Definitions

The stiffness of a material can be defined mathematically by various moduli (elastic, shear, or bulk), each one describing the resistance to deformation of a material in response to different types of stresses or applied pressures: tensile forces, shear forces, and volumetric compressive forces, respectively. The moduli are mathematically related to each other, and with some simplifying assumptions about underlying physical properties of tissues, can be interconverted.

The moduli most relevant to shear wave elastography are the complex shear modulus and the elastic modulus (Young modulus), described here. The complex shear modulus, G^*, describes the overall resistance of a material to an applied shear stress. It is a complex function of frequency, $G^*(\omega) = G'(\omega) + iG''(\omega)$, where ω is the shear wave frequency and i is the imaginary component of the complex expression. As shown by the formula, the complex modulus has two components: an elastic component called the storage or shear elastic modulus, G', which is the real part of the complex shear modulus, and a viscous component called the loss or viscous modulus, G'', which is the imaginary part of the complex shear modulus. Conceptually, the elastic component represents springlike energy-preserving behavior (e.g., a firm bouncy rubber ball), whereas the viscous component represents dashpot-like energy-absorbing and -damping behavior (e.g., a ball made of modeling clay). Thus, a tissue has shear stiffness due to both elastic behavior and viscous behavior. As discussed later, one area of active research is to better understand how each contribution relates to specific pathophysiologic conditions. The magnitude of the complex shear modulus, G, which is equal to , is emerging as the standard parameter reported in current clinical implementations of MR elastography. It is commonly described in the literature as "shear stiffness."

The effective shear modulus (μ) is a parameter reported historically in the MR elastography literature. This parameter is related to the shear-wave speed (assuming linear elasticity behavior, tissue isotropy and incompressibility) by the following equation: $\mu = \rho c^2$, where μ is the effective shear modulus, ρ is the density of the tissue (assumed to be 1 g/cm^3), and c is the shear-wave speed. The shear-wave speed is a measure of the propagation velocity of a shear wave in tissue, and the shear-wave attenuation describes its dissipation (loss of energy or amplitude). The shear-wave speed and attenuation are mathematically related to the shear elastic modulus and shear loss modulus; in general, shear-wave speed increases as the elastic modulus increases, and shear-wave attenuation increases as the loss modulus increases. Not to be confused with the storage or shear elastic modulus is the elastic or Young modulus, E, which indicates the stiffness when a compression force (as opposed to a shear force) is applied perpendicular to the surface. Under certain assumptions, the Young modulus is approximately three times the storage modulus.

Frequency Dependence and Rheologic Models

Another key concept is that all the above terms are frequency dependent, meaning that the measured values of a tissue depend on the frequency of shear waves at the location of the measurements. To convert these frequency-dependent terms to frequency-independent terms (e.g., elasticity and viscosity), it is necessary to collect data at multiple frequencies (typically three or more) and then fit the observed data to one of several possible so-called rheological models describing the tissue (e.g., Maxwell model or Kelvin-Voigt model). These models make assumptions about the underlying structure of tissue, which allows them to derive frequency-independent terms from the frequency-dependent terms. A full discussion of these rheological models is beyond the scope of the current review, because the frequency-independent terms are not yet reported in clinical applications.

Parameters and Units Reported

Depending on what the U.S. Food and Drug Administration has required, elastographic imaging techniques usually report only a subset of the variables that can be measured. Some report shear wave speed (in meters per second), whereas others report the magnitude of the complex shear modulus (in kilopascals), or the Young modulus (in kilopascals).

TABLE S1: Parameters, Abbreviations, Units, Conversion Rules, and Concepts in Elastography

Parameters	Abbreviation	Units	Conversions	Concepts
Shear-wave speed	c	m/s	$c = \lambda \cdot f$, where λ is the wavelength and f is the frequency of the shear wave	The shear wave speed indicates the propagation velocity of shear waves in a medium.
Complex shear modulus	G^*	Pa	$G^* = G'(\omega) + iG''(\omega)$, where G' is the storage modulus, G'' is the loss modulus, ω is the angular frequency and i the complex number	The complex shear modulus represents the solution of the inverse wave field propagation problem. It has a real (G') and an imaginary component (G''), which are both a function of frequency. It takes into account both the elasticity and viscosity behavior.
Storage or shear modulus	G'	Pa	$G'(\omega) = \rho\omega^2 \frac{k'^2 - \alpha^2}{(k'^2 + \alpha^2)^2}$, where ρ is the mass density, ω is the angular frequency, and k' is the real part of the wave number (defined ω/c) and α is the imaginary part of the wave number (also called "attenuation"). When α = 0, as is commonly assumed in ultrasound elastography, then this equation can be simplified to $G' = \rho c^2$, where c is the shear-wave speed.	The storage (or shear) modulus represents the real part of the complex shear modulus. It indicates elasticity (springlike behavior)—that is, the ability of a medium to resist shear deformation without energy loss.
Loss or viscous modulus	G''	Pa	$G''(\omega) = -2\rho\omega^2 \frac{k'\alpha}{(k'^2 + \alpha^2)^2}$, where ρ is the mass density, ω is the angular frequency, k' is the real part of the wave number (defined ω/c) and α is the imaginary part of the wave number (also called "attenuation").	The loss (or viscous) modulus represents the imaginary part of the complex shear modulus. It indicates viscosity (dashpot-like behavior)—that is, resistance to movement or deformation.
Magnitude of the complex shear modulus	G	Pa	$G = \sqrt{(G')^2 + (G'')^2}$, where G' is the storage modulus and G'' is the loss modulus.	The magnitude of the complex shear modulus. It indicates stiffness.
Effective shear modulus	μ	Pa	$\mu = \rho c^2$, where ρ is the mass density and c is the shear wave speed.	The effective shear modulus is calculated at a particular frequency.
Elastic (Young) modulus	E	Pa	$E = 2(1 + \nu) \cdot G'$, where ν (also known as Poisson ratio) is 0.5 for soft tissue, hence: $E = 3G'$. When a tissue is assumed to be purely elastic, the equation can be rewritten as: $E = 3\mu = 3\rho c^2$, where μ is the effective shear modulus calculated at a particular frequency, ρ is the mass density and c is the shear-wave speed.	The elastic modulus, known as Young modulus. It indicates elasticity (spring-like behavior)—that is, the ability of a medium to resist normal (perpendicular) deformation.