is a contraction. (Recall that $R^{*,K_{\rm per}}_{\{n_k\}}$ is the return map to the set $\mathcal{T}_{\{n_k\}}$ in the absence of clustering and $K_{\rm per}$ is the period of the cluster size distribution $\{n_k\}$). As it is formulated, the proof of this statement is incorrect because it uses an ill-defined quantity, namely, $|x^{s_\delta}|^{1/s_\delta}$, which does not make sense in our context. Besides, the proof is also incomplete because it implicitly assumes that either both x and y simultaneously satisfy the condition (A1) or they both do not. However, it may happen that x satisfies this condition and y does not (or vice versa.) So, these cases would need to be addressed too.

At this stage, we do not know whether Lemma 2 holds. However, one can prove the following weaker statement [1], which holds for every norm $\|\cdot\|$ in $\mathbb{R}^{K-1}$.

Lemma. [1] For every $\{n_k\}_{k=1}^K$ and $0 < \epsilon_\eta \leqslant 1$, there exist $C > 0$ and $\rho \in [0,1)$ such that for every $x, y \in \mathcal{T}_{\{n_k\}}$ so that $(R^{*,K_{\rm per}}_{\{n_k\}})^t x, (R^{*,K_{\rm per}}_{\{n_k\}})^t y \in \mathcal{T}_{\{n_k\}}$ for all integers $t \geqslant 1$, we have

$$\left\| \left(R^{*,K_{\rm per}}_{\{n_k\}}\right)^t x - \left(R^{*,K_{\rm per}}_{\{n_k\}}\right)^t y \right\| \leqslant C\rho^t \|x - y\|, \quad \forall t \geqslant 1.$$

Although this statement is weaker than the previously claimed Lemma 2, it suffices to support the phenomenological results of the paper; viz., the behavior of every orbit must be asymptotically periodic and the asymptotic cluster distributions are determined by the associated periodic orbits.

Actually, because of the discontinuous character of the dynamics, the claim that every trajectory must asymptotically approach a periodic orbit is not always true. It is true for all $\epsilon_\eta \in (0,1]$, except finitely many values for each N. The exceptional parameter values where it does not hold correspond to periodic orbit bifurcation points $\zeta(\{n_k\})$.

When $\epsilon_\eta = \zeta(\{n_k\})$, the periodic orbit in $\mathcal{T}_{\{n_k\}}$ does not exist. Indeed, in this case, the corresponding periodic point lies in the boundary of $\mathcal{T}_{\{n_k\}}$, not in the interior (see Lemma 3 in the paper and its proof), and the return dynamics sends this point to another set $\mathcal{T}_{\{n'_k\}}$ where the distribution $\{n'_k\}$ has fewer clusters. Yet, there are initial conditions $x \in \mathcal{T}_{\{n_k\}}$ for which $(R^{*,K_{\rm per}}_{\{n_k\}})^t x \in \mathcal{T}_{\{n_k\}}$ for all $t \geqslant 1$ (and which must approach the periodic point on the boundary of $\mathcal{T}_{\{n_k\}}$).

In the opposite case, for any $\epsilon_\eta \neq \zeta(\{n_k\})$, either the orbit stays forever in $\mathcal{T}_{\{n_k\}}$ and approaches the periodic orbit [case $\epsilon_\eta < \zeta(\{n_k\})$] or two clusters must eventually merge and then the orbit leaves $\mathcal{T}_{\{n_k\}}$ to enter another set $\mathcal{T}_{\{n'_k\}}$. The latter case must occur for the orbit of every initial condition in $\mathcal{T}_{\{n_k\}}$ when $\epsilon_\eta > \zeta(\{n_k\})$.

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[1] A. Blumenthal and B. Fernandez, Population dynamics of globally coupled degrade-and-fire oscillators, available online at http://hal.archives-ouvertes.fr/hal-00986128.

